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(SHALER MEMORIAL SERIES.)

THE GEOLOGY OF ASCENSION ISLAND.

By Reginald A. Daly.

With Twenty-one Plates.

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Introduction; Acknowledgments.

THE islands of the deep oceans have geological importance out of all proportion to their individual areas or their total area, when compared with equal areas of the continents. Small as it is, a deep-sea island tells us practically all we shall ever directly ascertain concerning the nature of the solid suboceanic material throughout a much more extensive region. Thus, from a few thousand relatively minute patches of dry land geologists must glean most of their detailed data for about one half of the earth. Investigations in geodesy, seismology, the earth's magnetism, paleogeography, isostasy, diastrophism, and petrology cannot be completed until the information from the deep-sea islands have been brought to book.

All of these islands seem to be volcanic. Their rocks indicate the composition of subcrustal or intercrustal magmas. Since the island cones have been built up from the bottom of the sea, 3000 to 6000 meters below the surface, the petrologist and volcanologist have here the advantage of being able to study the products of each truly enormous volcano at a late stage in its growth. None of the purely volcanic cones on the continents is comparable in height or volume with any one of many deep-sea volcanoes. When the explorer lands on an oceanic island, he begins his study at a contour line between 3000 and 6000 meters above the base of the pile, without the discomforts and expense which usually accompany exploration at similar altitudes on the lands. The study of the late stages in the growth of great volcanoes is, then, specially favored among the islands of the deep sea.

Moreover, in a few cases, oceanic eruptions have brought to view fragments of the solid crust on which the respective cones have been built. In this way precious information regarding the evolution of the ocean-basins, paleogeography in general, and other problems of the first rank has been secured. If for no other reason, an intensive study of every oceanic island is amply warranted. Since the geological mapping of this half of the earth demands much less work and expense than the other half, there is no good reason why the detailed mapping of the deep-sea islands should not be completed within a few decades. This systematic and comprehensive work is not likely to be undertaken by governments. It demands private enterprise, and, ideally, private endowment on a scale so large that the study shall be continuously pursued by experts consecrated to the working out of a great synthesis. Meantime the less satisfactory process of getting information by individual studies pursued during independent expeditions is slowly but surely proving the significance of the oceanic islands.

Charles Darwin, Renard, Melliss, Prior, Reinisch, and others have illustrated the point from Ascension and Saint Helena Islands. No adequate geological map of either island has, however, been published. The opportunity of making geological reconnaissance maps of both was accorded to the writer during the year 1921. This paper summarizes his observations on Ascension; a second paper, on Saint Helena, will, it is hoped, be printed in these Proceedings after the lapse of a few months.

The geological part of the map of Ascension (Plate I) is the product of twenty-seven days of field-work (October 30 to November 25). Much more work could, of course, have been profitably devoted to the work, but steamer sailings and other considerations prevented the more thorough study.

The travelling and field expenses were defrayed from the Shaler Memorial Fund of Harvard University. They were low because the writer was the guest of Major C. A. Tennyson, Commandant of the island, which had long been under the control of the British Admiralty and noted on their books as a battleship. In 1922 the administration of Ascension was transferred to the Governor of Saint Helena, acting for the Colonial Office. Owing to the unlimited kindness of Major Tennyson and his staff of marines, the field-work was greatly facilitated. To him and to Rear Admiral F. C. Learmonth, Hydrographer to the Royal Navy, who donated charts of the island, the writer's thanks are due. As noted below, Dr. J. S. Flett, Director of the Geological Survey of Great Britain, generously presented a number of thin sections of Ascension rocks. The writer is specially grateful to Dr. H. S. Washington, Mr. E. G. Radley, and Miss Helen E. Vassar for the care which they have taken with the seven new chemical analyses of rocks.

The base-map used was the official chart, the quality of which is specially good. Its scale is about 1: 36,500. It shows relief by shading, supplemented by numerous indications of local heights, which were controlled by triangulation. Adding barometric data, material was secured for the drawing of a sketch-contour map, Plate I. Rough and imperfect as it is, this map gives an idea of the general relief. It bears a representative selection from the many soundings shown on the chart.

Previous Work.

In 1501, on Ascension Day, the island was discovered by de Nova, the Portuguese navigator. For more than three centuries it received little notice. In 1815, when Napoleon was sent to Saint Helena, the British Government annexed Ascension, and under naval control the first settlement, Georgetown, was founded. The Eastern Telegraph-Cable Company has here long had one of its most important stations, relaying transatlantic messages.

During 1819 the island was mapped by Lieut. R. Campbell, and in 1838 it was again mapped, and the surrounding waters sounded, by Lieut. G. A. Bedford of H. M. S. *Raven*. This second edition has been superseded by that made (1898–1901) by Capt. E. Y. Daniel, the chart now sold by the Admiralty and copied by the United States Hydrographic Office (chart No. 538; price, 40 cents).

As early as 1804 Bory de Vincent (Voyage dans les Principales Iles des Mers d'Afrique, 1801–2, 3 vols., Paris, 1804) published gossipy notes on Ascension and Saint Helena; these have now only historical interest. Of somewhat similar quality is the account given by R. P. Lesson, on pages 489–506 of the first volume on Zoölogy, resulting from the "Voyage autour du monde sur la corvette *La Coquille*, 1822–5" (Paris, 1826). The "Narrative of a Voyage to the Southern Atlantic Ocean in the years 1828–30, performed in H. M. Sloop, *Chanticleer*" (2 vols., London, 1834), by the naval surgeon, W. H. B. Webster, contains a little known but valuable description of the geology and natural history of Ascension. Webster anticipated Darwin in many important discoveries relating to the geology.

The classic pages of Darwin's "Geological Observations on the Volcanic Islands, etc." (pp. 40–82, 2d ed., London, 1876) give the best account of the island. The first edition (1844) was illustrated with Campbell's map; later editions with that of Bedford. Darwin's Journal of Researches (Voyage of H. M. S. *Beagle*, 1831–6) contains a few pages devoted to Ascension.

The *Challenger* Expedition (1873–6) resulted in several publications dealing with the island:

Narrative of the Cruise, by C. Wyville Thomson, vol. 1, part 2, pp. 927–9, 944, London, 1885;

Summary Report, by J. Murray, pp. 1237–41, London, 1895;

Report on the Petrology of Oceanic Islands (in vol. 2 of Report on Physics and Chemistry), by A. Renard, pp. 39–74, London, 1889;

The Atlantic, by C. W. Thomson, vol. 2, pp. 221–229, New York, 1878;

Notes by a Naturalist, by H. N. Moseley, pp. 487–489, new ed., London, 1892.

An entertaining, popular description of the island is given in Mrs. David Gill's "Six Months in Ascension" (London, 1878). Brief accounts are to be found in the Africa Pilot, Part 2, 5th ed., London, 1901; in the Encyclopaedia Britannica, 11th ed., article "Ascension," 1910; and in volume 13 (L'Afrique meridionale) of E. Réclus' Nouvelle Géographie Universelle (pp. 24–30, Paris, 1888).

The petrography of Ascension has been discussed by Darwin and Renard in the books already cited. G. T. Prior (Mineralogical Magazine, **13**, 257, 1903) gives additional notes. Another modern, but much more comprehensive, statement of the petrography has been published by R. Reinisch, who studied the specimens collected by members of the Deutsche Südpolar Expedition of 1901–3, volume on "Geologie und Geographie," Teil 2 (pp. 646–654, Berlin, 1912); a number of analyses are given.

In the April number of the Geological Magazine, 1922, the writer published a preliminary report on his exploration of the island.

Finally, note may here be taken of Faye's measurement of the force of gravity at Ascension, the result of which is given in the Comptes Rendus of the French Academy of Sciences (vol. 102, p. 651, 1886). As at Saint Helena, Fernando Noronha and Saint Thomas, he found gravity to be greatly in excess of its theoretical value.

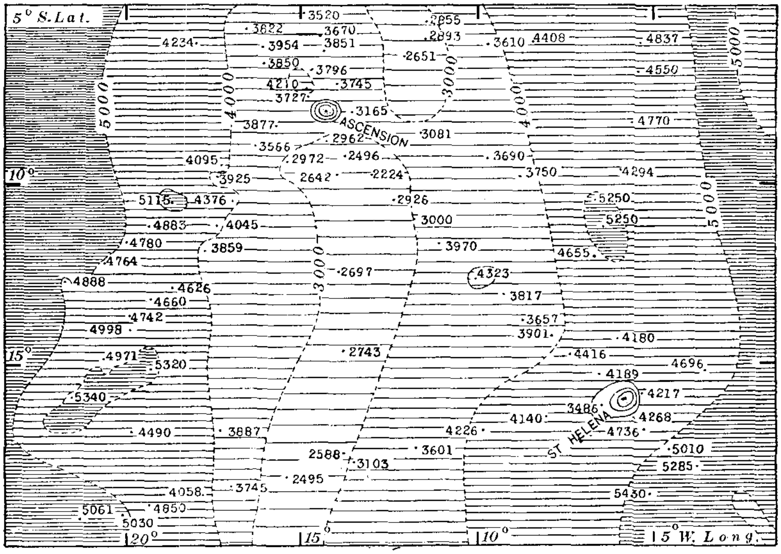
Physiography.

Ascension Island lies just north of the parallel of eight degrees, south latitude, and centers on the meridian of fourteen degrees, twenty-four minutes, west longitude. In plan it approximates to an equilateral triangle, measuring 12 kilometers by 9.5 kilometers. The circumference is about 35 kilometers; the area, about 97 square kilometers.

The base of the island-cone is near the top of the mid-Atlantic swell at the depth of nearly 3,000 meters below the surface of the sea (Figure 1). To east and west of the swell the Atlantic bottom has depths greater than 5600 meters. The Saint Helena cone, situated near the eastern foot of the swell, 1250 kilometers southeast of Ascension, has its base at the depth of about 4200 meters.

On a manuscript chart, kindly supplied by the Eastern Telegraph

Company, Ltd., of London, indicated soundings show the existence of a broad, dome-shaped, relatively shallow area of the bottom. The minimum depth found in this area is 830 fathoms, or 1518 meters, at a point bearing about N. 30° W. from Georgetown and distant 16.5 kilometers therefrom. Three kilometers to the northwest of that point is another sounding, of 860 fathoms or 1573 meters. Between the shallowest point and the shore of Ascension, three soundings, at 1100, 1018, and 1178 fathoms, seem to prove that the dome-shaped area rises from the general surface of the mid-Atlantic swell, quite independently of the Ascension cone; another volcanic center may be indicated.


5° S. Lat.
10°
15°
20°
15
10°
15 W. Long.
ASCENSION
ST HELENA
5000
4000
3000
3520
3622
3670
3954
3851
3850
3796
4210
3745
3727
3165
3877
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3690
3750
4294
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5115
4376
4883
4045
4780
3859
4764
4888
4626
4660
4742
4998
4971
5320
5340
4490
3887
2926
3000
3970
2697
4323
3817
3657
3901
2743
5250
5250
4655
4180
4416
4189
4696
4217
3486
4268
4140
4736
4226
3601
2588
3103
2495
3745
4058
4850
5061
5030
5010
5285
5430

FIGURE 1. Position of Ascension and Saint Helena Islands in relation to the mid-Atlantic swell. Depths in meters.

To the westward of Ascension the soundings are sufficiently numerous to give an idea of the submarine slope on that side. From the shore at Georgetown to the apparent base of the cone to the northwestward, a distance of about 13 kilometers, the average slope is close to 10°. From sea-level at Georgetown to the top of the Peak the average slope is 6° 40′. From the summit of the Peak to the base of the cone to the northwestward the slope is about 8° 30′. Off Southeast Head the submarine slope is much steeper; a sounding at 39 fathoms or 71 meters is spaced only 1100 meters from a sounding at 400 fathoms or 731 meters, giving a slope of 31°. Probably, however, the mean slope of the cone as a whole is not very different from that of a typical basaltic volcano rising from the floor of the ocean.[1]

As usual the horizontal distance between the shore-line and the 50-fathom isobath is considerably greater than that between the 50-fathom and 100-fathom isobaths. The break-of-slope at the outer edge of the coastal shelf is located close to the 50-fathom line. In illustration the slopes off Georgetown may be noted:

0–50	fathoms	1:40
50–100	"	1:10
100–200	"	1:6
200–500	"	1:3
500–700	"	1:5

The island lies in the trade-wind belt. The mean temperature of the air at the shore (Georgetown) is 85° F. and at the elevation of 2000 feet (610 meters) on Green Mountain, about 75° F. In spite of these temperatures the field-worker is kept comfortable by the strong, persistent wind. In fact Ascension is widely known in naval circles for its health-giving climate. The conditions are arid to semi-arid. The mean annual rainfall is not far from 20 inches (50 centimeters), but most of the rain falls during the autumn months of March and April, often in the form of "cloud-bursts." The porous nature of the rocks prevents any significant storage of water except by the use of catchments surfaced with cement, at high cost of labor and material. These catchments, totalling several acres in extent, are all situated near the top of the highest point, the summit of Green Mountain, some eight kilometers from Georgetown. At the very top is a large dew-pond, kept filled by the drip from planted bamboos; the precipitation of water from the humid air is rapid enough to preserve a depth of a meter or less throughout the dry season. The water piped from Green Mountain has proved insufficient for the inhabitants of Georgetown, who have long relied on the distillation of sea water for a large part of their supply.

The famous "rollers," breaking on the shores of Ascension and

---

[1] See O. Krümmel, Handbuch der Ozeanographie, Stuttgart, **1**, 97, 1907; F. Dietrich, Untersuchungen über die Böschungsverhältnisse der Sockel oceanischer Inseln (Inaug. Diss., Greifswald, 1892); G. W. Littlehales, Paper 95, United States Hydrographic Office, Washington, 1890.

Saint Helena, are explained in the Hydrographic manuals as due to swells set going by storms in the North Atlantic; accumulating evidence tends to confirm this theory.

Thin deposits of phosphate, on the basaltic flows, occur at many places. These will be described by Professor E. S. Larsen and G. Richards in a special paper.

Ascension is entirely volcanic, except for some small patches of beach material thrown up by storm waves. From the time of its discovery no sign of volcanic activity, not even a hot spring, has been reported, though geologically the island is extremely young. The chart shows the sites of forty-four vents, including a distinct vent probably represented in Boatswain Bird islet, near the eastern end of the main island. Adding other, smaller vents, the total number is at least sixty.

The highest point is the top of The Peak (2817 feet or 858 meters above sea), which itself is the highest part of the complex called Green Mountain (Plate II, *A*). Southward from The Peak to sea-level the mean slope is three times as rapid as that on the opposite side; to eastward the mean slope is nearly twice that on the west. The asymmetry of the island's profile is further expressed by the special development of high cliffs and cliffy slopes in the eastern and southeastern parts of the island. Even before the visits of Lesson, Webster, and Darwin, the intelligent commandants of Ascension had remarked on the lack of symmetry in the individual cones, which in many cases are steeper to windward (the southeast) than in the opposite direction. The strong trade-wind is the obvious direct cause of the more rapid, initial accumulation of ash and cinder on the leeward side of each vent. The initial asymmetry has been increased by the erosion of the windward slopes and drifting of the finer material to leeward. This thinning and weakening of the structure on the windward side has caused the lava, later rising in the vent, to break through or over the rim on that side. Typical breached cones have been thus produced.

More than half of the island is surfaced with scoriaceous flows of basaltic or trachydoleritic composition; pyroclastic beds, though common, are subordinate in volume. There is no reason to doubt that the great composite cone is also chiefly basaltic at all depths below the present land-surface. Important trachytic masses are intercalated with the femic flows and pyroclastic beds or rest upon these. One may assume similar interruption of the basaltic complex by other local, subordinate bodies of trachyte, down to some depth below sea-level. Clearly, however, trachyte is not the rock on which the visible island is based. Darwin speaks of the trachyte as "fundamental" and the basalt as characteristically "overlying" the trachyte, having been erupted "at the base of the great central mass of trachyte." The true relations of the two types of lava are described below.

For the convenience of the reader the following table (Table I) has been compiled, giving a brief, preliminary description of the named cones and vents, which are classified according to their essential features. Heights have been taken from the chart or from notes of barometric readings.

TABLE I.

Basaltic Masses and Other Femic Masses.

(Masses of composition other than basaltic are specially designated.)

Exogenous lava-dome (due to local eruption without definite crater):
    Bears Back, 800 feet. (Plate IV.)

Fissure-eruption:
    On Southeast Head (several small craters opened on the fissure) — trachyandesite. (Plate VIII.)

Dominantly or wholly lava-formed cones:
- Dark Slope, 763 feet — breached. (Plate XI.)
- Driblet west of Dark Slope.
- Table Crater, 640 feet — breached.
- Lady Hill, 1181 feet.
- Slag cones 800 meters W. S. W. of Lady Hill summit.
- Three driblets, northeast foot of Sisters Peak.
- Landing Pier cone, Georgetown, 80 feet (scoria cone) — trachydolerite.
- Hayes Hill, 106 feet — trachydolerite. (Plate XXI, *A*.)
- Cat Hill, slag-cone, 280 feet.
- Twelve driblet cones, culminating at the west end of the series in Booby Hill.

Composite cones, composed of flows and pyroclastics:
- Saddle Crater, 422 feet.
- South Gannet Hill, 749 feet.
- Round Hill, 450 feet.
- Horseshoe Crater, 394 feet.
- Mountain Red Hill, 1786 feet. (Plate X, *B*.)
- Sisters Peak, 1460 feet. (Plate III, *A*.)
- 1012-foot cone N. W. of Sisters Peak.
- 1187-foot cone S. E. of Sisters Peak.
- Travellers Hill, 1174 feet.
- Thistle Hill, 1080 feet.

Butt Crater, 740 feet.
Street Crater, 787 feet.
Three unmapped craters between Street and Hollands craters.
Sisters Red Hill, 905 feet.
Hollands Crater, 654 feet. (Plate IV.)
East Crater, 743 feet. (Plate IV.)
Breached cone, north foot of Bears Back, about 500 feet.
Upper Valley Crater, 803 feet.
Crater Cliff cone, 186 feet.
Southeast Crater, 1146 feet.

Dominantly or wholly pyroclastic cones:

South Red Crater, 546 feet.
Southwest Red Hill, 731 feet.
613-foot cone N. W. of Dark Slope (has effluent flows).
Perfect Crater, 1020 feet.
The Peak, 2817 feet. (Plate II, *A*.)

TRACHYTIC MASSES.

Endogenous domes or crater-fillings, not visibly affected by axial subsidence:

Pillar Bay dome (probable).
Cocoanut Bay body, a relic, perhaps eviscerated by explosion.
Ragged Hill dome, 944 feet.
Green Mountain dome, 2490 feet; rose in a large basaltic caldera; thick overflows of trachyte; on the east affected by a second caldera explosion, The Peak cone of basaltic tuff and ash being built in this cavity.
Weather Post dome, 1990 feet; thick overflows of trachyte; affected by a major explosion at the Devils Cauldron (caldera) and probably by a still greater explosion, which formed the depression between Weather Post and the ridge running north from White Hill. (Plate XV, *A*.)
White Hill dome, 1723 feet; thick, stubby overflows of trachyte.
Little White Hill dome, 552 feet; rose in center of older explosion-crater of which much of the basaltic rim is well preserved. (Plate VII.)
Wig Hill dome, 475 feet; veneered with basaltic scoriae. (Plate IX.)
Southeast Head dome, 479 feet; probably extended by thick overflows of trachyte; fissured and flooded by a very young flow of trachyandesite. (Plates VIII and IX.)
Boatswain Bird Islet, 323 feet; probably an independent dome of monolithic trachyte. (Plates XIX and XX.)
Dome (?) at southeastern foot of Bears Back; small exposure.
Stubby flow 500 meters southeast of 1187-foot summit on Sisters Peak ridge.
"The Crags" dome (1 km. N. N. W. of Sisters Peak), about 300 feet; largely covered with younger basaltic ash and lavas.

Cross Hill dome, 868 feet; rose inside an older basaltic crater-rim; at least one outflow of trachyte; after solidification covered by basaltic, scoriaceous flows and cinders, which issued through the body of the dome. (Plates II and X, *A*.)

Endogenous domes or crater-fillings, deformed by axial subsidence:

Riding School dome, 800 feet; rose in older basaltic caldera-rim; thick, stubby overflows. (Plates X, *B* and XI.)

"Drip" dome, 900 feet, at southern foot of Sisters Peak ridge; rose in older basaltic crater or caldera; overflows of trachyte perhaps represented in outcrops of this rock east and northwest of Thistle Hill summit.

The many cones and domes, like the lava flows, have forms almost ideally constructional (Plates II–V; X, *A*; XIV, *A*; XVII). Several vents and flows have such freshness of aspect as to suggest their origination perhaps since the beginning of the Christian era; in any case the island is manifestly very young. Erosion has affected constructional forms of some of the trachytic bodies and also the older basaltic flows at Southwest Bay. The imposing sea-cliffs at the eastern side of the island, as well as those bounding Boatswain Bird islet (Plates XIX and XX), imply wave-action during many centuries. However, the cliffed trachytes and underlying tuffs are comparatively weak rocks, so that even for this part of Ascension one need not assume an age greater than a few tens of thousands of years. If the long escarpment at Southwest Bay is a true sea-cliff, that erosion must have taken several millenia, for the rocks truncated by the cliff are massive flows of strong basalt. In spite of these exceptions the physiographic development of the island, its march in the erosion cycle, has just begun. It is possible that the whole mass above sea-level has been erupted during post-Glacial time; the age of the much greater mass of the cone, below sea-level, is quite unknown.

It may be added that the writer found no definite evidence of either crustal uplift or crustal subsidence at Ascension; nor clear proof that this island was affected by the six-meter, probably eustatic, lowering of sea-level, registered at Saint Helena. The shore rocks of Ascension may be younger than that shift of sea-level; or, antedating it, the marks of the higher stand of the sea may have been drowned by later subsidence of the island. These and other unanswered questions bearing on the stability of the crust in this region invite further study.

General Structure.

In the field one cannot usually make sure distinction between basalts and trachydolerites. The latter are demonstrated at Hayes Hill, at the Landing Pier, Georgetown, and probably at the ridge a few hundred meters north of the summit of Cross Hill, but doubtless there are other occurrences. On the map (Plate I) no attempt has been made to indicate by a special design the distribution of the trachydolerites, which are mapped along with all the other types more femic than trachyte as "rocks of basaltic habit." It should be noted also that the non-stippled area on Southeast Head, Plate I, represents the flood of trachyandesitic lava there flooding the trachyte.

On Plate I the trachytes are shown by a stippled pattern except in the case of each of the smaller bodies, for which a design in solid black was chosen in order to facilitate reading of the map; this difference of design for the trachytes has no other significance.

BASALTIC CONES.

The femic masses entering into the constitution of those composite piles which also include trachyte will be considered on later pages dealing specially with the trachytic bodies. Most of the purely basaltic cones present no unusual characteristics, so that a detailed account of each does not seem warranted. Some data relating to them are given in Table I; only a few additional notes will be added.

Reference has already been made to the prevailing asymmetry of the cones and to the related development of breached cones and craters. Good examples of these are found at Hollands Crater, East Crater (Plate IV), and Table Crater. In an analogous manner effluent discharges of lava have tended to be particularly abundant on the windward sides of Cross Hill, Spoon Crater cone, and South Gannet Hill.

The breached East Crater affords an example of axial subsidence which is unique among all the cases where basaltic lava has risen in the craters of the island. From the foot of the breaching overflow its surface rises at an angle of about 20 degrees to the rim of the crater, where the chaotic surface of the same flow sinks rapidly, about 25 meters, to the center of the crater. This late subsidence of the crater-floor was probably due to withdrawal of the magma in depth, though mere thermal contraction of the freezing magmatic column may have played a part. Similar axial subsidence has taken place at the Riding School and "Drip" domes of trachyte.

Special note may also be made of the Bears Back (Plate IV). This plateau-like mass is not fully understood. It seems to be monolithic and not made up of several flows of basalt. The lateral scarps are rather steep on all sides, indicating high viscosity at the time of eruption. The whole eminence may, indeed, represent a flattened dome, formed by a single exudation of very stiff basalt. Since, however, trachyte crops out at the foot of the Bears Back, it is not possible to exclude the hypothesis that the mass is a low, broad dome of trachyte which has become veneered with a thick flow of basalt.

Small, steep-sided cones, wholly composed of driblet flows of basalt, are well represented in the island. The best examples were found in two different sets, each set aligned as if the respective vents were opened along a single fissure.

One of these groups is situated near the 550-foot contour, 600 meters north of Sisters Peak (1460-foot summit) and an equal distance W. N. W. of Sisters Red Hill. Here three driblet cones, ranging from three to twelve meters in height, are arranged on a straight line, 60 meters in length; this line passes through the top of Sisters Peak. The well-like vents are cylindrical down to the levels where they are choked with débris, but in one case at a depth of about nine meters the pipe widens into a spacious, domed chamber. Evidently during activity the vent occupied by the molten lava enlarged downward for some distance. The highest cone, in the middle, is a composite, made up of blobs of lava that issued from three adjacent pipes, of which the centers lie on the line joining the other two simple cones. The external slopes of the little cones are steep, from 30 to 60 degrees, compared with the horizontal plane.

The other set of driblets includes the main crater of Booby Hill, southwest of Green Mountain. East of it are eleven other driblets distributed along a line about 500 meters long. The Booby Hill (its top 30 meters above the surroundings) and two adjacent craters are relatively large and have been breached by strong flows of basalt. The other nine cones, more symmetrical, range from two to seven meters in height. The fissure which fed probably all twelve vents lines up with a major vent south of Spoon Crater cone, suggesting a genetic connection.

BASALTIC FLOWS.

Most of the areas covered by the young and rough lava flows of basaltic habit — locally called the clinker-fields — are shown on the naval chart by a special design. The complete mapping of all the individual flows exposed, if possible at all, would demand much more than a month of field-work. Those which could be mapped with some accuracy may be listed:

1. An extensive, fan-shaped flow from a small crater close to the most easterly of "The Crags," north of Sisters Peak. The length and width are each about three kilometers. The flow entered the sea along a wide front, from Bates Point to English Bay (Plate III, *B*). This may be called the Comfortless Cove flow.
2. A 2.5-kilometer flow from the eastern flank of Sisters Peak, which has certainly been one of the centers most recently active. This flow, cliffed by the sea-waves on both sides of Porpoise Point, east of East Crater, may be called the Porpoise Point flow.
3. A one-kilometer flow from a well-defined crater just north of the Bears Back plateau.
4. A one-kilometer flow, directed northward from a crater (not indicated on the chart), 700 meters W. S. W. of the top of Lady Hill. Near the site of the Wireless Station (now dismantled) this scarps on an older flow, probably from the same vent. These may be called the Wireless Station flows.
5. A series of flows emanating from an opening at the foot of Dark Slope cone and reaching the sea, 2 kilometers to the westward. The youngest of them cascaded over the 60-meter cliff facing Southwest Bay. These basalts are distinguished among all the greater flows of the island in being markedly porphyritic, with phenocrysts of feldspar; they may be called the Southwest Bay group of flows.
6. A broad flow from the southern flank of South Gannet cone, measuring about 1.5 kilometers in length, as well as in width. It vies with the Porpoise Point flow in being the roughest, most chaotic, on the island.

The average exposed flow of basaltic lava has much smaller volume than any of the first six flows above listed. Each individual output of lava at these great heights from the base of the Ascension pile was thus comparatively small; the behavior of the mechanism was normal for the closing stage of the life of a great volcano.

About nine-tenths of the visible flows may be described as of the chaotic type. *By fracture, shear, and rotation each rugged mass has* yielded to the complex tensions set up in the upper, more solidified shell, which slid on, or was sheared across, the less rigid, hotter, lower part. The forms developed are of two kinds. (1) The flow shows a succession of transverse, gaping fissures or trench-like depressions, separated by flat-topped, sharp ridges or by lines of spires — the whole simulating the crevasse-serac system of an alpine glacier. Or (2) the flow exhibits the even more irregular surface of typical block-lava (Plate V, *A*). In this case the blocks tend to be smaller than those of the Hawaiian flows and have dimensions more like those of the blocks in Vesuvian lava flows.

A few of the Ascension flows approach the smoother, ropy or pahoehoe type, though even these become chaotic where the underlying surface has strong irregularities. Perhaps the best illustrations of the smooth type are to be found in the Southwest Bay group of flows. These are characteristically jointed, in rough columnar fashion, to the depth of five to ten decimeters. Weathering has opened the joints somewhat, giving the appearance of a floor composed of large, closely set boulders. The very massive lava constituting the northern slope of the Riding School cone is bouldery, on a cyclopean scale. A flow from the foot of Mountain Red Hill, northwest side, is similarly bouldery, the round masses measuring three to ten meters in diameter. This flow measures 200 by 100 meters. Its initial viscosity was high, causing the constructional scarp to be steep. While differential weathering may be partly responsible for the peculiar habit of this particular flow, the cause is by no means apparent.

The basaltic flows are seldom as much as 20 meters thick; a common average thickness is 6 meters. The thickest observed body of basalt constitutes the lower, greater part of the cliff surrounding Cricket Valley (Figure 7). From the top of this monolithic mass to the ash-covered floor of the caldera is a vertical distance of 60 meters; the total thickness of the sheetlike monolith must be greater. Except at and near the top it is trappean and almost devoid of vesicles. Its mode of emplacement is not clear. In the sections of Figure 7 it is represented as a thick flow, but possibly it may be the remnant of a lava-lake in a crater, a lake which was frozen in situ and later in part destroyed by the caldera-explosion.

Some of the flows bear well individualized hillocks of two kinds; neither type is very abundant.

The one kind is the tumulus, turtle-shaped or shield-shaped, one to five meters high, ten to twenty-five meters long, and five to ten or fifteen meters wide. These tumuli are essentially like those of Hawaii and were caused by local, hydrostatic or gaseous pressures exerted by the still fluid material against the solidified, surface-shell of the flow. In some instances the uplifting pressure may have been due to steam generated by the volatilization of water which was overrun by the hot lava, but it seems clear that the liquid lava itself has also carried the pressure.[2] As usual the surfaces of the elliptical domes are fissured because of breaking tensions developed during their growth. The resulting crevasses are from one to three meters wide on the tumuli seen on the flows of the Southwest Bay group.

The other type was described by Darwin. It is best illustrated on the older of the two Wireless Station flows, on the flat east of Cross Hill. They are steep-sided cones, composed of highly scoriaceous basalt which is everywhere discrete, after the fashion of some scoriaceous agglomerates (Plate V, *B*). Their general appearance is that of a driblet-cone or hornito. In no case, however, could a central pipe be discovered.

Every part of a moving lava-flow shares the properties of ideal liquids and solids. Even the most fluent part is doubtless characterized by some elastic stress and corresponding strain. The chilled surface-phase has a much higher elastic limit, but this stiff part of the flow is also elastico-viscous. Its rigidity is higher, the decay of its rigidity slower, than in the case of the interior phase of the moving flow. The displacement of the surface-phase is therefore usually accomplished by bodily sliding over the more fluent lava beneath — a kind of landsliding. As in ordinary landsliding, the more rigid surface-shell downstream tends to be folded and broken by transverse shears. Back of the frontal scarp, itself largely caused by the forward thrust of the surface-shell, are found isolated pressure-ridges, grouped anticlines and synclines, anticlinoria, synclinoria, overthrusts, and underthrusts. Upstream the surface-shell is regularly broken by many transverse fissures, caused by the tensions set up by the displacement, sliding, on the inclined, uneven floor.

Figure 2 is a diagrammatic section of one of the flows near Comfortless Cove. It illustrates the combination of compressional and tensional features, just described. Similar features characterize basaltic flows in many other fields of lava. Their bearing on the new hypothesis of continental displacement was specially studied. A digression on this important question may be permitted.

The hypothesis of continental migration is probably best phrased in the sense of the following question: Have the cordilleras and the great basins of the Atlantic or Arctic type been developed by continental *sliding?* Is the continental crust analogous to the surface-shell of a lava flow which has moved? Is the earth-shell on which the crust rests an elastico-viscous substance, analogous to the hot, glassy, interior phase of the moving lava flow? Has the continental crust become deleveled by secular processes, so far that continental sliding became inevitable? Was the friction, static and kinetic, opposing the

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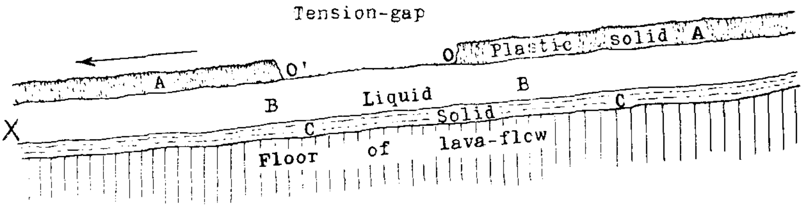
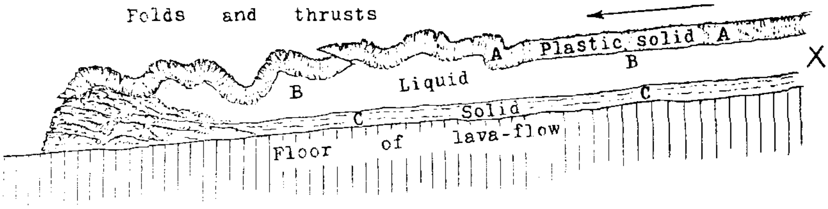
[2] Cf. R. A. Daly, Igneous Rocks and Their Origin, p. 134, New York, 1914.



Upstream: Chilled, solid layer, A, slides on hotter, viscous layer, B, which rests on chilled floor-layer, C. Under the tension layer A parts at O. Sliding causes cliffed depression O — O'



Downstream: Folds and thrusts in layer A, caused by its sliding on layer B through the distance O — O'.

FIGURE 2. Longitudinal section (diagrammatic) illustrating development of anticlines, synclines, thrusts, and tension-gap in superficial, plastic-solid phase of a basaltic flow, because of the sliding of that phase on the "liquid" lava of the interior of the flow. Arrows show direction of flowing and sliding.

The upper part of the section is continuous with the lower part at the point "X." Below the gap, O–O′, the layer B was quickly chilled to rigidity.

Section not quite in true scale; clifflet at "O" or "O′," from 3 to 5 meters high.

slow movement of continental blocks, so low that there remained energy sufficient for the folding of geosynclinal sediments downstream? One may go further and ask if the parallel between continental displacement and the flowing of physically heterogeneous lava is not a true homology rather than merely an analogy.

In any case the writer believes that the daring hypothesis of extensive continental migration in past times would be clarified if it were phrased in terms of continental sliding rather than in terms of continental flotation. A sliding theory seems much more feasible than a "drift" theory. If the continents are, and always have been, truly floating, they could have been displaced only by a dragging or pushing force other than pure gravity; an efficient force of that kind has not yet been discovered. Sliding means the operation of pure gravity, displacement due to the dead weight of the continent involved. In passing, it may be pointed out that the imagined secular deleveling, which gives potential, is not incompatible with the rule of isostasy.

Leaving these difficult but fundamental questions, suggested by the characteristics of many Ascension flows, the writer will note a detail which he has seen nowhere so well illustrated as in the Comfortless Cove flow. Not far from the cove itself one can find places where the surface of scission between the sliding "crust" of the flow and the underlying, weaker, hotter phase is now exposed to the air. The lower phase, still largely vitreous, is there grooved and striated. Its surface is roughened by serrate ridges and points, resembling sharks' teeth. These many projections point downstream and may be explained as due to elastic and viscous reactions in the glassy lava where relieved of the weight of, and elastic stresses induced by, the heavy *"crust" which had just slid away. The points of the projections* stand two to five centimeters above the general surface of the striated, glassy basalt (Figure 2*a*).

Contrasting with flows of the kind just described are a number of others in Ascension, which show a prolongation of liquid flow after the chilled surface-shell had become laterally anchored. Lava tunnels of the familiar sort were thus formed, though not in great number or of large size. In most instances their roofs have collapsed, except for short distances. One of the roofed relics, about 20 meters long, 3 meters wide, and 1–2 meters high, was found just below a small driblet cone at the western foot of the Dark Slope cone. This tunnel plunges southward at the unusually high angle of 30 degrees. Other lava tunnels, along the shore, are the loci of spouting horns which are quite spectacular during times of heavy surf.

Locally the tunnel streams of hot, fluent lava have worn pronounced channels or gutters, reaching as much as a meter in depth, in the older rocks. Such runnels seem to be represented on the brink of the cliff facing Southwest Bay, where, however, the old cliff is largely mantled with lava, frozen in the act of cascading westward over the cliff. Other runnels, three to ten meters in width, were seen along the path from Cat Hill to the cemetery at Georgetown.

The depression between Little White Hill and the Southeast Head dome is the bed of a very young flow. This flow had attained a considerable depth in the depression when the supply at the vent was stopped. A chilled surface-shell was formed and anchored to the sides of the valley. Then the still fluid lava beneath continued to flow in the direction of Southeast Bay. The anchored crust, thus left without support, settled in the middle, leaving on the valley sides moraine-like patches of itself, two to six meters above the general surface of the sunken, chaotic chill-phase (Figure 3). This flow deserves further study, but it seems to furnish another example of the principle described already by Glangeaud, working in the volcanic district of central France. He also found evidence of valley-flooding by lava, followed by subsurface draining of the fluid magma and consequent formation of terrace-like masses of lava on the sides of the valley.[3]

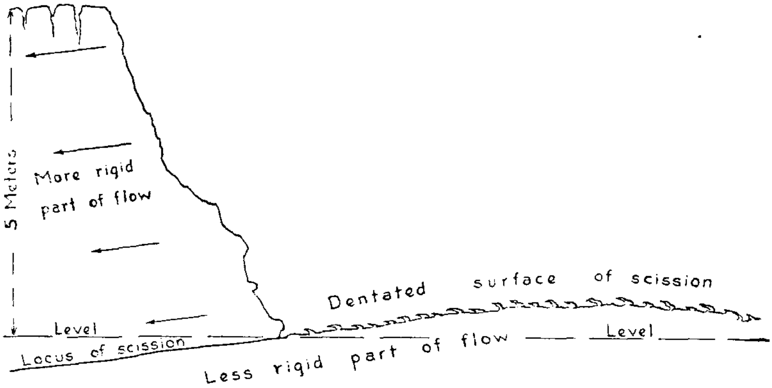


FIGURE 2a. Diagrammatic section illustrating the sliding of the upper, more chilled and rigid part of a lava flow on the underlying, hotter, and less rigid part; with resulting shark's-tooth projections on the grooved surface of scission (the glass here rapidly solidified by chilling).

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[3] P. Glangeaud, Bull. 135, Service carte géol. France, p. 57, 1913.

A similar process may account for a moraine-like ridge of lava skirting the northern slope of Southwest Red Hill.

Generally the basaltic flows of Ascension are initially black to dark gray at surface and floor as well as at intermediate levels. Among the more extensive flows this rule seems to have no exception. To depths of a few centimeters the brown tint of weathering is characteristic of the older flows, but apparently at no place has ordinary weathering produced the deep red color, so common in the scoriaceous phases of once deeply buried, basaltic flows, as in Saint Helena, the Hawaiian Islands, Samoan Islands, and elsewhere.

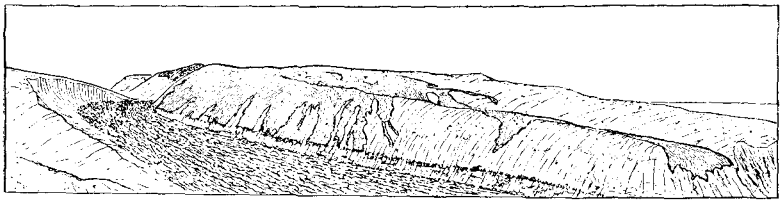

FIGURE 3. Sketch of a young flow of basaltic habit which occupies the floor of the valley between the Southeast Head plateau, flooded with trachyandesite (background), and the Little White Hill crater rim (left end of picture). The subsurface draining of the lava in the valley seems to have thinned that flow, leaving ragged, terrace-like patches on the slope of Southeast Head.

On the other hand, some short, thin flows, especially noted around Mountain Red Hill and along the southern side of the old caldera-rim of Green Mountain, are distinctly red and seem to have had that color from the time of consolidation. Their tint is much like that of many of the tuff-ash cones of the island. The specially high oxidation of these pyroclastics seems referable to the prolonged influence of steam passing through the vents where the material had temporarily rested. That well recognized effect of fumarolic emanation may likewise explain the magmatic reddening of the small-volume flows just mentioned. The gray and black flows escaped prolonged steaming, except where they happened to cross the sites of fumaroles or steam vents, active after the flows had come to rest. At such points local reddening of the rocks is to be expected.

The rules regarding color, stated for Ascension, seem to apply to basaltic regions in general. Hence the striking red phases of formerly *buried flows of basalt, seen in the cliffs of Saint Helena, Samoa, Hawaii*, etc., were probably not so colored at the times of their respective eruptions. Neither is explanation by weathering satisfactory. Perhaps in many of these cases the reddening is an effect of the passage of hot steam, permeating the piles of lava, especially the scoriaceous surface-phases and floor-phases, for a long time after the flows were solidified. That deeply buried flows have actually been charged with steam, after the fashion of a steam-pack, is indicated by explosions of the caldera type.

TRACHYTIC MASSES.

Though the existence of alkaline trachytes in Ascension has long been known, their mode of occurrence and structural relations have not been hitherto described in detail. These features of the geology are worthy of close attention and it is fitting that the data of Table I should be considerably expanded in the present paper. As shown in the table, most of the trachytic bodies are crater-domes or outflows from domes. One may well doubt that so many endogenous domes are anywhere else to be found in an area so small, and that, throughout the world, the completeness of exposure and of preservation of initial forms in such number can be surpassed.

The simpler domes will be first described; then those more complex; finally certain small masses of more obscure relations.

Ragged Hill Dome. The Ragged Hill dome (alt. 944 feet, 288 meters) is one of the most perfect in the island. It rises from beneath the bedded ash-scoria-driblet cone of Southeast Crater (alt. 1146 feet, 350 meters), south of Green Mountain (see Plate I and Figure 4). The base of the dome is concealed also by younger lava flows from Green Mountain, so that the height of the dome above these flows is only 30–40 meters on the north and about 80 meters on the south. The visible part is nearly circular, with diameters of 200 and 250 meters. Through erosion (insolation, wind, and rain) the dome has lost substance to the average depth of a few meters. This trachyte is charged with unusually large and abundant phenocrysts of feldspar. Fluidal banding is not conspicuous, but the dome structure is clearly indicated by the presence of a pronounced, concentric rifting; the plates dip away on all sides from the top of the dome, the angle of dip reaching as much as 35 degrees at the border of the mass. The platy structure appears to be due to thermal contraction. The trachyte contains many inclusions of vesicular lava, probably common basalt. The weathered surface is carious (Plate VI, A), and the hardened (silicified) shells and points are specially sonorous under the hammer. In hollows on the dome fresh feldspars, released by weathering, are concentrated.

From its position the Ragged Hill dome might be taken to be a lateral eruption from the Southeast Crater vent. On the other hand, the trachyte may have risen through an independent vent, as indicated, speculatively, in Figure 4. The underground relations are too obscure for certainty on the point. More evident is the steepness of the constructional slopes of the dome, showing the very high viscosity of the trachytic magma.

Little White Hill Dome. Like the last this dome (alt. 552 feet, 168 meters) has an initial form not complicated by effluent tongues, and it has not been greatly altered by erosion (Plate VII). The ground-plan is nearly circular; the diameter, about 150 meters. Below the normal, carious surface, due to weathering, the cliffy slopes display a fluidal structure. On the west, north, and northeast the light gray dome is surrounded by a continuous, crescentic, curving ridge of reddish, brown, and black ash and scoriae, evidently the rim of an older, basaltic crater, in the midst of which the trachytic dome arose. To the eastward the basaltic rim abuts against the independent dome of Southeast Head. Alternative explanations of this relation suggest themselves. Conceivably the Southeast Head dome is older than the explosive or other process responsible for the wide crater and represents part of its rim. On the whole, however, it seems more probable that the Southeast Head dome is younger than the basaltic crater, and during eruption overwhelmed the rim of the crater on that side. The second view finds support in the apparent absence of that amount of trachytic débris in the rim which might be looked for if the crater had been opened by explosion which affected the Southeast Head trachyte. On the south the waves of Southeast Bay have cut away the basaltic rim.

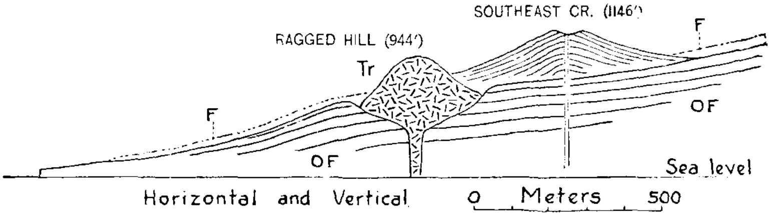

FIGURE 4. Section through Ragged Hill dome of trachyte (*Tr*) and the basaltic cone of Southeast Crater, both resting on older flows of basalt (*OF*). A young basaltic flow from Green Mountain (*F*) has partly submerged dome and cone.

Wig Hill Dome. The remarkable Wig Hill body of trachyte (alt. 475 feet, 145 meters), faced by a grand sea-cliff, is the remnant of another steep-sided dome, which seems to have risen in the southern part of the same wide crater (Plates VIII and IX). The remnant, not yet devoured by the waves, is veneered to a depth of 5 to 20 or more meters with basaltic, scoriaceous agglomerate and driblet flows of vesicular basalt. The contrast in color of the veneering "wig" and the underlying dome is very striking. Their relative ages were not determined with finality. Viewed from a distance, the "wig" looks like a layer of older basalt, lifted and domed by the trachyte as it rose to assume the usual dome form. Close investigation shows the absence of the corresponding tensional effects in the rock of the "wig." Hence it seems better to assume the reverse age-relation, the "wig" representing basaltic eruptions through one or more vents penetrating the trachytic dome. The Wig Hill composite would thus be analogous to that at Cross Hill.

Cross Hill. From Georgetown Cross Hill (alt. 868 feet, 265 meters) has all the appearance of being an ordinary ash or cinder cone of basaltic habit (Plate X, *A*). The northern, western, and eastern slopes are largely underlain by yielding lapilli to depths locally reaching several meters. The southern slope and the upper part of the cone in general are superficially composed of a somewhat cemented, bedded ash or tuff, containing irregular spatter-bombs. The beds of tuff have been differentially eroded by the persistent wind and the occasional showers, especially on the southern side, where a prism approximating 60 meters in maximum thickness has been thus removed ($a-b$, Figure 5). If the existing cone ever had a crater, this has been eroded away. On the same slope one sees a few, thin, steep flows of scoriaceous lava interbedded with the pyroclastics. The tuff carries occasional small fragments of pale gray trachyte, conspicuous among the dominant black, brown, and reddish lapilli.

The trachytic fragments were probably derived from a dome of trachyte which underlies the mantle of tuff and flows (Figure 5). The depth of the mantle is about 75 meters at the top of the cone. On the northeastern side the basaltic mantle has been completely removed by erosion, over a considerable surface, from the 600-foot contour down to about the 200-foot contour. Small outcrops of the trachyte are seen also at the foot of the cone on the same side, and at the foot of the northwestern slope.

Just northeast of Government House on the slope of Cross Hill, a large ridge-shaped outcrop of lava, probably trachydolerite, overlain by south-dipping beds of lapilli and moderately cemented tuff, forms part of an old crater-rim. The curving rim is also represented by another ridge to the eastward. Another, smaller part of the rim is probably to be seen at the foot of the southwestern slope of the cone. Elsewhere the old wall of the crater has been buried under younger pyroclastics or by lava flows from the interior of the island.

The history of Cross Hill seems, then, to be as follows: A trachydoleritic cone was built and eviscerated by explosion. At the center of the resulting crater or caldera, highly viscous trachyte was erupted, forming a typical dome, much like that at Ragged Hill or at Little White Hill. A short, stubby outflow carried some of this stiff lava to the site of Georgetown, but most of the trachyte remained inside the rim, solidifying there with slopes of 20 to 30 degrees and with a height above the crater floor of nearly 200 meters. Then the central vent was again opened, probably by the cracking of the new dome; through explosion and outflow the trachytic dome was deeply veneered with the contrasted, femic lava.

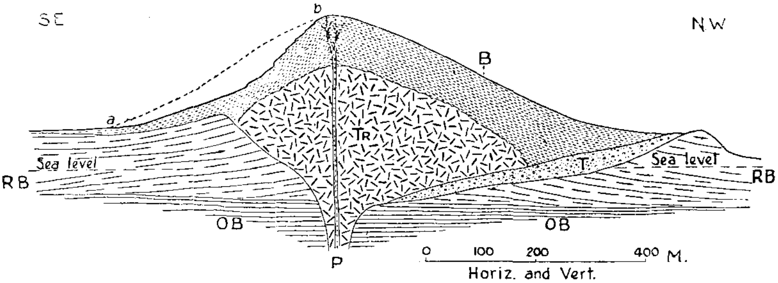

FIGURE 5. Section through Cross Hill, showing older basalts (*OB*) under a younger basaltic (trachydolerite) cone (*RB*), partly destroyed by explosion. In the resulting caldera tuff (*T*) was deposited. Later a monolithic dome of trachyte (*Tr*) was formed in the caldera. A thick cap of scoria, tuff, and breccia of basaltic habit (*B*) was then erupted upon the trachyte, through one or more narrow vents of the type shown diagrammatically at *P*.

The trachyte of the dome is a massive monolith, without notable flow-structure, crumbling at the top, and weathering to a rugged surface.

Riding School. The Riding School massif (alt. 800 feet, 244 meters) is an unusual volcanic type, which has claimed the attention of all visitors to the island. It consists of a basaltic-lava cone bearing a relatively large crater which has been filled nearly to the highest point of its rim with monolithic trachyte. The basalt is black, dark red, or dark brown (weathered). Its viscosity on eruption must have been high, since the outer, constructional slopes are steep, measuring 30 degrees or more. The northwest slope is underlain by a single, very thick and massive, vesicular flow, which is broken into bouldery masses of cyclopean proportions. In fact, from the partial exposure of the old crater-rim, one gets the impression that it may be essentially the product of the same outflow. If that be true, an axial subsidence must be assumed, for a well defined crater-shaped depression had already been formed before the next important addition to the composite mass was made. The central depression could hardly have been formed by explosion; the outer slopes are not covered with the abundant pyroclastic deposits expected on that hypothesis. However formed, this crater-like depression was nearly circular and had a diameter of about 500 meters.

At the center of that depression a large body of viscous trachyte arose. This may have formed, initially, a true dome, but, if so, the new structure was unstable. A large fraction of the risen or rising trachyte flowed over the rim of the basaltic crater, eastward and northeastward, making a thick flow about 700 meters long. This flow, like many others in the island, is stubby, with steep terminal scarps. A much smaller outflow took place on the southwest side.

Perhaps partly because of the outflows, which may have drawn magma from levels below the surface of the central body of trachyte, this part of the surface subsided. Or withdrawal of magma at still greater depth may have operated. Or, thirdly, crystallization and purely thermal contraction of the vertical column may have had some importance. For one or more of these reasons the surface of the trachytic crater-filling became basined (Figure 6).

Ash, tuff, and lapilli, probably derived from The Peak or other eruptions of the Green Mountain region, formed well stratified beds in the new hollow (Plates X, *B* and XI). The beds themselves are strongly basined and it looks as if the axial subsidence continued for some time after the effluent trachyte had crystallized.

The monolithic floor of trachyte is comparatively impervious to water. According to Darwin some of the upper, finer-grained beds are lacustrine deposits. If these were properly identified by the present writer, they underlie a three-meter layer of basaltic lapilli which are very like those constituting so much of The Peak of Green Mountain. Neither that superficial layer nor the lake-beds beneath seem to have been appreciably basined. The centripetal dips of the older, tuff-ash beds are commonly ten to fifteen degrees and have a maximum of about thirty-five degrees. Erosion has caused the projection of the stronger beds as crescentic ridges or cuestas, and has been specially active at the periphery of the basin; hence the slightly sunken "race-course" which has suggested the name of the Riding School.

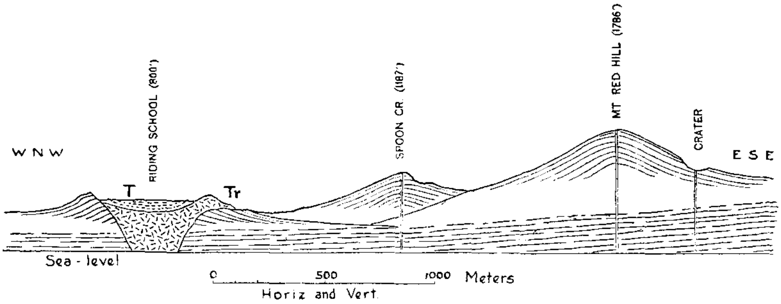

FIGURE 6. Section through basined dome of trachyte (*Tr*) at the Riding School, the corresponding "crater" filled with basined tuffs (*T*); section continued through Spoon Crater and Mountain Red Hill, both of basaltic habit.

The bedded rocks of the central depression were studied in the wall of wet-weather gulches and in a deep hole which had been opened by a prospector. Below the superficial layer of partially cemented, black to dark brown lapilli are the well-laminated silicious "lake-beds." These are white to pale gray and pale brown and are made up of trachytic or rhyolitic dust and volcanic sand. Some layers carry many angular fragments of white pumice. About ten meters is the exposed thickness of the more silicious beds; calculated from the dips the total thickness appears to be twice that amount.

The writer was unsuccessful in finding the bed of infusorial earth that Darwin reported from the Riding School basin.[4] The same result attended the effort of Mr. G. V. Douglas, geologist of the Shackleton-Rowett Expedition, to find it (verbal communication).

Silicious concretions, described by Darwin, occur in the layers of acid tuff and dust. One of these, collected by a member of the

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[4] C. Darwin, Journal of Researches, p. 499, new ed., London, 1901; cf. C. G. Ehrenberg, Quart. Jour. Geol. Soc. London, **2**, part 2, 71, 1846, and Berichte k. Akad. Wissen., p. 140, Berlin, 1845.

Deutsche Südpolar Expedition, has been analyzed, as reported by Reinisch (page 648 of his memoir):

SiO_2	90.07
$Al_2O_3 + Fe_2O_3$	2.85
MgO	0.04
CaO	0.09
Alkalies	0.03
Loss on ign.	7.31
	100.39

When quite fresh the Riding School trachyte is a pale greenish gray. By exposure to the weather it is decolorized to a pure light gray or a pale yellowish gray, though the leading mineral constituents show no obvious changes. The pitted or carious surface of weathering is striking, especially on the windward side (Plates VI, *B* and XIII). Hoodoo-like towers, spires, and mushroom-shaped irregularities on the 20-meter to 30-meter cliffs have been developed because of the great difference between the strength of the normal trachyte and the strength of the silicified "veins," so characteristic here as in nearly all the trachytic bodies of Ascension. The origin of these veins will be discussed on a later page. The weaker parts of the trachyte weather to white dust and sand, chiefly composed of soda-orthoclase and anorthoclase.

Under the mantle of tuff and loose ash which covers the effluent trachyte on the Green Mountain side, the initial surface of the trachytic flow has been preserved. To the depth of from one to two meters this lava is strongly brecciated, because of the tensions which were set up in the superficial shell during flow. Even in that shell the trachyte is not vesicular in the sense of carrying rounded gas-pores like those of the neighboring basalt. The minute, angular spaces between the dominant feldspars give a porosity of a quite different type. The same characteristic applies also to the main body of the Riding School trachyte, as it does generally to the Ascension trachytes. It may be added that a stubby flow from the direction of Middleton Peak, and situated northeast of the Riding School, has likewise had its initial surface preserved by moderate burial under pyroclastics. Later erosion has exposed that surface, which has features like those of the flow just described from the Riding School mass itself.

The Riding School trachyte carries many angular and rounded inclusions of vesicular basalt.

"Drip" Dome. North of the road in the col between Travellers Hill and the Sisters Peak group of cones, there is a small slaggy, basaltic cone with a crater, 100 meters wide and opening to the west. This crater is not shown on the Admiralty chart. On its eastern side the cone passes under a large body of cariously weathering trachyte. The rock is largely covered with a thick layer of alluvial ash and lapilli, obscuring the form as a whole, but the relations seem to be that of a small dome, the surface of which has sunk at the central axis. The trachyte has a platy-fluidal structure, the plates dipping centripetally at angles of from two to five degrees, along a crescent-shaped area. On the eastern side a broad tongue of the dome trachyte, 200 meters long, flowed out and froze at an angle of about ten degrees. Then axial subsidence basined the surface of the dome. The result appears to be a homology with the basined dome at the Riding School.

Going north from the effluent tongue, one follows the edge of the main body of the dome trachyte, exposed in a cliff about twelve meters high. The trachyte is here monolithic and transgresses the red scoriaceous rim of an old basaltic crater, in which the dome arose. Lumps of the basalt are enclosed in the massive trachyte, which is curiously weathered into forms suggesting sections of a bee's comb and also concretionary shapes of great variety.

For convenience this trachytic mass may be called the "Drip" dome, one of the few drips (springs) of the island being located at this locality.

Flow of Trachyte North of the "Drip" Dome. Near the 1000-foot contour on the southeast slope of the 1187-foot cone of the Sisters group is a basaltic crater, not shown on the Admiralty chart. This crater is breached by a 175-meter flow of trachyte, which issued from this small crater itself. No true dome is represented. The flow is directed toward the south-southeast. It viscosity was very high; it consolidated on slopes varying from 20 to at least 35 degrees. The trachyte seems petrographically like that of the "Drip" dome.

"The Crags" Dome. A point one kilometer northwest of the 1490-foot Sisters Peak is near the center of a group of massive outcrops of trachyte. That farthest to the northwest is particularly conspicuous, showing a 30–40-meter cliff on the northern side. This whole assemblage of trachytic outcrops may be called "The Crags" (Plate I). Between the individual crags is a thick mantle of lapilli, bombs, and wind-blown volcanic sand, and the trachyte is also partly submerged under young, heavy, basaltic flows which issued from vents along the northern base of Sisters Red Hill cone. The relations of "The Crags" trachyte are therefore not obvious, but it probably belongs to a single, relatively old, and hence considerably weathered dome, centering near the point first noted in this paragraph.

The trachyte has the habit normal for Ascension; it is of pale gray or yellowish-gray color, is monolithic at each crag, weathers cariously, and closely resembles the Cross Hill trachyte.

At one of the most southerly crags a 10-centimeter, angular inclusion of hornblende granite was found in the trachyte. Time failed for a thorough search for other inclusions of the kind; they cannot be numerous. The one actually discovered shows that the granitic fragments brought up at the Ascension vents are not confined to the explosion-breccias and tuffs. Apparently also it indicates the considerable depth from which the trachytic magma has come.

Bears Back Trachyte. At the foot of the southeastern slopes of Bears Back is a large outcrop of trachyte which underlies, and is older than, the basalt of the Bears Back plateau. The true form and relations of the mass represented could not be determined. As already observed, it may possibly represent the edge of a much larger, dome-shaped body centering under the Bears Back basalt. This trachyte is a common gray variety, not studied in thin section.

Green Mountain Dome. At the western end of Green Mountain the highest exposure of trachyte in Ascension is found, at the height of 2490 feet or 759 meters. This point is close to the vertical axis of one of the most voluminous domes of the island. Thence the stiff magma of the dome flowed out, westwards, northwards, northeastwards, and southwards. The longest of the flows, directed toward the west, is about two kilometers in exposed length. An outcrop of trachyte northwest of Spoon Crater cone, at its base, may be part of the flow just described; if so, the Spoon Crater cone is younger than, and in part built upon the lower end of the flow. In any case this basaltic cone is younger than the trachyte at its base.

Plate XIV, *A*, illustrates the habit of the trachytic overflows from the Green Mountain Dome center. The high initial slopes and the steepness of terminal and lateral scarps prove the viscosity to have been high, as usual.

The eastern part of the dome and much of the effluent trachyte still farther to the eastward were torn out by one or more great explosions, which developed an elliptical caldera, measuring 1500 meters by 1100 meters. In this depression later explosive eruptions built the steep sided Peak, which is chiefly made up of basaltic tuff, ash, and breccia. The contact of this young cone with the caldera-wall of trachyte is shown in Plate XIV, *B*.

Along the Invalids Path from the main road to the Sanatorium, the young basaltic tuff, dipping 20–30 degrees away from Green Mountain summit, is seen to rest with strong unconformity on well-bedded, trachytic tuff. These sections indicate a somewhat prolonged interval of erosion between the two periods of explosion.

The Green Mountain trachyte is normal in preserving a marked fluidal structure. At and near the foot of a flow crossing the mountain road on the north side of the mountain, the banding dips 40–50 degrees southward, that is, in the direction from which the flow came. The shear-planes of the sliding magma were developed with this high upstream dip because of the specially high viscous resistance that was offered by the chilled, terminal part of the advancing flow. Such upturning of the shear-planes at the lower ends of the trachytic flows is very common in the island. (Compare Plates XV, *B* and XVII; Figure 8.) The flow is brecciated to a depth of three meters. It lacks ordinary vesiculation and also any systematic jointing. The floor phase has likewise been brecciated by movement.

Similar features characterize a very thick overflow of trachyte west of the Mountain Farm. This tongue occupied a radiating valley in the old, basaltic cone of Green Mountain, and is probably at least 150 meters thick at maximum. Its surface slope varies from ten to twenty-five degrees. In a cliff the brecciated floor-phase is well exposed; it measures five meters in thickness.

The trachyte of Middleton Peak ridge probably came from the Green Mountain vent, but the evidence is not perfectly clear.

Some of the Green Mountain outflows of trachyte inclose many angular, vesicular fragments of basalt, evidently derived from the walls or floor of the dome-vent.

Weather Post Dome. Most extensive of all the bodies of trachyte is that of the eastern part of Ascension, including Weather Post (alt. 1990 feet, 606 meters), White Hill, and the great flow north of the Devils Cauldron (Punchbowl). Possibly the composite mass is the product of eruption from a single center, but the general topography rather suggests that there were two chief eruptive centers. (See Plate XV, *A*.) One of these is under Weather Post; the other, under White Hill.

Near the most northerly point of the rim of Cricket Valley, the

Weather Post trachyte is seen to have welled out over a series of brown basaltic tuffs, dipping southwest, that is, away from the Weather Post-Cauldron massif. So far as it goes this observation lends color to the supposition that the Weather Post trachyte was erupted within the rim of an older basaltic crater or caldera — the case being like that illustrated at the Riding School, at Green Mountain, or at the analogous Cross Hill. A little farther to the southeast, thick basaltic flows are seen to dip at a low angle toward the Weather Post and apparently underlie a thick effluent tongue of the Weather Post trachyte. These flows may have come from Green Mountain, flooding the older crater-rim. On account of lacking exposures elsewhere the existence of the buried crater or caldera could not be definitely proved.

Among the causes of obscurity is a heavy mantle of coarse breccia, the débris from the Cricket Valley explosion, which covers much of the Weather Post dome and the wide trachyte flow on the north, hiding contacts.

At the head of that flow, which forms a rough, sloping plateau stretching nearly to Northeast Point is the Devils Cauldron, a remarkable explosion-crater or caldera (Plate XVI). The Cauldron is broadly elliptical, approximately 200 meters in length, 30 to 60 meters in depth, except on the northwest side, where the wall is only some ten meters high. The walls are everywhere steep, if not actually vertical. The trachytic débris from this explosion is deep on the surrounding trachyte. The fluidal structure of the trachyte in the walls of the Cauldron, developed as usual, shows planes of shear that are irregularly disposed, though often nearly vertical. They tend to strike parallel to the rim of the Cauldron, an indication that the Cauldron may possibly be the locus of a distinct dome-extrusion. Even in that case, however, the Cauldron dome might have been satellitic to the Weather Post dome.

That the Cauldron trachyte was erupted through older rock of basaltic habit is shown by its inclosure of many fragments of vesicular and trappean basalt or trachydolerite (probably the former). Here again a thick shell underlying the original surface of the trachyte has been brecciated by the flow to a more or less chaotic condition. Locally a few, small gas-pores were observed in that shell, but pronounced vesicularity in the ordinary sense was nowhere to be seen.

The well-bedded tuff and agglomerate on the rim of the Cauldron reaches 7 or more meters in thickness; on Weather Post the pyroclastic overburden has a maximum thickness approaching 40 meters. These beds are chiefly composed of trachytic pumice, with some basaltic fragments, and many black to dark green, lustrous fragments of obsidian. The glass shows the common banding caused by devitrification in some layers, which, perhaps in consequence of local crystallization, are in some degree vesicular (operation of the "second boiling-point?").

On the southwest side, the Weather Post has certainly lost substance because of the powerful explosion at the so-called Cricket Valley (Figure 7). Probably there has been partial destruction of the dome by another major explosion on the eastern side, located in the deep depression between the Weather Post and the Powers Peak ridge. To this explosion may well be referred the trachytic débris, reaching a thickness of about 70 meters, on the col between Weather Post and White Hill. The wild, canyon-like depression opens to the sea at Spire Beach. It has been somewhat deepened by erosion, but its origin as a deep caldera, rather than as primarily a valley of erosion, seems probable; further study on the point is needed.

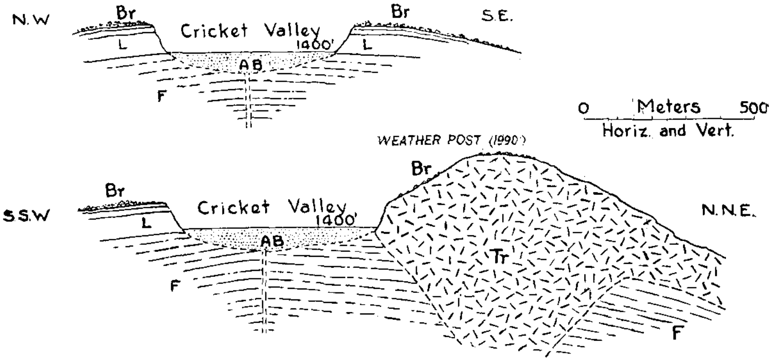

FIGURE 7. Sections through Cricket Valley, showing this caldera opened in basaltic flows of ordinary thicknesses (*F*), with one unusually thick flow of olivine basalt (*L*), and filled to an unknown depth with ash and tuff (*AB*). The explosion also affected the older trachyte (*Tr*) of the Weather Post. Débris of the explosion shown on the surface.

The high cliffs bounding Weather Post on the south and southeast give exposures showing that the outflows from the dome, like those from the White Hill dome, range from 50 to 200 meters in thickness. They are separated by brecciated phases and thin beds of yellow, trachytic tuffs. On the southwest side of the Weather Post dome the tuffs dip away from the dome at angles of 60–70 degrees, suggesting that they have been upturned by continued rise of the viscous dome-magma.

On the map (Plate I) two patches of basaltic rocks, at the shore northeast of Weather Post, are shown. They were not visited. From a distance the more northerly patch looks like a young lava flow which issued from an opening through the older, already solidified trachyte. The relation of the other patch is still more uncertain.

White Hill Dome. Petrographically the trachyte of the White Hill dome is like that of Weather Post, and, as above noted, sure evidence of any structural separation between the two massifs was not discovered.

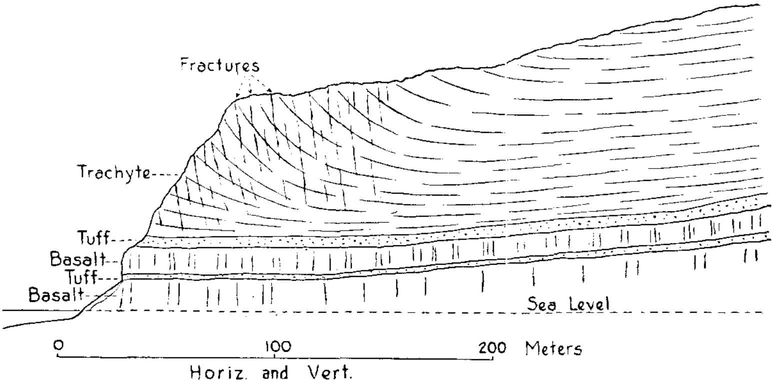

FIGURE 8. Longitudinal section through a thick flow of trachyte on the main island opposite Boatswain Bird islet; illustrating the common upturning of the trachyte flows near their lower extremities. Fractures due to tension.

On the walls of the grand amphitheatre opened above Spire Beach, the great thickness of the tongues apparently effluent from White Hill, and their separation by trachytic tuff and breccia, can be observed. The superficial appearance of the outflows is illustrated in Plate XVII, which serves also to show the form and structure of the trachytic flows in the whole island. The fluidal structure, the upturning of the shear-planes at the foot of the flow, here 30 to 70 meters thick, and the characteristic ragged, tensioned phase at the surface are all tolerably apparent in the photograph. Like many others, this flow is young enough not to have been greatly damaged by erosion.

The high cliffs southwest of Boatswain Bird islet exhibit a fine section through a White Hill outflow, illustrated in Figure 8. The sketch had to be made from a distance, and no opportunity for a close study of the cliff was afforded; yet the drawing is believed to be correct in principle. The glacier-like upturning of the shear-planes downstream was particularly clear. Crossing these planes at high angles are joint-like fissures, shown in the drawing. They are presumably connected with the horizontal tensions developed in the steeply inclined flow as it gradually solidified.

At a point southwest of White Hill its effluent trachyte rests on a series of basaltic flows; the trachyte is charged with angular inclusions of vesicular and compact basalt. Hence the White Hill trachyte is younger than some of the basalts. Lack of exposures forbade a more specific reading of the relation of this trachyte to its foundation. At the contact just mentioned the floor-phase of the trachytic flow is well exposed. It has the usual brecciated appearance and also a special abundance of angular blocks of lustrous black obsidian, reaching as much as one meter to nearly two meters in diameter. The glass is much like that seen in the surface-phase of the trachyte at the Devils Cauldron, showing devitrified and vesicular bands with light gray and greenish tints.

Southeast Head Dome. The trachyte of Southeast Head has the topography of a flat dome. It is supposed to be an endogenous dome on a scale larger than the adjacent Little White Hill body and perhaps considerably widened in outcrop by overflows. Actual proof of that assumption is not to be had: the contacts of this trachyte with older rocks seem everywhere hidden from sight, under the sea or under younger lavas or ejectamenta. The hypothetical structure underground is given in the section on Plate I. The highest point of the monolithic body is charted as 479 feet (146 meters) above sea. Except on the western side the trachyte has been cliffed by the waves.

After the solidification of the dome it was split by a vertical fissure, which runs nearly due east and west through the summit of the dome. From the foot of White Hill the fissure was followed nearly 1200 meters. Mr. Hedley Cronk, superintendent of the Government Farm, on Green Mountain, states that the fissure continues to Bottle Point. At intervals throughout most of its length, dark gray to black, scoriaceous trachyandesitic lava flowed out over the pale gray trachyte in the form of broad and narrow tongues. The contrast of color between the trachyte and the younger flood is nothing short of spectacular. (See Plate XVIII.) The photograph fails to show fully the difference. The tracing of the photograph (reduced), given in the more diagrammatic Figure 3, expresses the relation of the two rocks perhaps more vividly. The tongues of trachyandesite are thin and yet their final viscosity must have been great, for they hang on slopes as high as fifteen or twenty degrees. (See Plate VIII.)

Near the 479-foot point the fissure has been widened explosively and a double crater, from 30 to 50 meters in diameter, developed. On the crater-walls the pale gray, monolithic trachyte underlying the trachyandesitic flow is well seen. Evidently the last explosion, at least, followed the outflow of the trachyandesite. Between the two parts of the crater this rock forms a bridge, strongly sagged in the middle. A half dozen other and smaller vents of the central type have been formed along the fissure. From the western part of the fissure the trachyandesite ran southward, veneering the eastern foot of Little White Hill dome and the floor of the old crater in which that dome was extruded. One of the south-running flows, which flooded the valley between Little White Hill and Southeast Head has already been noted as showing terrace-like remnants left after the slumping of the chilled surface-phase because of subsurface draining. (See Figure 3.)

Visiting geologists should not fail to study the impressive features of the whole Southeast Head complex, and it is to be hoped that they will improve on the detail of mapping, which had to be done very sketchily in the time allowed during the writer's reconnaissance.

Cocoanut Bay and Pillar Bay Masses. No opportunity to sample these was given. From distant views the writer concluded that the bold cliffs at Cocoanut Bay are composed of trachyte, which probably forms a breached dome, partly buried under basaltic flows from Green Mountain. Whether the crater-like bay is due to wave-cutting or to explosion can not yet be stated. The nature and relations of the rocks at Pillar Bay are still more uncertain.

Boatswain Bird Islet. The topographic relations make it difficult to consider the picturesque monolith of this islet as connected with flows from either the Weather Post or the White Hill center. More probably it represents a strongly cliffed, independent dome, that issued from a special vent beneath (Plate XIX, *A*, *B*). Though the powerful waves have driven back the cliffs on all sides to considerable distances, the upper surface, now culminating about 323 feet (98 meters) above the sea, may not be very different from the original surface on this remnant of the dome. The perfect exposures on the vertical cliffs show the mass to be monolithic, cariously weathering, and altogether similar to the normal trachytes of the main island.

The rock has the usual weakness of the Ascension trachyte, so that the waves have been able to prolong the profiles of the vertical cliffs to depths of from ten to fifteen meters below low-water level. A fine natural bridge is a detail of the wave-cutting (Plate XX).

Pyroclastic Deposits.

The great explosions at Cricket Valley and the Devils Cauldron delivered projectiles reaching one to two meters in diameter, but in general the average pyroclastic deposit of the island is a rather coarse tuff or cindery ash. Many deposits are made of material wholly of basaltic habit. Other basaltic tuffs carry sporadic fragments of trachyte or trachytic pumice. In Green Mountain, and east and southeast thereof, beds of tuff and agglomerate, chiefly made of trachyte, are rather common. Large trappean projectiles which were thrown out of Cricket Valley caldera inclose trachytic xenoliths, evidently included when the basalt was molten.

The unweathered basaltic tuffs are variable in color, from black and dark gray to brown and deep red; all types weather to brown tints. The fresh trachytic tuffs are white, pale gray or pale greenish gray, weathering to light brown. Nowhere has weathering gone to the stage of important lateritization, such as is so general in Saint Helena. Ascension is too young for that.

Table I suggests the distribution of the principal bodies of pyroclastic material, but of course the finer débris of the more recent explosions veneers the surface far and wide, especially to leeward of each vent. Later winds and rains have redistributed the looser materials, which on the lower ground form long, local fans and streamlines. The tuffs in situ are best exposed on windward slopes; on that of Green Mountain torrential rains have cut steep-walled gulches, from 20 to 60 meters deep, in the basaltic tuffs. The Devil's Ashpit is a deep, cirque-like gulch cut in The Peak tuff. The Peak itself has many radiating gorges and revett edges, developed by erosion in somewhat coarser material. The sand-like débris flooring Cricket Valley is the favorite home of thousands of brilliantly colored land-crabs, which have there found deep, loose material suitable for their burrowings.

Typical bread-crust bombs are rare in Ascension. Spindle-shaped bombs with smooth sides are fairly common; examples may be collected on Cross Hill, Sisters Peak, South Red Crater, and South Gannet Hill. Basing his thought on similar specimens which he collected on Ascension, Darwin originated the true explanation of the form and structure of spindle-bombs.[5]

On the flanks of many tuff-breccia cones one finds scoriaceous blocks, a few decimeters in length, that roughly resemble spindle-bombs in shape. These appear, however, to have been formed in quite a different way — as tear-like exudations, rhythmically pinched off as small driblets of lava burrowed their way out to the air through the mobile fragmental deposits of the cones.

The trachytic tuffs are essentially composed of angular pieces of the ordinary, bubble-free trachyte, mixed with varying proportions of light pumice, in some places also with small fragments of black or deep green, lustrous obsidian. In most cases, if not in all, where the lower contacts of the thick outflows of massive trachyte can be seen, each flow is directly underlain by a bed of pumice-rich trachytic tuff. This rule suggests that the upper part of each ascending column of trachytic magma was exploded into the air before the main body rose high enough to cause lateral outflows or overflows.

Usually the trachytic tuffs are quite friable at the outcrop, but those widely exposed on the plain between Thistle Hill and the Bears Back are superficially hardened to a depth of two to five centimeters. This surface-shell is resonant under the hammer and recalls the irregular, silicified "veins" or interfaces of the trachytic domes and flows. The superficial silicification is here a phase of weathering under sub-arid conditions.

The Ascension tuffs show the centrifugal or outward dips and also centripetal or inverse dips, usual at vents with crater-rims. Successive beds may be unconformable, either because of erosion or because of explosion, taking place between the periods of pyroclastic action. In the second case the younger tuff may be seen to have been plastered on crater slopes at angles well above the angle of rest for even coarse, dry material; a good example was noted in the double crater opening near the foot of South Gannet Hill, south side.

DIKES.

Since neither deep erosion nor strong dislocations have affected the island, intrusive bodies visible at the surface are not to be expected. A half-dozen small dikes are the only bodies of the kind observed.

On the main road southwest of Cross Hill an irregular basaltic dike cuts slaggy breccia. With maximum width of about 25 centimeters, it strikes north 70 degrees east. Strong grooves were impressed on the dike-walls as the stiffening magma was forced past the rugosities of the breccia, east-northeastward, in a direction inclined 30 degrees from the vertical; the magma acted like a plastic solid, rather than like a purely viscous liquid.

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[5] C. Darwin, Geological Observations, p. 42. 2d ed., London, 1876.

A second, vertical, 15-centimeter dike of frothy, basaltic glass cuts a trappean flow at the north end of the old cliff at Southwest Bay.

The best exposed dike cuts basaltic ash on the western slope of Spoon Crater cone. This dike is vertical, is one to nearly two meters wide, and stands above the more easily eroded ash for a distance of about 150 meters. It strikes north 70 degrees west and may be connected with the Spoon Crater center.

A fourth vertical dike of basalt, with similar strike, cuts the base of a thick trachytic flow, 250 meters southwest of the top of Middleton Peak. This dike, more resistant to the weather than the trachyte, here brecciated by flow, stands up as a wall which is three to four meters high and about two meters wide. Three hundred meters to the westward a nearly parallel wall-dike of about the same width cuts trachytic tuff and breccia. In the absence of any other proof, these two dikes alone would show that some of the basaltic eruptions are younger than trachyte in Ascension.

Calcareous Deposits.

Though the physical conditions for the growth of coral reefs appear ideal at both Ascension and Saint Helena, neither island is fringed with reefs; the few species of corals now living along these shores are not reef-builders. The dominant and prevailing currents prevent colonization by larvae issuing from the reefs of the western Atlantic. On the other hand, both islands are in the "shadow" of Africa, so that west-bound currents cannot bring larvae from the Indo-Pacific region. There is no evidence that the conditions were essentially different at any time since the much older Saint Helena rose above sea-level.

Strong beaches of nearly pure calcareous sand have been formed in Clarence Bay and Southwest Bay. These are made up of fragments of shells, corals, and abundant calcareous algae, including Lithothamnion, a genus which seems to thrive as well as it does on a Pacific island. Darwin and Renard described these beaches, the former noting the advanced lithification of some of the deposits. (See Plate III, *B*.)

An older limestone at Southwest Bay represents a relatively ancient storm-beach or dune-deposit, so lithified as to make building-stone, ringing under the hammer. In it the waves are now cutting low cliffs and it can be seen to extend outwards, well beyond the low-tide level of the sea. In this case the cementation looks like that which is so common in calcareous dune-sands within the tropics and caused by the evaporation of water, either sea-spray or rain-water.

At Georgetown the beach sands have been well cemented into "beach-rock," which is almost entirely confined to the vertical interval between high-water level and low-water level or a little deeper level. The calcium carbonate cementing the beach-rock came directly out of the sea-water; as noted elsewhere this mode of the lithification of beach sands in the tropical belt seems best explained by the action of alkaline carbonate which is generated during the decay of animal matter, specially concentrated in the sands and muds.[6]

The beach-rock at Georgetown is remarkably perforated by parallel worm-borings; the sand is most lithified along the walls of the bore-holes.

FAULTS.

Displacements by ordinary faulting are rare in Ascension. The only cases observed refer to faults in tuffs and tuff-breccias.

In the valley about 600 meters south of the 1187-foot point on Spoon Crater, a wet-weather stream has cut a fine gorge about 30 meters deep, where pyroclastic basaltic deposits are well exposed. Some 20 meters of deep red to brown tuff overlie one meter or more of light brown tuff, which itself rests on ten meters of dark gray tuff and breccia. This series dips to the southeast and seems to represent a relic of an old, elsewhere buried crater-rim, centering at or near the vertical axis of the Spoon Crater cone. The tuffs are cut by a complex of nearly vertical faults with throws ranging from three to six meters. The faults are probably all of the normal type.

Several faults cut the tuff-breccia series along the road up Green Mountain, near the 5-mile post and between the 1500-foot and 2000-foot contours. Both normal and reverse faults were found, with maximum observed throw of about two meters. All of these local displacements may be connected with the settling of the composite Green

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[6] R. A. Daly, Year Book No. 18, Carnegie Institution of Washington, for 1919, p. 192. See also the writer's "Geology of American Samoa" in Pub. No. 340 of the Carnegie Institution of Washington, 1924.

Mountain cone, either because of the compacting of its material under dead weight or because of internal magmatic movements. Possibly the reverse faults are due to the upthrust exerted by the stiff trachytic magma as it rose to form the great dome of the mountain. Along some of the fault-planes traversing black, basaltic tuff, pale brown trachytic sand has been injected in a manner suggestive of sandstone dikes (earthquake jars).

Petrography.

Renard, Prior, and Reinisch have indicated the main rock-types in Ascension. The collections of rocks studied by them were made by others, and in many instances neither the localities nor the field relations of the critical specimens could be stated with exactness. The sampling of 1921 showed that Darwin and the members of the Deutsche Südpolar Expedition had found the main volcanic species constituting the island. The following description is thus supplementary to the petrography already published by experts, whose essential results are summarized. Three new analyses of the trachytes are given, and also four new analyses of the basalts, of which the chemical nature had not been made sufficiently clear. The thin sections from the 1921 collection are 157 in number. Besides, the writer has studied a score of thin sections (without specified localities) kindly presented by Dr. J. S. Flett, Director of the Geological Survey of Great Britain.

Named in the order of decreasing volumetric importance, the volcanic rocks include: basalts, trachytes, trachydolerite, trachyandesite, and rhyolitic obsidian, the last being intimately associated with the trachytes. The dominant basalts will first be described, then the trachydolerites, trachytes, and obsidian; finally the granular, plutonic rocks which occur as fragments in tuffs and breccias or as xenoliths in the lavas.

BASALTS.

Neither picritic nor limburgitic flows were seen in the island. The most mafic volcanic species collected is the abundant olivine basalt. It forms flows interbedded with, or overlying, flows of the even more common olivine-free or olivine-poor basalt. Some of the superficial, youngest flows, exemplified by the South Gannet flow and by the Bears Back flow or dome, are composed of olivine basalt. This kind of basalt forms rugged, tensioned flows as well as those so smooth as to approximate the pahoehoe or ropy lava; as a rule they are not so scoriaceous as the olivine-free basalts.

Two typical fresh specimens were selected for analysis.

One of these, taken from the surface flow of the Southwest Bay group, at a point on the road, south 85 degrees east from McArthur Point and west of Dark Slope Crater, has been analyzed by Miss Helen E. Vassar, chemist in the Department of Mineralogy and Petrography at Harvard University. The flow is relatively smooth and has the "bouldery" habit above described. The rock is dark gray, with the usual appearance of common basalt. Fairly numerous phenocrysts of feldspar, reaching eight millimeters in length, are embedded in the compact base, which is moderately vesicular. Under the microscope the phenocrystic feldspar is seen to be acid bytownite, Ab_{25} An_{75}. Olivines, generally rounded and reaching maximum diameters of about one millimeter, are subordinate phenocrysts, composing about 12 per cent. of the rock by weight. The groundmass carries abundant pale green augite, much iron oxide, probably both magnetite and ilmenite, and apatite. Less than 5 per cent by weight is glass, darkened by skeleton crystals of iron oxide.

The analysis of this specimen (No. 2731) gave the results shown in column 1, Table II. In the Norm classification the analysis is that of an ornose, in the salfemane class and order, gallare; it is alkali-calcic and persodic, and near camptonose, the dosodic subrang of the same rang.

TABLE II.

BASALTS OF ASCENSION ISLAND.

	1	2	3	Norms.	1	2	3
SiO_2	47.69	48.64	52.87	Orthoclase	3.33	7.23	11.68
TiO_2	2.79	3.52	2.01	Albite	24.63	30.39	40.30
Al_2O_3	16.23	15.54	16.68	Anorthite	29.47	22.52	18.90
Fe_2O_3	2.20	5.31	4.54	Diopside	13.60	15.14	11.28
FeO	9.93	7.73	4.79	Hypersthene	8.83	9.14	6.05
MnO	.17	.17	.37	Olivine	10.48	.15	...
MgO	7.15	4.96	3.92	Magnetite	3.25	7.66	6.50
CaO	10.02	9.03	7.32	Ilmenite	5.32	6.69	3.80
Na_2O	2.87	3.60	4.63	Apatite	1.24	1.24	1.24
K_2O	.64	1.24	2.06	$CaCO_3$	.10	...	...
H_2O-	.09	.16	.40	Water	.28	.37	.70
H_2O+	.19	.18	.30				
P_2O_5	.59	.64	.52		100.53	100.53	100.45
CO_2	.04	.03	n.d.				
	100.60	100.75	100.41				
Sp. gr.	2.99	2.97	2.84				

1. Olivine basalt, surface flow of the Southwest group, specimen No. 2731; analysis by H. E. Vassar.
2. Olivine-poor basalt, flow, north wall of Cricket Valley, specimen No. 2850; analysis by H. E. Vassar.
3. Trachydoleritic basalt, flow from Mountain Red Hill, specimen No. 2839; analysis by E. G. Radley.

The second specimen, also analyzed by Miss Vassar, was taken near the 1600-foot contour in the north wall of Cricket Valley and about 300 meters due east of the 1884-foot point marked on the Admiralty chart. It represents a flow overlying the much thicker body of basalt exposed on the wall of the "valley" (Figure 7). The specimen is a compact, very slightly vesicular, fresh basalt, with a medium gray color. Occasional phenocrysts of feldspar, up to 5 millimeters in diameter, break the monotonous, trappean surface.

The phenocrysts are: bytownite, in some cases faintly zoned, with a medium labradorite on the exterior; and colorless, generally anhedral olivines, which make up about seven per cent. of the rock by weight.

The groundmass is a confused, granular, somewhat fluidal composite of labradorite, colorless, diopsidic augite, another pale green augite, iron oxide, and apatite.

The analysis of this specimen (No. 2850) is stated in column 2, Table II; the norm is that of a camptonose.

A specimen (No. 2839), probably not far from being representative of the most voluminous lavas of Ascension, has been analyzed by Mr. E. G. Radley, chemist of the Geological Survey of Great Britain, London. It was collected at a point 100 meters north of the summit of South Red Crater, from a flow that issued from Mountain Red Hill and wrapped around the cone of South Red Crater. The moderately vesicular, dark gray, compact rock shows rare phenocrysts of bytownite. Other rare phenocrysts of augite in stout prisms reaching one millimeter in length, are seen under the microscope. The groundmass is a composite of labradorite, very pale, greenish augites, magnetite, probably ilmenite, and apatite. Occasional, small grains of olivine represent an unimportant accessory. No glass was observed.

The specific gravity, 2.84, lower than that of either of the olivine basalts, is correlated with the less mafic nature of the Red Hill basalt.

The analysis of No. 2839 is given in column 3, Table II. The norm places it in the dosodic subrang andose, of the alkali-calcic rang andase, of the order germanare, of the dosalane class.

The rock is clearly on the border line between true basalt and trachydolerite, and may be called a trachydoleritic basalt.

TRACHYDOLERITES.

Hayes Hill, a low scoriaceous cone at the water-front of Georgetown, and a yet smaller cone of scoriae immediately north of the Landing Pier (Plate XXI, *A*, *B*) have each yielded a specimen analyzed by Reinisch, who has shown these lavas to be trachydolerites in the sense of the Rosenbusch classification.

Reinisch describes these specimens as "strongly vesicular, red-brown to brownish-black fragments of slag or bomb-fragments. . . . They carry in places a superficial layer of glass, 1-2 millimeters thick, and occasionally show small phenocrysts of plagioclase, though generally no mineral is macroscopically visible. . . . In thin section a groundmass of poorly transparent, dark brown glass, full of opaque flecks and minute grains of iron oxide, is seen to be charged with laths of plagioclase ranging from labradorite to andesine. . . . Neither pyroxene nor olivine appear."[7]

In a characteristic specimen of the Hayes Hill rock, No. 2784, the present writer has found some very small grains of augite and much rarer olivine; its specific gravity is 2.66. The specific gravity of a specimen, No. 2761, of the Landing Pier cone is 2.58. This rock is largely glass charged with granules of magnetite and larger crystals of medium labradorite and andesine, which are also the essential feldspars of No. 2784.

The analyses given by Reinisch are quoted in the following table (Table III).

TABLE III.

Trachydolerites of Ascension Island.

			Norms (Washington's Tables)		
	1	2		1	2
SiO_2	54.04	51.18	Orthoclase	20.57	20.57
TiO_2	.94	1.34	Albite	39.82	32.49
Al_2O_3	19.58	21.41	Anorthite	21.96	26.97
Fe_2O_3	5.09	4.61	Nephelite	. . .	3.41
FeO	3.75	3.32	Diopside	2.87	2.16
MgO	1.99	1.75	Hypersthene	3.43	. . .
CaO	5.54	6.56	Olivine	.90	2.38
Na_2O	4.70	4.72	Magnetite	7.42	6.73
K_2O	3.48	3.53	Ilmenite	1.82	2.58
H_2O	1.16	1.08	Apatite	.67	1.01
P_2O_5	.31	.48	Water	1.16	1.08
	100.58	99.98		100.62	99.38

1. Trachydolerite from Hayes Hill (cf. specimen No. 2784); analysis by R. Reinisch.
2. Trachydolerite from the Landing Pier cone (cf. specimen No. 2761); analysis by R. Reinisch.

As stated, both analyses fall in the dosodic subrang andose, of the alkali-calcic rang andase, of the perfelic order germanare, of the dosalane class. The accuracy of the alumina and magnesia determinations may well be questioned.

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[7] R. Reinisch, Deutsche Südpolar-Expedition, Bd. 2, Geographie und Geologie, p. 652, 1912.

Reinisch describes basaltic trachydolerite "from two different beds at Cricket Valley." In these he found, besides phenocrystic plagioclase (labradorite to andesine), augite, and olivine, a groundmass composed of andesine, augite, olivine, and magnetite, with a cement of alkali-feldspar and anorthoclase. Mica and a barkevikitic hornblende and apatite are accessories.

He also refers to trachydolerite a lava which incloses fragments of trachyte which have been partly melted. This flow occurs nearly half way up the side of Green Mountain. Reinisch mentions trachydoleritic lapilli on The Farm, northeast slope of Green Mountain. In neither of these last two cases does he give evidence that the mafic rock is other than an ordinary basalt.

The monolithic ridge, culminating at the charted 193-foot point north of Cross Hill — a basaltic, pretrachyte product of eruptivity at the Cross Hill center — appears to carry a little alkali feldspar in the groundmass, which contains accessory biotite. The grain is so fine and the crystallization so confused that a precise determination of the alkali feldspar and of its amount have not been possible. Pending analysis this rock may be tentatively called a trachydoleritic basalt. The basaltic scoria and thin flows overlying the core trachyte of Cross Hill itself may be trachydoleritic; they have not been specially studied in thin section.

ANDESITE?

Renard, on page 58 of his report on the *Challenger* collection, mentions "certain rocks, much resembling basalts, which may be classed as andesites . . . met with in various parts of the island, particularly on Red Mountain." He referred the microlites of essential feldspar to andesine or oligoclase and the nominative mineral to bronzite. "The mineral identified as bronzite is always altered, and the decomposition shows itself by the deep red tint which clothes the section." He adds: "The red colour produced by alteration makes these little prisms resemble certain olivines, but the outlines of the sections and the elongated form of the prism do not confirm this supposition." (P. 59.) On the other hand, the present writer found in several sections of the basaltic rocks clear cases of olivine partly altered to a deep reddish-brown substance (probably iron-stained serpentine) and also fresh olivines so elongated as to simulate an orthorhombic pyroxene. To him it seems likely that the pseudomorphs described by Renard really represent olivine. Further study of this Red Mountain lava is needed. It was not sampled in 1921, and no true andesite was then elsewhere found in the island.

TRACHYANDESITE.

The lava of the fissure-eruption at Southeast Head is of andesitic habit. Specimen No. 2864 was selected for analysis. It was taken from a tongue of the ragged flow on the western slope of the Southeast Head plateau, about 200 meters from the shore of Southeast Bay. It is a dark gray, in places almost black, vesicular rock, less scoriaceous than most of the flow. It contains a few xenoliths of a labradorite-augite lava, probably an olivine-free basalt.[8] Rare glints from microphenocrysts of feldspar form the only breaks in the dense groundmass, as seen with the unaided eye. The phenocrystic feldspar is largely andesine, $Ab_{60}\,An_{30}$, with occasional external shells of oligoclase-andesine. Other, untwinned, squarish phenocrysts have a very small optical angle and the extinctions of sodiferous orthoclase. A few small, automorphic, pale greenish phenocrysts of augite appear in thin section. The microcrystalline to cryptocrystalline groundmass is an aggregate of oligoclase-andesine, orthoclase, colorless to pale greenish diopside, euhedral to dust-like ilmenite or titaniferous magnetite, a little apatite, and interstitial glass. The proportion of glass is not easy to determine; it is estimated at from 10 to 15 per cent of the rock by weight.

The analysis of No. 2864, by Dr. H. S. Washington of the Geophysical Laboratory, Carnegie Institution of Washington, gave the results shown in column 1, Table IV; the norm places it in the dosodic subrang akerose, of the domalkalic rang monzonase, of the order germanare, of the dosalane class.

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[8] The xenolithic character of this material was not recognized until it was studied in thin section; hence one reason for assuming in the field that the flow is an olivine-free basalt, as wrongly stated in the preliminary report (Geol. Mag., **59,** 149, 1922).

TABLE IV.

TRACHYANDESITE OF ASCENSION ISLAND.

	1	2		NORM OF No. 1
SiO_2	58.00	57.72	Quartz	2.76
TiO_2	3.38	.37	Orthoclase	16.68
Al_2O_3	14.92	17.64	Albite	49.78
Fe_2O_3	1.73	4.47	Anorthite	5.84
FeO	5.78	2.78	Diopside	9.98
MnO	.11	.03	Hypersthene	4.55
MgO	2.23	1.01	Magnetite	2.55
CaO	4.50	4.36	Ilmenite	6.38
Na_2O	5.88	5.50	Apatite	1.55
K_2O	2.76	3.90	Water	.40
H_2O-	.09	. . .		
H_2O+	.31	1.65		100.47
P_2O_5	.71	.57		
	100.40	100.00		
Sp. gr.	2.68	. . .		

1. Flow from fissure, Southeast Head, specimen No. 2864; analysis by H. S. Washington.
2. Average of nine trachyandesites, reduced to 100 per cent.

Column 2 of Table IV gives the average analysis of nine trachyandesites noted in Rosenbusch's "Elemente der Gesteinslehre." The Southeast Head flow seems to be best classified as a trachyandesite.

TRACHYTES.

The study of about fifty new thin sections of specimens, collected from nearly all the different bodies of trachyte, has confirmed the impression, won in the field, that the Ascension trachytes are on the whole fairly uniform in chemical composition. Prior and Reinisch have already described most of the varieties. Without exception they contain strongly dominant alkaline feldspar, commonly about 80 per cent by weight, which is regularly soda-orthoclase with variable proportions of anorthoclase. The *nominative minerals* are aegirite, diopsidic augite, riebeckite, aenigmatite (cossyrite), and other brown amphiboles. Of these aegirite is by far the commonest constituent, apparently failing only in certain trachytic projectiles found in the Green Mountain region. True, undoubted riebeckite seems to be confined to the dome-rock of Green Mountain, and to the trachytic tuffs and projectiles of that region. A closely allied amphibole in small amount characterizes the rock of "The Crags" dome. No mica has been discovered in any specimen.

This qualitative, mineralogical evidence of comparative uniformity is well matched by the chemical analyses. In addition to those published by Renard, Prior, and Reinisch, three new analyses are available. The description of the Ascension trachytes may well center around these analyses, old and new.

Trachyte of the Ragged Hill Dome. The specimen (No. 2855) collected at the top of this dome, but some meters below the initial surface, is of a fairly dark, greenish-gray color. It has the usual porosity of trachyte and is further normal in lacking bubble-vesicles of the kind common in basalts. The only phenocrysts are feldspars, from one to three millimeters in length and so abundant as to give the rock a somewhat syenitic look. Study of the thin section and of the rock-powder has showed that both soda-orthoclase, with extinction of eight degrees on (010), and anorthoclase are represented. The groundmass carries the same kinds of feldspar; considerable diopside, much aegirite, magmatically altered in large part to an opaque material resembling magnetite; deep greenish-brown to opaque material which suggests riebeckite or an allied amphibole; anhedra of magnetite, probably titaniferous. Neither quartz nor apatite was identified in the section, though both appear in the norm of the rock. No glass was seen.

The analysis of No. 2855, by Miss Vassar, gave the values shown in column 1, Table V. The analysis enters the dosodic subrang nordmarkose, of the peralkalic rang nordmarkase, of the perfelic order canadare, of the persalane class.

TABLE V.

TRACHYTES OF ASCENSION ISLAND.

	1	2	3	4	5
SiO_2	65.18	66.98	66.12	63.98	67.05
TiO_2	.44	.89	. . .	.28	.10
Al_2O_3	15.91	14.30	15.51	16.00	15.43
Fe_2O_3	4.41	3.85	3.27	2.57	3.25
FeO	.98	.33	.93	2.12	1.25
MnO	.17	.21	. . .	. . .	. . .
MgO	.10	.30	.17	.64	.16
CaO	.81	.83	1.05	1.58	1.06
Na_2O	6.24	6.76	6.31	6.45	6.12
K_2O	4.60	4.34	5.40	5.18	5.32
H_2O-	.45	.08	1.98	.61	.56
H_2O+	.53	.44			
P_2O_5	.08	.22	. . .	. . .	.04
BaO	. . .	.04	. . .	. . .	. . .
ZrO_2	. . .	.13	. . .	. . .	. . .
CO_2	.09	none	. . .	. . .	. . .
	99.99	99.70	100.74	99.41	100.34
Sp. gr.	2.64	2.54	. . .	. . .	. . .

1. Trachyte of Ragged Hill dome, specimen No. 2855; analysis by H. E. Vassar.
2. Trachyte of Southeast Head dome, specimen No. 2863; analysis by H. S. Washington. Special tests for S and Cr_2O_3 had negative results.
3. Trachyte from a point "north of Dark Slope crater"; analysis by R. Reinisch.
4. Trachyte of Cross Hill, quarry; analysis by R. Reinisch.
5. Trachyte from a point "half-way up Green Mountain"; analysis by R. Reinisch.

TABLE V.

NORMS OF TRACHYTES, Table V.

	1	2	3	4	5
Quartz	10.14	11.70	8.34	3.60	10.62
Orthoclase	27.24	25.58	31.69	30.58	31.14
Albite	52.92	49.26	49.78	53.45	49.78
Anorthite	1.67	. . .	. . .	. . .	. . .
Acmite	. . .	6.93	3.23	.92	1.39
Diopside	.43	1.73	.86	6.18	.86
Wollastonite	.12	. . .	1.74	.23	1.74
Titanite	. . .	.78	. . .	. . .	. . .
Ilmenite	.76	1.06	. . .	.61	.15
Magnetite	2.55	. . .	3.02	3.25	3.94
Hematite	2.72	1.44	. . .	. . .	. . .
Apatite	.31	.31	. . .	. . .	. . .
Zircon	. . .	.18	. . .	. . .	. . .
$CaCO_3$	.20	. . .	. . .	. . .	. . .
Water	.98	.52	1.98	.61	.56
	100.04	99.49	100.64[9]	99.43[9]	100.18[9]

Trachyte of Southeast Head. The third of the new analyses of trachyte was made from specimen No. 2863, collected at the western edge of the Southeast Head dome, about 100 meters from the shore at Southeast Bay. The rock is non-vesicular, compact, light brownish gray in color, and almost perfectly fresh. Rare glints of light betray the presence of phenocrysts of anorthoclase, generally less than one millimeter in length. The groundmass is a microcrystalline mass of soda-orthoclase, perhaps anorthoclase, micropoikilitic quartz, and colorless glass (the last about 8 per cent of the rock by weight), the whole sprinkled with a rather thin cloud of deeply colored aegirites in minute, corroded or skeletal forms, some magnetite, a few grains of colorless diopside, rare apatites, and very rare zircons. One small area of a black, opaque, moss-like mineral may be altered riebeckite.

The micropoikilitic quartz (nearly 10 per cent of the rock by weight) occurs in roundish, ragged individuals from 0.5 to 1.0 millimeter in diameter. It has precisely the same habit as the free quartz in the

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[9] The norms of Nos. 3, 4, and 5 are taken from Washington's Tables.

"silicious veins" soon to be described, and in the trachyte of Tutuila, Samoa, recently studied by the writer. Prior found it in the trachytes of Ascension, British East Africa, and the Aden peninsula.[10] Renard saw it in his specimens from Ascension, and, like Prior, doubted that the quartz originated in ordinary weathering. More probably it is of late-magmatic origin and was deposited by residual solutions of the magma after the crystallization of the other constituents.

The analysis of No. 2863, by Dr. H. S. Washington, is stated in column 2, Table V; its norm places it in the dosodic subrang kallerudose, of the peralkalic rang liparase, of the quardofelic order britannare, of the persalane class. The rock is a somewhat vitrophyric aegirite trachyte.

Riding School Trachyte. The glass-free, main part of the Riding School trachyte is a light gray, non-vesicular, minutely porous rock with rare phenocrysts of anorthoclase and soda-orthoclase up to three millimeters in length, and one-millimeter needles of aegirite. The microcrystalline to cryptocrystalline groundmass is essentially made up of the same minerals, together with a little aegirite-augite. A specimen from the northeastern overflow is free from quartz, but another, broken from the edge or rim of the basined dome itself, carries from five to ten per cent of micropoikilitic quartz by weight. A third thin section, from a typical "silicious vein" contains at least fifteen per cent of the micropoikilitic quartz. (See a later section on silicified interfaces.)

Reinisch (p. 649 of his paper) gives an analysis of a glass-rich diopside trachyte, collected at the northern foot of Dark Slope cone. At this locality the present writer found no trachyte in place and the specimen in question may be a transported fragment of the Riding School trachyte. Its analysis is quoted in column 3, Table V.

Reinisch found 68.31 per cent of silica and 2.47 per cent of water in a specimen of an "Alkali-Trachyt-Perlit," collected in the stream course north of the Riding School. He also gives analyses of a trachytic pumice from the foot of the Riding School cone, north side, and of a seal-red trachytic tuff from the southwest corner of the Riding School. These are quoted in Table VI.

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[10] G. T. Prior, Miner. Mag., **13**, 242, 255, 257, 259, 1903.

TABLE VI.

PUMICE AND TRACHYTIC TUFF FROM THE RIDING SCHOOL.

	1	2
SiO_2	63.02	55.10
Al_2O_3	15.75	18.56
Fe_2O_3	.52	6.80
FeO	3.15	.03
MgO	.38	.62
CaO	1.49	.70
Na_2O	6.11	3.17
K_2O	5.21	4.00
H_2O	4.83	8.30
Sol. in water	. . .	3.17
	100.46	100.45

Trachytes of the "Drip" Dome and the Mass at the Northwest Base of Thistle Hill. These are aegirite trachytes, hardly to be distinguished from the Riding School rock. The microphenocrysts include anorthoclase and rare aegirites. The groundmass carries essential soda-orthoclase, aegirite, and a diopsidic pyroxene. Quartz fails in the three sections made from the trachyte of the "Drip" dome; the Thistle Hill mass has a little micropoikilitic quartz. As usual the specially fine-grained phases are more sonorous under the hammer than the coarser phases, and even rival the "silicious veins" of the Riding School and other bodies, in this respect.

Trachyte of "The Crags" Dome. Beneath the pale brown, weathered shell, the fresh rock of "The Crags" dome is gray to fairly dark greenish-gray in color. It shows occasional phenocrysts of anorthoclase and soda-orthoclase, reaching maximum lengths of about four millimeters. The groundmass is essentially a microcrystalline aggregate of soda-orthoclase and aegirite, with subordinate diopsidic augite, magnetite, and much green to dark bluish gray, highly pleochroic amphibole of moss-like appearance, like that of riebeckite.

Trachyte of Cross Hill. This rock has a striking variety of color. At the quarry cut in the base of the western slope of the cone, the trachyte is a very pale gray, almost white. The same color characterizes the outcrop 800 meters to the north and also much of the dome-rock on the northeastern slope of the hill. Light brownish and greenish phases are, however, common in the rock of the dome itself; on the path from Bates Cottage to the Wireless Station the tint is a decidedly dark greenish gray. The cause of this variation is not apparent; it can hardly be referred to weathering, since it is found in rock which is almost perfectly fresh.

The whitish rock of the quarry is speckled with very minute, dust-like grains of amphibole and ragged needles of aegirite-augite and aegirite. These can be distinguished only under the microscope, which also shows microphenocrysts of soda-orthoclase, anorthoclase, and rare individuals of a twinned feldspar, probably oligoclase-albite. The groundmass contains soda-orthoclase, possibly some anorthoclase, aegirite-augite, micropoikilitic quartz, and magnetite.

The very ragged, embayed amphibole has the pleochroic scheme: **a**, light yellowish green; **b**, deep brown to black; **c**, dark olive green. The extinction **c** : *c*, on the clinopinacoid is about 18 degrees; that on cleavage pieces is about 17 degrees. The optical angle is large. This amphibole thus resembles one described by Osann as occurring in the sanidinite of San Miguel.[11] It seems not to be the catoforite, reported by Reinisch from the quarry rock. Aenigmatite was not demonstrable by the present writer; Reinisch states it to occur in small amount.

Reinisch made an analysis of the quarry rock. His result is quoted in column 4, Table V. The norm of the analysis places it in the subrang nordmarkose, where the two analyses of the Riding School trachyte also fall.

The darker phase of the Cross Hill trachyte lacks essential amphibole; its groundmass bears a few deeply colored anhedra of an obscure brown mineral which may be aenigmatite.

Trachytes of Green Mountain. At least two types of trachytes are represented in Green Mountain: one with aegirite as the only essential mafic constituent; the other containing both aegirite and alkaline amphibole.

The aegirite trachyte, of the usual light gray to whitish tints, composes a thick flow outcropping on the road up the mountain at the 1400-foot contour. A similar variety composes the craggy mass, probably also an overflow from the central dome, the exposed part

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[11] See the Rosenbusch-Wülfing "Mikroskopische Physiographie," **1**, part 2, 237, 1905.

of which extends from the sanatorium to, and including, Monkey Rock north of the summit. These trachytes are free from quartz.

Amphibole-bearing phases seem to be quite abundant in the mountain. One of these, forming at the Valley Tank the trachytic rim of the caldera in which The Peak tuff-cone was built, has been specially studied. (See Plate XIV, *B*.) It is light gray and compact, with rare and small phenocrysts of soda-orthoclase and anorthoclase. The groundmass contains the same feldspars along with aegirite, aegirite-augite, and considerable riebeckite. Cossyrite (aenigmatite) was not identified.

The overflow exposed in a cliff-section 300 meters south-southwest of The Farm buildings has phenocrysts of soda-orthoclase, anorthoclase, and rare aegirites, in a groundmass containing, in addition, riebeckite, a diopside-like augite, many grains of an amphibole which may be aenigmatite, and micropoikilitic quartz. A dark brown to opaque, moss-like amphibole is probably magmatically altered riebeckite.

Reinisch reports aenigmatite-arfvedsonite trachyte from Green Mountain, taken at the level of the sanatorium; and again from the watercourse north of Donkeys Plain. He gives an analysis of a "catoforite-aenigmatite trachyte" collected half way up Green Mountain. See column 5, Table V. The analysis falls in the dosodic subrang, nordmarkose.

Large and small fragments of riebeckite trachyte are tolerably abundant in the breccias crossed by the road up Green Mountain and in the breccias on Middleton Peak ridge, a spur of the mountain. Many others are strewn over the floor of the valley followed by the Green Mountain road, westward and northwestward from Travellers Hill; many of these have been moved down the valley by freshets, but some, if not most of them, are projectiles from the explosive center of Green Mountain.

Three of the thin sections cut from these projectiles appear to contain, besides riebeckite, at least three other varieties of amphibole. All three have the same ragged, moss-like habit, as well as sensibly the single and double refraction, of the adjacent riebeckite. One of the varieties is strongly pleochroic: **a**, deep blue green; **b**, deep blue green to opaque; **c**, gray green. The extinction angle *c* : **c** is nearly zero. This mineral approximates true riebeckite. A second variety is highly pleochroic in brownish green and very deep brown colors. A third is nearly colorless, with a greenish cast. In long sections its extinction is nearly parallel to the cleavage, which is poorly developed in all these amphiboles. The nearly colorless variety occurs isolated and also is seen to merge into the green amphibole. The former has all the appearance of being an iron-poor or iron-free analogue of riebeckite, as if it might be the chemical equivalent of the jadeite molecule among the sodic pyroxenes. With the material in hand this speculative suggestion cannot be properly tested.

The whole group of moss-amphiboles has the corroded look of minerals which have been subject to the reactions due to residual water-gas or other fluids of the late magmatic period. All of the dark-colored varieties, probably including many types intermediate between those described, have apparently been acted upon by water vapor or water gas, which has removed silica and concentrated iron oxides, rendering the remaining solid more or less opaque. This change parallels that so often observed in the case of aegirite.

Reinisch reports aenigmatite in all his specimens of amphibole trachyte. The present writer has had such difficulty in proving this mineral in the dense specimens of his own collection that he hesitates to list it in several instances where the mineral is suspected.

QUARTZ TRACHYTES.

Quartz Trachyte of White Hill Dome. Another mass from which no analysis has hitherto been reported is the great White Hill dome. A specimen (No. 2861) from the lower end of one of its outflows (Plate XVII) is highly fluidal or platy, with an alternation of thin, pale bluish gray and nearly white layers. This eutaxitic structure is connected with the varying degree of crystallization of the original glass. To the naked eye the rock is aphanitic throughout; the microscope shows microphenocrysts of soda-orthoclase. For the rest the rock is a layered composite of colorless glass, charged with minute, ragged needles of aegirite and skeleton-crystals of alkaline feldspar, chiefly soda-orthoclase. The only other crystallized minerals are magnetite, in minute specks, and about ten per cent of micropoikilitic quartz.

The analysis of No. 2861, by Mr. E. G. Radley, is given in column 1, Table VII. The analysis falls in the sodipotassic subrang liparose, of the peralkalic rang liparase, of the order britannare, of the persalane class.

Its chemical composition places the rock among the rhyolites. However, it is probable that the silica percentage has been increased by infiltration during the late magmatic period, and that the magma was more nearly a typical trachyte than a typical rhyolite. To indicate the very close relation between this White Hill rock with the true trachytes of the island, the former may be called a quartz trachyte.

Trachytes of Little White Hill and Wig Hill. These have not been microscopically studied. In the field they appeared closely similar to the White Hill rock.

TABLE VII.

QUARTZ TRACHYTES (RHYOLITES) OF ASCENSION ISLAND.

				NORMS	
	1	2		1	2
SiO_2	71.88	70.99	Quartz	23.40	25.86
TiO_2	.25	. . .	Orthoclase	28.36	13.90
Al_2O_3	12.85	14.84	Albite	38.78	50.30
Fe_2O_3	3.60	3.76	Anorthite	. . .	3.01
FeO	.05	.35	Corundum	. . .	1.33
MnO	.29	tr.	Acmite	5.08	. . .
MgO	.18	.14	Diopside	.86	. . .
CaO	.60	.60	Hypersthene	. . .	.40
Na_2O	5.32	5.94	$CaSiO_4$	.70	. . .
K_2O	4.78	2.40	Ilemite	.46	. . .
H_2O-	.18	.40	Magnetite	.46	1.16
H_2O+	.17		Hematite	1.66	3.04
P_2O_5	.05	. . .	Apatite	.09	. . .
	———	———	Water	.35	.40
	100.20	99.42		———	———
Sp. gr.	2.58	. . .		100.20	99.40

1. Quartz trachyte, outflow from White Hill dome, specimen No. 2861; analysis by E. G. Radley.
2. "Augite trachyte" from Weather Post (reported by A. Renard, Petrology of Oceanic Islands, *Challenger* Reports, p. 52, 1889); analysis by Klement.

Weather Post-Devils Cauldron and Boatswain Bird Islet Trachytes. The trachyte continuously exposed from the top of Weather Post to the lower end of the great flow north of the Devils Cauldron seems to be a very homogeneous aegirite (-diopside) trachyte, bearing in the groundmass numerous ragged grains and short needles of black, opaque material which may represent magmatically altered cossyrite as well as aegirite. The feldspars of phenocrysts and groundmass alike are soda-orthoclase with less important anorthoclase. Quartz does not appear in the three thin sections available, one from the northern flow and two from the rim of the Cauldron. The diopside phenocrysts are rare and bear thick mantles of aegirite.

Renard (page 47 of his paper) gives an analysis of "augite trachyte" from Weather Post, column 2, Table VII. He explains the high silica by the infiltration of quartz, which is "probably of secondary origin." Taken as it stands, the analysis is that of the dosodic subrang, kallerudose, of the peralkalic rang, liparase, of the quardofelic order, britannare, of the persalane class.

The monolith of Boatswain Bird islet is an aegirite-diopside trachyte, poor in diopside, which is confined to the groundmass. Micropoikilitic quartz is rather abundant in the one thin section made from this rock; a little moss-like, black material may represent altered riebeckite or a closely allied variety of amphibole.

SILICIFIED INTERFACES ("SILICIOUS VEINS") IN THE TRACHYTES.

The weathered surface of practically every body of trachyte in the island is locally roughened with prominent, intersecting, rib-like ridges (Plates VI and XIII). They are spaced at intervals varying from a few centimeters to one or more meters. Intersections occur at intervals of the same orders of magnitude. Where vertical, the ribs usually project some centimeters or a few decimeters. Those more nearly parallel to the general surface of the ledge often form hard caps, standing on round, more or less slender necks composed of the weaker, more normal trachyte. The weathering of each composite may be described as carious, on a large scale.

Darwin's attention was actively drawn to the problem of these projecting ribs, which he called veins. His account of them is excellent and worth quoting: "They contain crystals of glassy feldspar, black microscopical specks and little dark stains, precisely as in the surrounding rock; but the basis is very different, being exceedingly hard, compact, somewhat brittle, and of rather less easy fusibility. The veins vary much, and suddenly, from the tenth of an inch to one inch in thickness; they often thin out, not only on their edges, but in their central parts, thus leaving round, irregular apertures; their surfaces are rugged. They are inclined at every possible angle with the horizon, or are horizontal; they are generally curvilinear, and often interbranch one with another From their hardness they withstand weathering, and projecting two or three feet above the ground, they occasionally extend some yards in length; these plate-like veins, when struck, emit a sound, almost like that of a drum, and they may be distinctly seen to vibrate; their fragments, which are strewed on the ground, clatter like pieces of iron when knocked against each other. They often assume the most singular forms; I saw a pedestal of the earthy trachyte, covered by a hemispherical portion of a vein, like a great umbrella, sufficiently large to shelter two persons. I have never met with, or seen described, any veins like these; but in form they resemble the ferruginous seams, due to some process of segregation, occurring not uncommonly in sandstones — for instance, in the New Red sandstone of England. Numerous veins of jasper and of siliceous sinter, occurring on the summit of this same hill, show that there has been some abundant source of silica, and as these plate-like veins differ from the trachyte only in their greater hardness, brittleness, and less easy fusibility, it appears probable that their origin is due to the segregation or infiltration of silicious matter, in the same manner as happens with the oxides of iron in many sedimentary rocks."[12]

Largely because of its sonority when struck with a hammer, Renard (page 60 of his paper) assumed that one of his specimens, collected by Dr. Maclean of the *Challenger* Expedition staff, and labelled "piece of clinkers," represented one of these "veins." The locality of the specimen, Southwest Bay, and its mineralogical composition show, however, that this is a normal, clinkery phase of the basaltic lava and has nothing to do with the hard "veins" in the trachytes.

Thin sections of the "veins" show, in fact, that Darwin was right in attributing them to the local silicification of the normal trachyte. The introduced silica is always quartz in micropoikilitic form, exactly like that seen in some of the more normal trachytes. The proportion of quartz by weight in the "veins" has been roughly estimated as from 15 to 25 per cent, or from five to ten or more times the amount found in the rest of the rock mass.

The free silica has not been introduced during the simple weathering of the trachyte; its formation is most probably to be ascribed to late-magmatic action. After the effluent trachyte came to rest, its glass cooled and crystallized, with the generation of tensions analogous to those causing columnar jointing in lavas. The magmatic steam escaped, preferably along the actual or potential partings so developed, and from the volatile solution the quartz was precipitated. A little hematite seems to have been simultaneously formed, giving the characteristic reddish tint, often seen on the "veins." The silicified sheets are not true veins; they are merely parts of the trachyte modified by fluids, presumably gaseous solutions, migrating along interfaces of the trachyte. Any fissures along the interfaces seem seldom to have had widths greater than a fraction of a millimeter.

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[12] C. Darwin, Geological Observations, pp. 51–52, 2d ed., London, 1876.

It is not unreasonable to assume that the gaseous solutions responsible for the silicification described were the same as those which have so notably affected the pyroxenes and amphiboles of the normal trachytes. The conversion of much or all of the aegirite or riebeckite into pseudomorphs of iron oxides and other substances has clearly been accompanied by the leaching-out of silica from the original minerals; its late deposition, as micropoikilitic quartz, at some distance in the rock might readily be expected.

OBSIDIANS.

The obsidians of Ascension occur as projectiles and as chilled, surface and floor phases of the trachytic flows. Independent bodies of glass of large size, either extrusive or intrusive, were not found. Whether in place or in the form of projectiles, the observed, non-pumiceous glass is confined to the region occupied by the greater domes of trachyte, including the Riding School, Green Mountain, Weather Post-Devils Cauldron, and White Hill. The projectiles were seen to be particularly abundant in the trachytic breccias and tuffs underlying the trachytic flows The largest individual bodies of glass are those constituting parts of the floor phases of the flows. These phases are composed of alternating glassy and lithoidal layers in the usual eutaxitic combination, obviously a flow-structure.

In his "Geological Observations" Darwin devoted half of his chapter on Ascension Island to a minute description of the massive, laminated, and spherulitic obsidians. His masterly account is accessible to every reader and need not be repeated.

The massive obsidian is lustrous black to greenish black by reflected light, and dark green by light transmitted through splinters. In every thin section the glass is charged with many slender, often line-like needles of diopsidic, green augite, with which, in some sections much rarer, small microphenocrysts of soda-orthoclase are associated.

Renard has quoted the essential facts from Darwin's description and has supplied the analysis of a Green Mountain specimen, given in column 1 of Table VIII. Column 2 gives an analysis, by Reinisch, of a "Rhyolithobsidian," collected in the stream-course north of the Riding School. Columns 3 and 4 give, respectively, Reinisch's determinations of silica and water in a spherulitic "Rhyolithobsidian" from the outer mantle of the Riding School, and in "Obsidianknollen" collected on the "Kegelmantel" of Green Mountain.

TABLE VIII.

	1	2	3	4
SiO_2	72.71	71.42	69.70	65.59
Al_2O_3	12.80	14.09	...	...
Fe_2O_3	2.64	1.41	...	...
FeO	1.48	2.32	...	...
MnO	Tr.	...	...	...
MgO	0.10	0.08	...	...
CaO	0.58	0.80	...	...
Na_2O	6.50	6.01	...	...
K_2O	3.87	3.52	...	...
H_2O	0.48	0.85	0.94	0.87
	101.16	100.50		

Washington's Tables states the norms of 1 and 2, as follows:

	1	2
Quartz	21.54	20.58
Orthoclase	22.80	20.57
Albite	44.02	50.83
Anorthite	...	1.11
Acmite	7.39	...
Sodium metasilicate	0.61	...
Diopside	2.45	2.45
Hypersthene	1.78	1.95
Magnetite	...	2.09
Water	0.48	0.85
	101.07	100.43

Analyses 1 and 2 both fall in the dosodic subrang, kallerudose.

The specific gravity of a 20-centimeter, non-vesicular projectile, composed of very homogeneous black obsidian, containing about two per cent of augite needles, was found to be 2.415 at 20° C. Tilley obtained the density (grams per cubic centimeter) of 2.435 for a specimen of Ascension obsidian, locality being unspecified.[13]

XENOLITHS AND PROJECTILES OF PLUTONIC-ROCK TYPES.

Even before the year 1828 members of the naval garrison of Ascension had observed pieces of granite among the volcanic projectiles of the island. The first published account of these seems to be due to the surgeon, Webster. On page 316 of his entertaining book occurs the following passage: "There are various specimens of fragments of granite, and other primitive rocks, found scattered indiscriminately among the lava, or lying in the beds of cinders, and they bear more or less the marks of the action of fire. Some of the species of granite are very perfect and complete; others are semi-calcined and brittle. The quartz rock appears converted into a mass like red sandstone. Argillaceous schist, and graywacke, and syenite are likewise found."[14] Neither schist nor graywacke were discovered by the present writer nor, apparently, by any other visitor to the island since Webster's time; hence it seems likely that Webster mistook certain tuffs for the rocks in question.

Darwin's later account of the granitic fragments is readily accessible in his "Geological Observations"; it is summarized, and new data resulting from microscopic study have been added, by Renard in his paper, already quoted.

Darwin collected his granitic specimens "in the neighbourhood of Green Mountain." Granitic projectiles are relatively abundant in the basaltic tuffs crossed by the switch-back road to the summit, between the 1350-foot and 1500-foot contours; that is, below the 5-mile post on the road. Artificial cuttings furnish good exposures of these tuffs, from which the present writer took a number of angular to subangular, coarsely granular fragments, mixed with larger blocks of porphyritic trachyte. The former range from two to ten centimeters in diameter.

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[13] C. E. Tilley, Mineralog. Mag., **19**, 275, 1922.

[14] W. H. B. Webster, Narrative of a Voyage to the Southern Atlantic Ocean in the years 1828, 1829, 1830, performed in H. M. Sloop *Chanticleer*, **2**, 316, London, 1834.

One specimen is light gray to white, speckled with many small lustrous black amphiboles. The grain is medium to fine. Under the microscope the rock is seen to be made up of microperthite, soda-orthoclase (a little oligoclase in one thin section), quartz, and a highly pleochroic amphibole. Magnetite, apatite, and rare zircons are accessories. The quartz occurs as isolated, interstitial grains and also to a considerable extent in micrographic and also poikilitic relations to the feldspars. The amphibole has enormous absorption, with the following scheme of colors for sections of the usual thickness: **a**, yellowish brown; **b**, very deep brown to opaque black; **c**, deep brown. $\mathbf{b} > \mathbf{c} > \mathbf{a}$. The colors and double refraction recall aenigmatite, but the extinctions measured on cleavage plates (110) are never more than two degrees. The absorption scheme and the abnormal extinction angle may possibly be connected with the reheating and oxidation of a normal hornblende after the granite became immersed in the basaltic magma in depth. C. Schneider and M. Belowsky have induced both kinds of change in iron-rich amphibole by artificial heating[15]

At the same locality other specimens of hornblende granite, pinker in color and less charged with micropegmatite, were also found.

On the trail which contours The Peak on the north side, specimens of augite-hornblende-quartz syenite, olivine-poor gabbro, and typical augite-biotite diorite were collected from the basaltic tuff of The Peak. The quartz syenite carries a small amount of oligoclase; like all the other syenites, it is nearly a quartz-poor equivalent of the granite above described.

The trachytic tuffs of Green Mountain likewise yielded many angular fragments of granular rocks, especially on the southern slope of the Middleton Peak ridge. The largest fragment collected measures about 10 centimeters in diameter. Microscopic examination showed representatives of the following types:

1. Alkaline hornblende-biotite granite; light pinkish gray, medium-grained, rich in microperthite.
2. Brownish to pinkish gray, strongly miarolitic hornblende syenite with accessory quartz — a quartz syenite.
3. Pinkish gray augite-hornblende-quartz syenite, transitional to granite.

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[15] See H. Rosenbusch and E. A. Wülfing, Mikroskopische Physiographie der petrographisch wichtigen Mineralien, 4te. Auflage, Bd. 1, 2te Hälfte, p. 234, Stuttgart, 1905.

4. Light greenish gray, medium-grained, salic diorite, transitional to monzonite; altered; probably carried much original augite and biotite, now represented by uralite and chlorite.
5. Light brownish gray, mottled, sugary, miarolitic augite-hornblende diorite.
6. Typical olivine gabbro.
7. Typical olivine-free gabbro.
8. An unusual olivine-free gabbro, made up of highly automorphic augite and labradorite, embedded in an opaque black cement, which is almost unaffected by a magnet and is probably ilmenite.
9. Typical coarse-grained wehrlite.

Granitic boulders of the kinds above described occur in the gravel streams of the main valleys north of Green Mountain.

In the massive trachyte of "The Crags" dome a single, angular inclusion of granite was found. It measured five centimeters in greatest diameter. This rock closely resembles the alkaline hornblende granite from the basaltic tuffs of Green Mountain. The feldspars are microperthite and orthoclase, the latter being surrounded by thick shells of soda-orthoclase. The amphibole is apparently identical with the abnormal variety, just described, in the projectile from the basaltic tuffs of Green Mountain. Brown biotite is a rare accessory in the xenolith.

Many angular and subangular xenoliths of gabbro were discovered in the scoriaceous flows of basalt which issued from Dark Slope crater. These fragments of plutonic character range from a few centimeters to 50 centimeters or more in diameter. All are dark gray to deep green-gray, medium-grained to rather coarse-grained even for gabbros, and more or less friable. This weakness of the material is not due to weathering, but doubtless to the loosening of the original fabric by the second heating. Probably more than half the xenoliths seen are typical olivine gabbros. Also numerous are olivine-free gabbros, which differ among themselves, some bearing essential diallage, others a diopsidic augite without the diallagic parting, and still others containing both kinds of pyroxene. Throughout the series the feldspar averages close to bytownite, $Ab_{25} An_{75}$, with but small variations from the mean.

The specimens from the Dark Slope cone were collected in the failing light of the dusk. Several very friable inclusions were then, under the poor conditions for observation, thought to be granitic. That conclusion was stated in the writer's preliminary account of his visit to the island.[16] When later the rock collection was unpacked, it was found that no quartz-bearing specimen from Dark Slope was included. Hence it is now doubtful that granitic xenoliths actually occur in these basaltic flows.

The whole list of granular projectiles and xenoliths in the writer's collection thus includes:

1. Alkaline amphibole granite; both with and without micropegmatite.
2. Alkaline hornblende-biotite granite.
3. Pyroxene-hornblende-quartz syenite.
4. Hornblende syenite, verging on quartz syenite.
5. Monzonitic diorite.
6. Typical augite-biotite diorite.
7. Typical augite-hornblende diorite.
8. Augite gabbro.
9. Abnormal augite gabbro with ilmenitic mesostasis (free from olivine).
10. Typical olivine gabbro.
11. Typical olivine-free gabbro.
12. Typical wehrlite (peridotite).

Renard describes the following additional types among the projectiles studied by him:

13. Biotite granite, bearing some micropegmatite.
14. Biotite-bearing "diabase."
15. Enstatite-bearing, olivine-free gabbro.

All of the salic fragments are fritted and brittle, and some of them carry small, irregular ribbons and droplets of brown glass, showing incipient fusion. In most cases the essential minerals are murky with fluid and glassy inclusions; cleavages are unusually well developed, probably because of the reheating.

If any of the granular rocks were notably strained, or gneissic through metamorphism in the solid state, one could be reasonably certain that those fragments were derived from an older terrane underlying the great cone of Ascension, at a depth of 2500 or more meters below sea-level. However, the visible evidences of strain — undulose extinction and moderate fracturing of the minerals, especially quartz — are of the kinds expected because of heating in the volcanic vents. Hence it is not easy to decide the question whether or not all of the granular rocks are deep-seated phases of the magma represented in the exposed part of the Ascension cone. The gabbroid, dioritic, and wehrlitic xenoliths and projectiles are most readily explained as plutonic phases or differentiates of that magma. The problem of the more salic fragments is more widely open.

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[16] See Geol. Mag., **59**, 149, 1922.

The granites and syenites are mineralogically and chemically allied, all of them being of alkaline types and usually rich in microperthite or soda-bearing potash feldspar. Their consanguinity is further suggested by an essential mineral in common, hornblende, and by the occurrence of transitional varieties, the quartz syenites. Probably, therefore, all these salic fragments originated in a single mass, or a number of syngenetic masses, of plutonic rock. All of them are chemically and mineralogically quite different from the trachytes and quartz-trachytes (rhyolites) of the island, and it is hard to believe that the granites and syenites are merely deep-seated differentiates, syngenetic with those salic phases of the surface lavas. Though many coarse-grained fragments of aegirite-bearing, porphyritic trachyte, approaching syenite porphyry, accompany the granular fragments, not one represents a transition between the true syenite and the trachyte. On the whole it seems most probable that the granitic and syenitic fragments were derived from an older terrane on which the Ascension cone rests.

If the granite is a "continental rock," as it appears to be, the early discovery of these quartz-bearing projectiles at Green Mountain, corroborated by Darwin and later visitors to the island, is obviously important. That discovery has been thought to support the views of those who believe the Atlantic basin was formed by the foundering of a vast block of a primitive continent. On the other hand, if granitic, "continental" terranes enter into the composition of the mid-Atlantic swell, on which Ascension is built, the swell itself might with as much plausibility be regarded as the zone of parting and crustal readjustment when America and Eurasia-Africa slid away from each other. Truly this newer explanation of the Atlantic basin faces difficulties, but they seem to be no more portentous than those facing the older hypothesis, which, for example, has never been brought into agreement with the principle of isostasy.

Quite recently Lacroix has published an account of the discovery of biotite granite, quartz-aegirite syenite, and nephelite syenite, occurring as pebbles and larger detached blocks on Kerguelen Island.[17] This island, dominantly basaltic but bearing some phonolite, is situated on the site of Gondwanaland in the open Indian ocean — a position analogous to that of Ascension in relation to the former land connection between Africa and South America. The lavas of Kerguelen, in which no quartz-bearing phases have been described, appear to be chemically more like those of Saint Helena than the lavas of Ascension, but the parallel between Ascension and Kerguelen is sufficiently close to suggest that the origin of the quartzose, granular rocks is in each case connected with those events that have led to the formation of a relatively young ocean basin.

Order of Eruption.

In many instances the trachytic bodies have been proved to rest upon, or to have penetrated, older rocks of basaltic habit. That this is a perfectly general relation is reasonably inferred from: (1) the stratigraphic facts ascertained at Green Mountain, Riding School, the southeast end of Sisters Peak group of cones, Weather Post, White Hill, and Little White Hill; and (2) from the presence of basaltic xenoliths in the trachytes of Cross Hill, Riding School, Green Mountain, White Hill, Devils Cauldron, the "Drip" dome, and Ragged Hill.

On the other hand, the trachytes are overlain by, and thus older than, some of the rocks of basaltic habit. Among the specially clear examples are those at Green Mountain (The Peak basaltic tuff built up on a trachytic wreck), Cross Hill (basaltic tuff and flows resting on a trachytic dome), the "Drip" dome (largely covered by basaltic tuff and lapilli), and Ragged Hill (dome flooded at its base by basalt flows). The young basaltic tuffs of Cross Hill, The Peak, and Green Mountain in general carry fragments of trachyte.

All of the visible bodies of trachyte were obviously erupted at a very late stage of the growth of the Ascension composite cone, of which at least 95 per cent is below sea-level. The absolute lengths of time separating the trachytic eruptions cannot be very great. Possibly, indeed, the visible trachytes were all generated during one, comparatively brief period in the history of the Ascension magma. Their differentiation may thus have been quite contemporaneous, though the actual eruptions may not have been contemporaneous. Definite proof of difference of age among the trachytes could not be found except in the fact that some of the domes appear to be more weathered than others. Allowing for this, one may still be justified in assuming that there may have been a distinct trachytic period in the evolution of the Ascension composite. The small amount of rhyolitic magma demonstrated in the island is clearly contemporaneous with the accompanying and much more voluminous trachytes.

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[17] A. Lacroix, Comptes Rendus, **179**, 113, 1924.

As already remarked, the xenoliths and tuff-fragments of plutonic rocks are doubtless of different ages. The gabbro and peridotitic xenoliths of the Dark Slope flows are most simply regarded as relatively young, deep-seated phases of the basaltic magma which has built up so great a part of the Ascension cone. The granitic fragments of the tuffs are considered to be derivatives from the basement on which that cone rests, and the same may be assumed with some probability in the case of the syenitic fragments, though one cannot definitely exclude the hypothesis that some of these represent deep-seated phases of the young trachytic magma.

The sequence of the volcanic rocks in Ascension is like that registered in many other regions where alkaline trachytes have been found. It is worth while to review some examples.

1. Glangeaud's notable monograph on the puys of the Auvergne conveniently summarizes the observations of many masters, including his own observations, on those celebrated volcanoes. In many respects the classic French group is like the assemblage of vents and masses in Ascension. There too basalt flows antedate the trachytic eruptions. At each of four of the puys extrusion of trachytic lava was followed by extrusion of younger basaltic lava. At the Puy des Gouttes pyroclastic beds of trachytic and basaltic lava material alternate.[18]

2. An analogy is seen in the Velay, where the trachyte of Queyrières forms flows between older basalt and younger, black andesite.[19]

3. In the Cantal the order of eruption is from basalt, through trachyte, to andesite, and, finally, flooding basalt.[20]

4. According to Harker the effusive trachyte of Skye has a stratigraphic position in the midst of the great Tertiary series of basalts of that island.[21]

5. In the German Westerwald the sequence, from oldest to youngest is: basalt, trachyte, andesite, basalt.[22]

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[18] P. Glangeaud, Bull. 135, Service carte géol. de la France, pp. 47–48, 1913.
[19] P. Termier, Bull. 13, Service carte géol. de la France, p. 6, 1890.
[20] M. Boule, Livret-guide, VIIIe Congrès géol. internat., part 10, Paris, 1900.
[21] A. Harker, Tertiary Igneous Rocks of Skye, pp. 56–57, Glasgow, 1904.
[22] G. Angelbis, Jahrb. preuss. geol. Landesanst., **3**, xlv, 1883.

6. According to Holmes the Tertiary lavas of the Sanhuti district of Mozambique were erupted in the following order: ordinary basalt, trachytoid phonolite, picritic basalt.[23] The sequence is thus analogous with the other cases.

7. The very extensive eruptions of Somali Land occur in the following order: oldest, porphyritic basalts; then pantellerites and soda-rich rhyolites; youngest, doleritic basalts.[24]

8. In central Madagascar eruptions of basalt precede and follow those of trachyte and phonolitic trachyte.[25]

9. The Cenozoic lavas of the state of Victoria, Australia, were erupted in the order: basalt, trachyte, basalt.[26]

10. Süssmilch states that the late-Tertiary phonolites, alkaline trachytes and allied types of New South Wales were preceded and followed by flows of normal basalt.[27]

Petrology.

Average Composition of the Types of Lava; some Comparisons. For convenience in discussing the genetic relations of the Ascension rocks the available chemical analyses have been assembled in Tables IX and X.

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[23] A. Holmes, Quart. Jour. Geol. Soc., **72**, 231, 1917.

[24] H. Arsandaux, L'étude des roches alcalines de l'Est-Africain, p. 16, Paris, 1906.

[25] A. Lacroix, Comptes Rendus, Acad. des Sciences, **154**, 315–316, 476, 1912.

[26] E. W. Skeats, Presidential Address, Australian Assoc. Adv. Science, **12**, 173, 1909.

[27] C. A. Süssmilch, Presidential Address, Jour. Roy. Soc. New South Wales, **57**, 36, 1923.

TABLE IX.

ANALYSES OF FEMIC ROCKS OF ASCENSION ISLAND.

	1	2	3	4	5	6
SiO_2	47.69	48.64	52.87	51.18	54.04	58.00
TiO_2	2.79	3.52	2.01	1.34	0.94	3.38
Al_2O_3	16.23	15.54	16.68	21.41	19.58	14.92
Fe_2O_3	2.20	5.31	4.54	4.61	5.09	1.73
FeO	9.93	7.73	4.79	3.32	3.75	5.78
MnO	0.17	0.17	0.37	. .	. . .	0.11
MgO	7.15	4.96	3.92	1.75	1.99	2.23
CaO	10.02	9.03	7.32	6.56	5.54	4.50
Na_2O	2.87	3.60	4.63	4.72	4.70	5.88
K_2O	0.64	1.24	2.06	3.53	3.48	2.76
H_2O-	0.09	0.16	0.40	1.08	1.16	0.09
H_2O+	0.19	0.18	0.30			0.31
P_2O_5	0.59	0.64	0.52	0.48	0.31	0.71
CO_2	0.04	0.03	. . .	. . .	. . .	. . .
	100.60	100.75	100.41	99.98	100.58	100.40
Sp. gr.	2.99	2.97	2.84	(2.58)[28]	(2.66)[28]	2.68

1. Olivine basalt, flow of the Southwest Bay group (Vassar, analyst).
2. Olivine basalt, Cricket Valley (Vassar, analyst).
3. Trachydoleritic basalt, South Red Crater (Radley, analyst).
4. Trachydolerite, cone at Landing Pier, Georgetown (Reinisch, analyst).
5. Trachydolerite, Hayes Hill (Reinisch, analyst).
6. Trachyandesite, fissure-eruption at Southeast Head (Washington, analyst).

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[28] Determined from specimens collected by the writer and not from those analyzed.

TABLE X.

ANALYSES OF SALIC ROCKS OF ASCENSION ISLAND.

	1	2	3	4	5	6	7	8	9	10	11
SiO_2	65.18	66.98	63.98	67.05	66.12	71.88	70.99	63.02	68.31	55.10	72.71
TiO_2	0.44	0.89	0.28	0.10	...	0.25	...	...	...	...	...
Al_2O_3	15.91	14.30	16.00	15.43	15.51	12.85	14.84	15.75	...	18.56	12.80
Fe_2O_3	4.41	3.85	2.57	3.25	3.27	3.60	3.76	0.52	...	6.80	2.64
FeO	0.98	0.33	2.12	1.25	0.93	0.05	0.35	3.15	...	0.03	1.48
MnO	0.17	0.21	...	...	...	0.29	Tr.	...	...	...	Tr.
MgO	0.10	0.30	0.64	0.16	0.17	0.18	0.14	0.38	...	0.62	0.10
CaO	0.81	0.83	1.58	1.06	1.05	0.60	0.60	1.49	...	0.70	0.58
Na_2O	6.24	6.76	6.45	6.12	6.31	5.32	5.94	6.11	...	3.17	6.50
K_2O	4.60	4.34	5.18	5.32	5.40	4.78	2.40	5.21	...	4.00	3.87
H_2O-	0.45	0.08	0.61	0.56	1.98	0.18	0.40	4.83	2.47	8.30	0.48
H_2O+	0.53	0.44				0.17					
CO_2	0.09	...	...	...	...	...	...	...	...	...	...
P_2O_5	0.08	0.22	...	0.04	...	0.05	...	...	...	...	...
BaO	...	0.04	...	...	...	...	...	...	...	...	...
ZrO_2	...	0.13	...	...	...	...	...	...	...	...	...
	99.99	99.70	99.41	100.34	100.74	100.20	99.42	100.46		100.45[29]	101.16
Sp. gr.	2.64	2.54	...	...	...	2.52	...	...	...	...	...

1. Trachyte of Ragged Hill dome (Vassar, analyst).
2. Trachyte of Southeast Head (Washington, analyst).
3. Trachyte of Cross Hill, quarry (Reinisch, analyst).
4. Trachyte from a point half-way up Green Mountain (Reinisch, analyst).
5. Trachyte from a point "north of Dark Slope" (Reinisch, analyst).
6. Rhyolite (quartz trachyte), flow from White Hill (Radley, analyst).
7. " Trachyte " from Weather Post (Klement, analyst).
8. Pumice from foot of Riding School cone (Reinisch, analyst).
9. " Alkalitrachyt-Perlit," from stream-course north of the Riding School (Reinisch, analyst).
10. Trachytic tuff, southwest part of the Riding School (Reinisch, analyst).
11. Obsidian from Green Mountain (Klement, analyst).

Column 1 of Table XI gives the average of the analyses of Ascension basalts, which may be compared with the average plateau basalt, with the average of 198 basalts (including some plateau basalts and many from central eruptions), and with the average basalt of Hawaii. Columns 1 and 2 of Table XII permit similar comparison with the average basalt of Tutuila Island in the Samoan chain of volcanoes (five analyses by H. S. Washington, given in the writer's "Geology of American Samoa," Pub. No. 340, Carnegie Institution of Washington).

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[29] Includes 3.17 per cent of salts soluble in water.

TABLE XI.

Comparison of Average Analyses of Basalts.

	1	2	3	4
SiO_2	49.68	49.35	49.87	49.04
TiO_2	2.76	2.59	1.38	2.92
Al_2O_3	16.13	14.05	15.96	13.64
Fe_2O_3	4.02	3.40	5.47	2.99
FeO	7.47	9.94	6.47	8.89
MnO	.24	.21	.32	.12
MgO	5.33	6.36	6.27	8.53
CaO	8.78	9.73	9.09	9.52
Na_2O	3.70	2.90	3.16	3.00
K_2O	1.31	1.00	1.55	.86
P_2O_5	.58	.47	.46	.49
	100.00	100.00	100.00	100.00

1. Average of three basalts from Ascension Island.
2. Average of fifty analyses of plateau basalts, including: eleven Deccan basalts, six Oregon basalts, and thirty-three Thulean basalts (from H. S. Washington, Bull. Geol. Soc. America, **33**, 797, 1922).
3. Average of 198 analyses of basaltic rocks, published before the year 1910; world-wide distribution.
4. Average of fifty-four analyses of mafic rocks in Hawaii, weighted as follows: equal weights given to respective averages of twelve from Kohala, twelve analyses from Mauna Kea, six analyses from Hualalai, thirteen analyses from Kilauea, and eleven analyses from Mauna Loa. (See H. S. Washington: reference under "2".)

Note: all averages calculated as water-free and to 100 per cent.

The degree of similarity of all these averages illustrates once more the relative uniformity of common basalt throughout the world. This material may vary in composition somewhat as it is traced from its position beneath the continental crust to its position beneath the sub-Pacific crust, but these variations are too slight to have yet been detected with full certainty. However, it seems clear that the eruptible part of the Sima is everywhere of basaltic character.

The proof of the existence of common basalt in Ascension is an important fact which must modify the impression given by a statement of Reinisch, who, working with a relatively small collection of specimens and without the advantage of a prolonged reconnaissance of the island, announced that all of the specimens collected in Ascension by the Deutsche Südpolar Expedition are "durchweg Alkaligesteine" (page 646 of his paper). According to the general consensus of opinion among petrographers, the basalts of Ascension have both chemical and mineralogical characters which must place them in the calc-alkaline or subalkaline group of rocks.

TABLE XII.

Comparison of Ascension Lavas with those of Tutuila, Samoa.

	1	2	3	4	5	6	7	8
SiO_2	49.68	48.44	52.65	52.34	65.71	66.38	71.73	70.80
TiO_2	2.76	4.29	2.00	2.99	.43	.65	.25	.26
Al_2O_3	16.13	13.27	16.61	16.66	15.40	17.00	12.82	12.73
Fe_2O_3	4.02	4.06	4.52	2.80	3.46	2.12	3.60	4.03
FeO	7.47	8.27	4.77	5.67	1.12	1.33	.05	.28
MnO	.24	.15	.37	.07	.22	.05	.29	.06
MgO	5.33	8.21	3.91	3.39	.27	.29	.18	.04
CaO	8.78	7.72	7.29	5.58	1.07	1.62	.60	.50
Na_2O	3.70	3.47	4.61	4.24	6.36	5.35	5.31	5.77
K_2O	1.31	1.55	2.05	2.38	4.96	4.47	4.77	5.28
H_2O	. . .	. . .	.70	2.63	.91	.64	.35	.25
P_2O_5	.58	.57	.52	1.25	.09	.10	.05	None
	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

1. Average of three analyses of Ascension Island basalts, calculated as water-free.
2. Average of five analyses of Tutuila Island basalts, calculated as water-free.
3. Trachydoleritic basalt from Ascension Island; one analysis.
4. Trachydoleritic basalt from Tutuila Island; one analysis.
5. *Average of five analyses of Ascension Island trachytes.*
6. Average of two analyses of Tutuila Island trachytes.
7. Quartz trachyte (rhyolite) from Ascension Island; one analysis.
8. Quartz trachyte (rhyolite) from Tutuila Island; one analysis.

In Table XII, columns 1, 3, 5, and 7, are entered the average analyses of the important types of lava in Ascension. The variation is systematic and is strikingly parallel to that shown in the averages of Washington's analyses of the rocks of Tutuila, Samoa (columns 2, 4, 6, and 8). The volcanic complex of the island in the western Pacific thus exemplifies the same process of differentiation as the process responsible for the variety of lavas in a mid-Atlantic island. The table suggests that trachydolerite and trachyandesite represent stages in the development of trachyte from basalt. Attention may therefore be concentrated on the problem of the origin of trachyte.

Leading generalizations which apparently should govern thought on this subject, including facts already noted, are:

1. The close time and space relations of eruptions of trachyte and common basalt;
2. The usual insignificance of trachyte in volume when compared with the accompanying basalt, and, a fortiori, when compared with any one of the basaltic plateaus;
3. The existence of trachydolerite and other volcanic species, transitional from basalt to trachyte;
4. The alternation of eruptions of trachyte and basalt at central vents;
5. The non-existence of great fissure eruptions of trachyte;
6. The observation that trachytic eruptions at central vents seem to be generally preceded by long periods of dormancy;
7. The common heralding of trachytic eruption by major explosions, which show that much hot gas had accumulated at the top of each column of trachytic magma;
8. The great rarity of ordinary bubble-vesicles in trachytes, except in quite subordinate, pumiceous phases;
9. The high viscosity of trachytes, with resulting tendency for the formation of endogenous domes or crater-fillings as well as short, thick flows;
10. The correlated fine grain of most trachytes;
11. Repeated instances of transition from trachyte to more rhyolitic lava, and, in other regions, to phonolite;
12. The rather common development of micropoikilitic quartz in trachytes;
13. The not uncommon association of trachytes with limburgites and other highly mafic lavas.

The rules numbered 1, 2, 3, and 4 are obviously obeyed at Ascension and have already been emphasized. Few other regions illustrate more clearly, or with so many examples, intimacy between trachyte and basalt.

In the nature of the case the fifth rule cannot be well tested by the data from a single island. On the other hand, practically all of the trachytic bodies of Ascension have issued from central vents, suggesting, here as elsewhere, that the formation of trachyte is connected with processes operating in volcanic pipes.

A word may be added concerning rules 6 and 7. Wherever the base of a trachytic flow could be seen in the island, that flow was seen to rest on an explosion-breccia, which includes fragments of both trachyte and the older basalts. The corresponding explosions are most reasonably dated at times immediately preceding the respective outflows of the massive trachyte. Similar examples have been recently mapped by the writer in the island of Tutuila, Samoa. The accumulation of sufficient gaseous pressure at each of these central vents logically demands an earlier, prolonged period of dormancy.

With few exceptions, rules 8, 9, and 10 appear to be general for the world.

From their field relations the rhyolitic obsidians of Ascension are thought to represent a segregation of small volumes of silica-rich magma at the tops of the columns of trachyte in the magmatic state. Thus, along with the abundant water-gas and other gases which were assembled just under the volcanic plugs during dormancy, silica was somewhat concentrated. The result has been the formation of a quartz-trachyte type of obsidian approaching the comendites and pantellerites in composition.

The local development of phonolitic, nephelite-bearing phases in trachyte, like that reported by Washington from Monte Ferru, Sardinia, has not been observed in Ascension.[30]

Even after eruption the Ascension trachyte was affected by the rise of silica-bearing solutions, which were responsible for the micropoikilitic quartz, so often seen in the main body of a trachytic flow and, still more abundantly, in the silicified interfaces already described. In largest part the volatile part of these solutions was probably water.

So far as known, the island bears no visible picritic, limburgitic, or other ultra-mafic lava. The wehrlitic fragments in the trachytic tuffs of Green Mountain represent the only analogy to these types yet discovered.

The foregoing brief review of the relative quantities and the chemical, structural, and temporal relations of the Ascension lavas can hardly fail to suggest a conclusion as to the origin of the salic types: basalt is the source magma; trachyte, its derivative. There appears to be no ground for assuming a special "alkaline" basalt, rather than common basalt, as the primary magma.

---

[30] H. S. Washington, Amer. Jour. Science, **39**, 514, 519, 1915.

The development of rhyolitic phases at the tops of the trachytic columns of magma indicates gravitative separation of the units of differentiation, whether solid or fluid. Gravity so clearly controls the differentiation of intrusive magmas that one is justified in postulating its great importance at Ascension, though, as at all volcanic centers, full and direct evidence is not to be expected.

The theory of pure fractional crystallization, which premises the gravitative removal of solid phases with fall of temperature, has usually been phrased in terms of an initially liquid system, uniformly cooled in all parts. However, magma resting in the earth's crust is not likely to cool at a uniform rate in either vertical or horizontal planes. The more rapidly chilled phase near the contact should undergo some fractional crystallization before the central part of the magma has been at all crystallized because of cooling. The residual liquid of the peripheral phase is less dense than the original magma, which therefore tends to replace the new liquid phase, driving it upwards in the magma chamber. The result is a kind of convection in the chamber, concentrating increasingly salic liquid at the top. This vertical motion of the residual liquid is a secondary but important consequence of the sinking of the early-formed crystals. The shape of the chamber, especially the inclination of its walls to each other and to the vertical, is likely to affect the speed of the concentration of the residual liquid.

If the magma chamber in which trachyte has been developed has in depth a cross-section considerably greater than the cross-section of the pipe filled with trachyte, the original magma vertically below the pipe would be relatively little affected by the sinking of crystals. The extrusion of the trachyte must be accompanied by the rise of this basalt, which may soon follow the trachytic magma to the earth's surface. Herein perhaps may be a basis for explaining the observed sequence — basalt, trachyte, basalt — at Green Mountain and at Cross Hill.

The trachyandesite flooding the trachyte of Southeast Head may represent the lower, more femic phase of the same body of differentiated magma which first welled out as trachyte; or the trachyandesite may be a new differentiate of basalt, following up the erupted trachyte. A choice between these alternatives does not seem possible. In either case, however, the relation of the trachyandesite to the trachyte does not offer any special difficulty for the theory of fractional crystallization.

Pyroxene andesites appear to be direct differentiates of common basalt and they are much more voluminous than trachyte. This fact has suggested that the differentiation of trachyte may be due to a condition not normally dominating at basaltic vents of the central type. The condition described has been speculatively found in the special concentration of volatile constituents, notably water and carbon dioxide, at trachytic vents. It is reasonable to suppose that water-gas and carbon dioxide may be absorbed by magma from the walls of its chamber, and the writer is inclined to favor the hypothesis that resurgent gas is a leading factor in the differentiation of highly alkaline phases.

That the trachytes are commonly, if not always, associated with the rise of much volatile matter into volcanic pipes is indicated by the high explosiveness of the upper part of each column of trachytic magma. Just before eruption the lower, greater part of each column is apparently quite poor in dissolved gas, as shown by the high viscosity and the fine grain of all the trachytes and by the general lack of bubble-vesicles in domes and flows of trachyte. Thus the gases seem to have continued to stream upward through the column. The common phenomenon of feldspathization of the country rocks at plutonic contacts suggests that feldspathic material will be dissolved with the gases; carried to the limit a trachytic magma would result.

If the Ascension trachytes originated in that way, their differentiation must have taken much time and the emanating gas must have had great total volume. The xenolithic granite in the trachyte of the Crags dome and the granitic fragments in the agglomerate of Green Mountain were probably derived from the terrane underlying the Ascension cone. If so, the trachytic magma stood in columns perhaps 2,000 meters high. Like most eroded volcanic necks, these columns doubtless had small cross-sections to considerable depths. To supply the material for any one of several domes and outflows, such as Weather Post and Southeast Head, the slender columns must have been long, possibly measuring more than 2,000 meters vertically.

To permit of such drastic differentiation on the large scale, in spite of the comparatively rapid loss of heat along the walls of each vent, an abundant upward stream of fluxing gas, active for a long time, seems to be essential. Moreover, to be effective, the rising gas must itself have had a decidedly high temperature. That all of this gas did not directly escape into the air is shown by the rule of explosion just before most, if not all, of the trachytic eruptions took place.

The study of the Ascension trachyte has thus led to a sympathetic reception of the following hypotheses: (1) that the trachyte is a differentiate of common basalt; (2) that the differentiation has been dependent upon the upward movement of fluids, both liquid and gaseous; and (3) that the active gas, probably water-gas, was largely of resurgent nature, that is, derived from the rocks surrounding the various vents. These hypotheses are consistent with the view that fractional crystallization has also played an important rôle in the generation of the salic magmas from basalt. No evidence of induced liquid immiscibility in the basaltic magma has been discovered, but the writer is not prepared to deny its participation. On the other hand, fractional crystallization under the conditions of nature (see page 77) seems to imply the progressive, gravitative separation of liquid phases, even if those phases are perfectly miscible.

Summary.

Darwin's account of Ascension Island is accurate and thorough to such a degree that all later descriptions must be in a sense but supplementary to his. Yet modern volcanology and petrology demanded a considerable addition to Darwin's picture. The staff of the *Challenger*, Renard, Prior, the staff of the Deutsche Südpolar Expedition, and Reinisch have filled many gaps, but more detailed mapping and further chemical study of this remarkable island have remained highly desirable. The reconnaissance map, Plate 1, and seven new rock analyses, by Washington, Vassar, and Radley, are among the principal contributions of this paper.

The types of lava are now seen to include a chemical series beginning with common olivine basalt and passing through olivine-free basalt, trachydoleritic basalt, trachydolerite, andesite (?), and trachyandesite to alkaline trachytes and alkaline quartz-trachyte ("rhyolite"). The series is in principle identical with that recently demonstrated for the volcanic composite at Tutuila, Samoa; thus illustrating the rule of one law in the differentiation of lavas, whether this took place at a mid-Atlantic center or at an almost antipodal center in the western Pacific.

The most striking features of Ascension are a dozen endogenous domes or crater-fillings of trachyte (with occasional quartz-trachyte and obsidian phases), some of which have magnificent outflowing tongues of the same lava. The recency and largely unmodified forms of these eruptions give them special importance in connection with the problems of origin and emplacement of trachytes. The intimacy of the association between the trachytes and common basalt is very evident. A possible mode of the derivation of the trachyte from basaltic magma, founded on a modified form of the fractional-crystallization theory, is briefly sketched.

Among the more detailed results of this investigation may be mentioned: the evidence of pronounced vertical, axial subsidence of the lava columns in the Riding School, "Drip," and East Craters; the suggestive sliding phenomenon connected with the basaltic flows; the description of the spectacular fissure-eruption of trachyandesite at Southeast Head; the listing of fifteen plutonic types, including quartz-bearing species, among the volcanic projectiles; the discovery of xenolithic granite in the trachyte of "The Crags" dome, with its suggestion of the considerable depth at which trachyte is differentiated in a volcanic pipe; the enforced emphasis on the problem of silicified interfaces and of micropoikilitic quartz in the trachytes; the repeated proofs that ordinary vesicles of bubble form are absent from the greater part of each trachytic mass; many new illustrations of the high viscosity characterizing trachytic magma at the time of its eruption; and the repeated occurrence of "rhyolitic" obsidian and pumice in such relations as to show that the trachytic magma in each vent was capped by a specially silicious and hydrous, explosive solution.

The compilation of significant data from the writings of Renard, Prior, and Reinisch along with the new findings have lengthened this paper, but have clearly increased its value for petrologists who desire a record of all trustworthy chemical analyses of the Ascension rocks.

HARVARD UNIVERSITY
Cambridge, Massachusetts

ILLUSTRATIONS.

PLATE I.

Geological Sketch Map of Ascension Island.

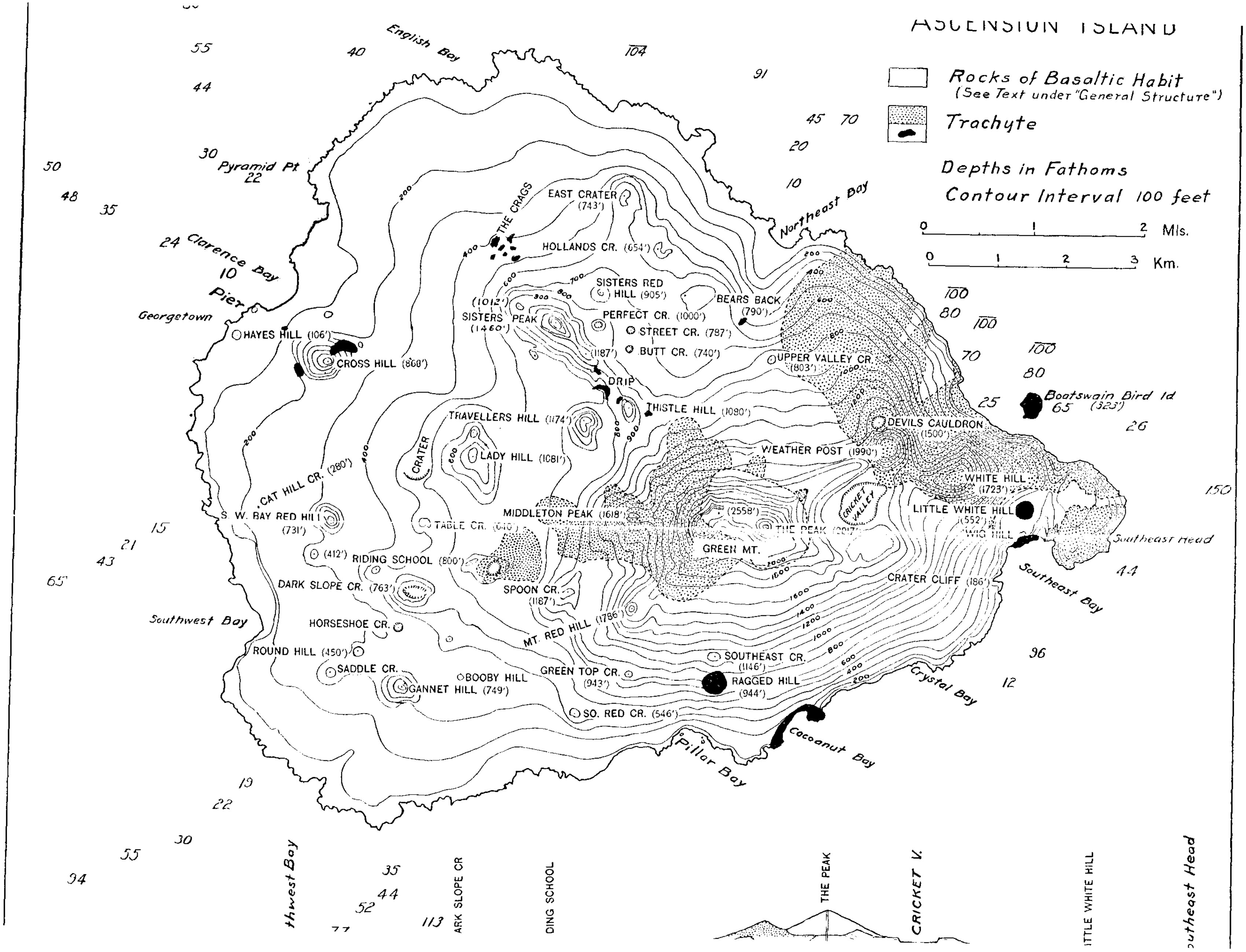

ASCENSION ISLAND
Rocks of Basaltic Habit
(See Text under "General Structure")
Trachyte
Depths in Fathoms
Contour Interval 100 feet
0 1 2 Mls.
0 1 2 3 Km.
English Bay
Pyramid Pt
Clarence Bay
Pier
Georgetown
Northeast Bay
Boatswain Bird Id (323')
Southeast Head
Southeast Bay
Crystal Bay
Cocoanut Bay
Pillar Bay
Southwest Bay
EAST CRATER (743')
THE CRAGS
HOLLANDS CR. (654')
SISTERS RED HILL (905')
BEARS BACK (790')
PERFECT CR. (1000')
STREET CR. (787')
BUTT CR. (740')
SISTERS PEAK (1460')
(1012')
(1187')
UPPER VALLEY CR. (803')
DRIP
THISTLE HILL (1080')
DEVILS CAULDRON (1500')
WEATHER POST (1990')
WHITE HILL (1723')
LITTLE WHITE HILL (552')
WIG HILL
CRICKET VALLEY
THE PEAK (2817')
(2558')
GREEN MT.
MIDDLETON PEAK (1618')
TABLE CR.
TRAVELLERS HILL (1174')
LADY HILL (1081')
CRATER
HAYES HILL (106')
CROSS HILL (868')
CAT HILL CR. (280')
S. W. BAY RED HILL (731')
(412')
RIDING SCHOOL (800')
DARK SLOPE CR. (763')
SPOON CR. (1187')
MT. RED HILL (1786')
CRATER CLIFF (186')
HORSESHOE CR.
ROUND HILL (450')
SADDLE CR.
GANNET HILL (749')
BOOBY HILL
GREEN TOP CR. (943')
SOUTHEAST CR. (1146')
RAGGED HILL (944')
SO. RED CR. (546')
200
400
600
700
800
900
1000
1200
1400
1600
2000
55
44
40
104
91
45
70
20
10
30
22
50
48
35
24
10
100
80
100
70
100
80
25
65
26
150
44
96
12
15
21
43
65
19
22
30
55
94
35
44
52
113
thwest Bay
ARK SLOPE CR
DING SCHOOL
THE PEAK
CRICKET V.
ITTLE WHITE HILL
utheast Head

PLATE II.

FIGURE *A*. Panorama of Ascension Island, looking southeast from anchorage at Georgetown; from photograph.

FIGURE *B*. Looking west from Green Mountain road; left, lower slope of Lady Hill; middle, Cross Hill; right, young basaltic flow which issued from base of Sisters Peak.

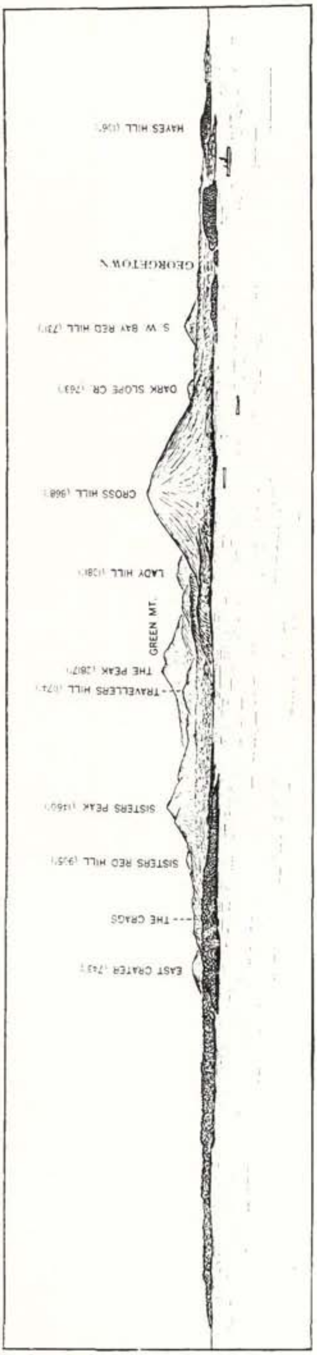

HAYES HILL (106')
GEORGETOWN
S. W. BAY RED HILL (731')
DARK SLOPE CR. (763')
CROSS HILL (968')
LADY HILL (1081')
GREEN MT.
THE PEAK (2817')
TRAVELLERS HILL (1274')
SISTERS PEAK (1460')
SISTERS RED HILL (957')
THE CRAGS
EAST CRATER (743')

Fig. A

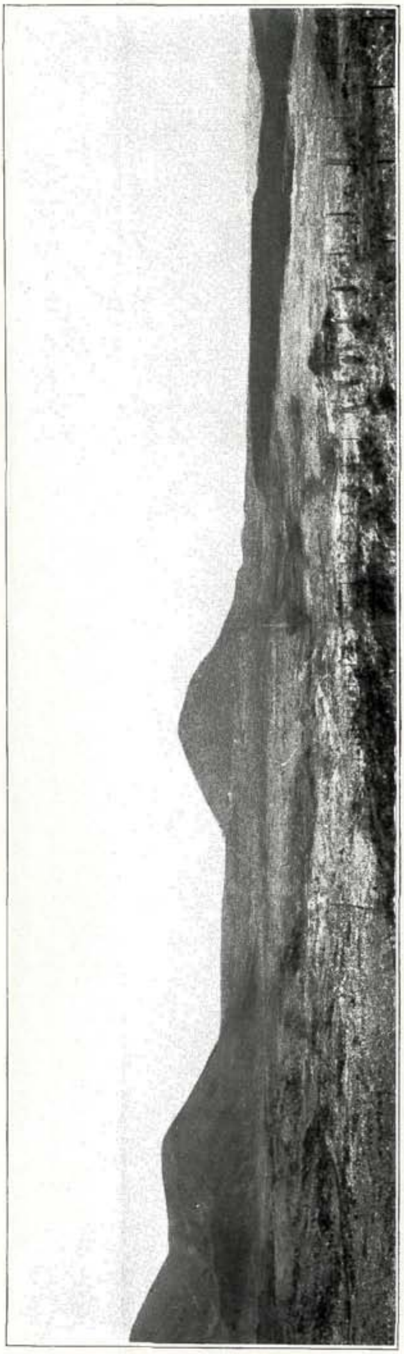


Fig. B

PLATE III.

FIGURE *A*. Sisters Peak (right) and East Crater (left), seen from Governor's house, Cross Hill.

FIGURE *B*. Young basaltic flow which issued from a vent west of East Crater. Looking north from Cross Hill; Pyramid Point in the distance; calcareous-sand beach of Clarence Bay in foreground.

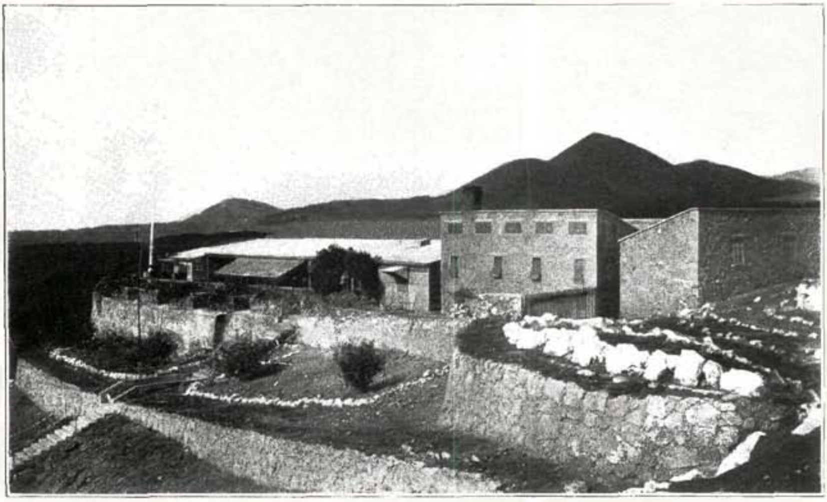


FIG. A

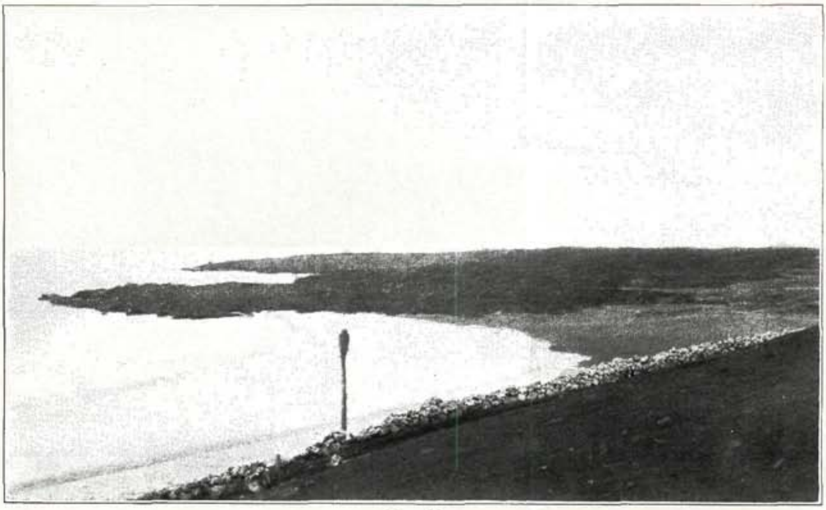


FIG. B

PLATE IV.

Panorama from a low level on Green Mountain, showing various craters, Bears Back, and a young basaltic flow from Sisters Peak (midground); slope of Thistle Hill, left, foreground.

Plate IV.

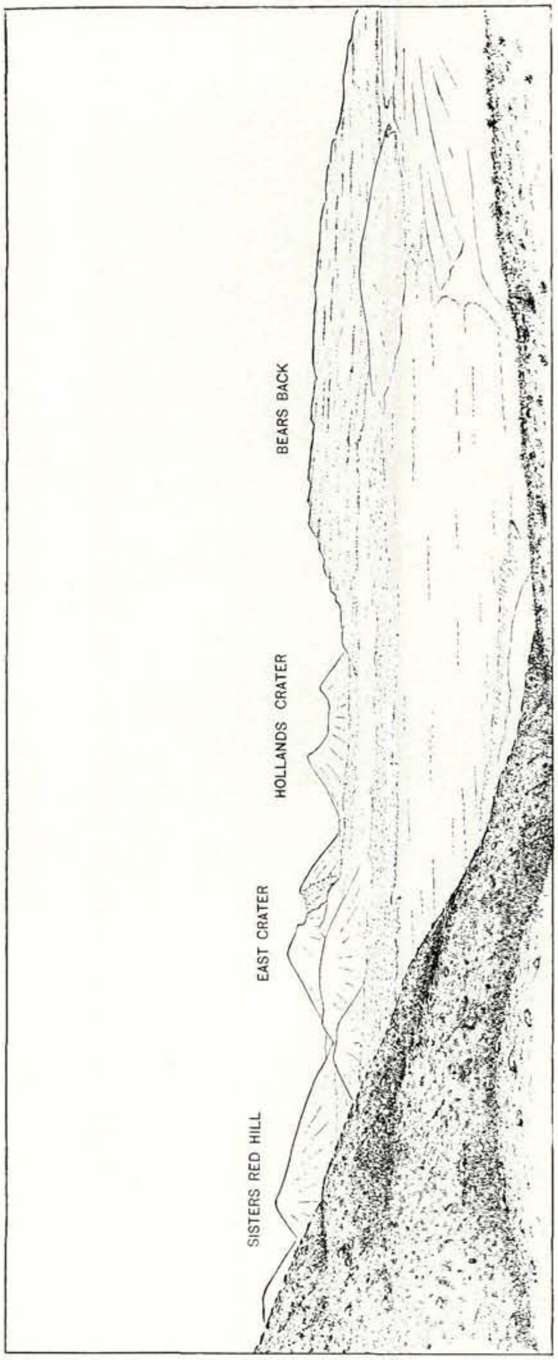

SISTERS RED HILL
EAST CRATER
HOLLANDS CRATER
BEARS BACK

PLATE V.

Figure *A*. Characteristic rough, constructional surface of young basaltic flow, cliffed by the waves.

Figure *B*. Hornito-like conelets on back of young basaltic flow, near Wireless Station.

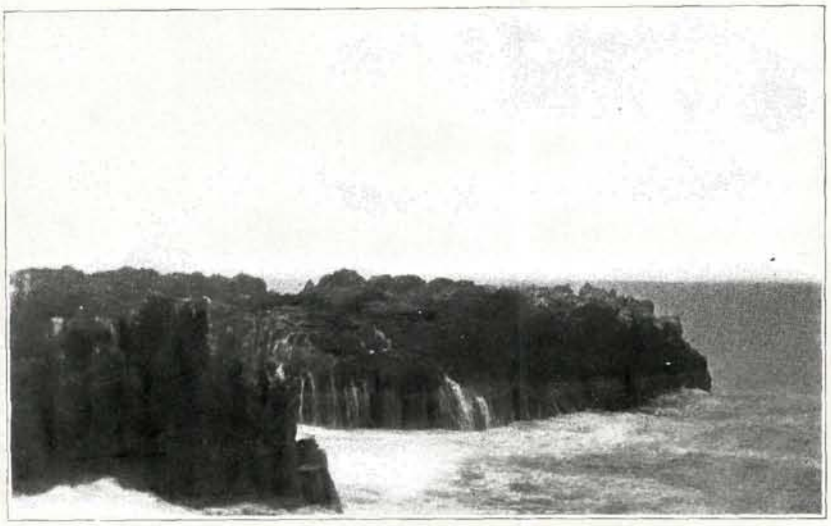


FIG. A

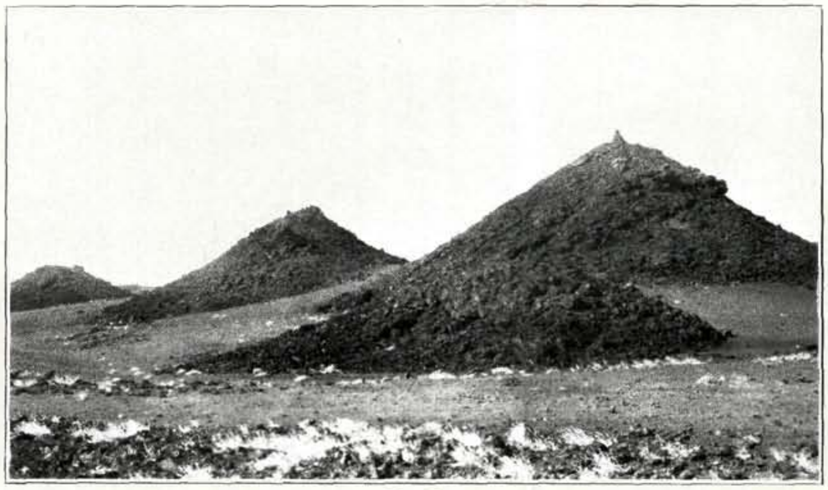


FIG. B

PLATE VI.

FIGURE *A*. Weathered-out silicious "veins" in trachyte, top of Ragged Hill dome; undercutting of the softer trachyte by the wind.

FIGURE *B*. Rough, flat-lying, silicious "vein" projecting from the softer trachyte, Riding School; the hoodoo-like form is about two meters high.

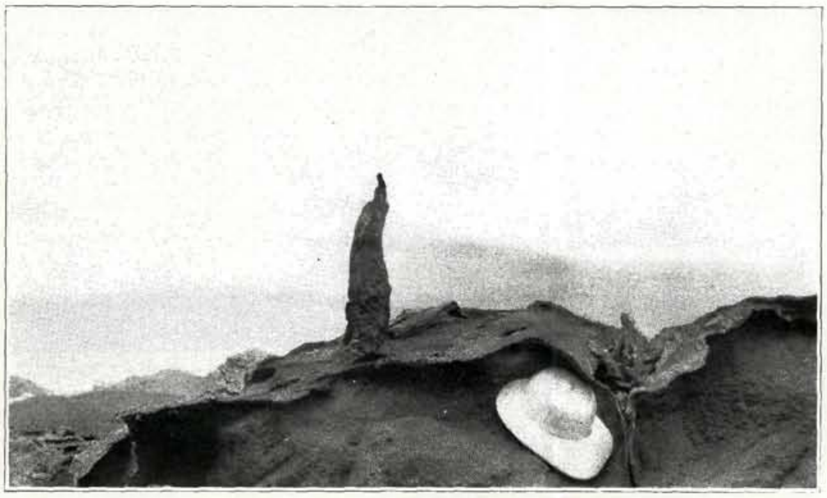


FIG. A

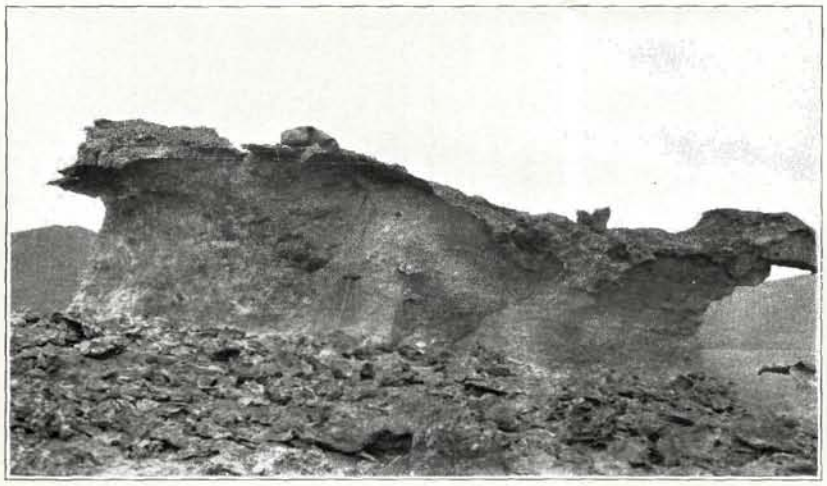


FIG. B

PLATE VII.

Little White Hill dome of trachyte, standing in nearly circular crater-rim of basaltic material (midground and foreground). White Hill dome of trachyte in the background, left. Traced from photograph.

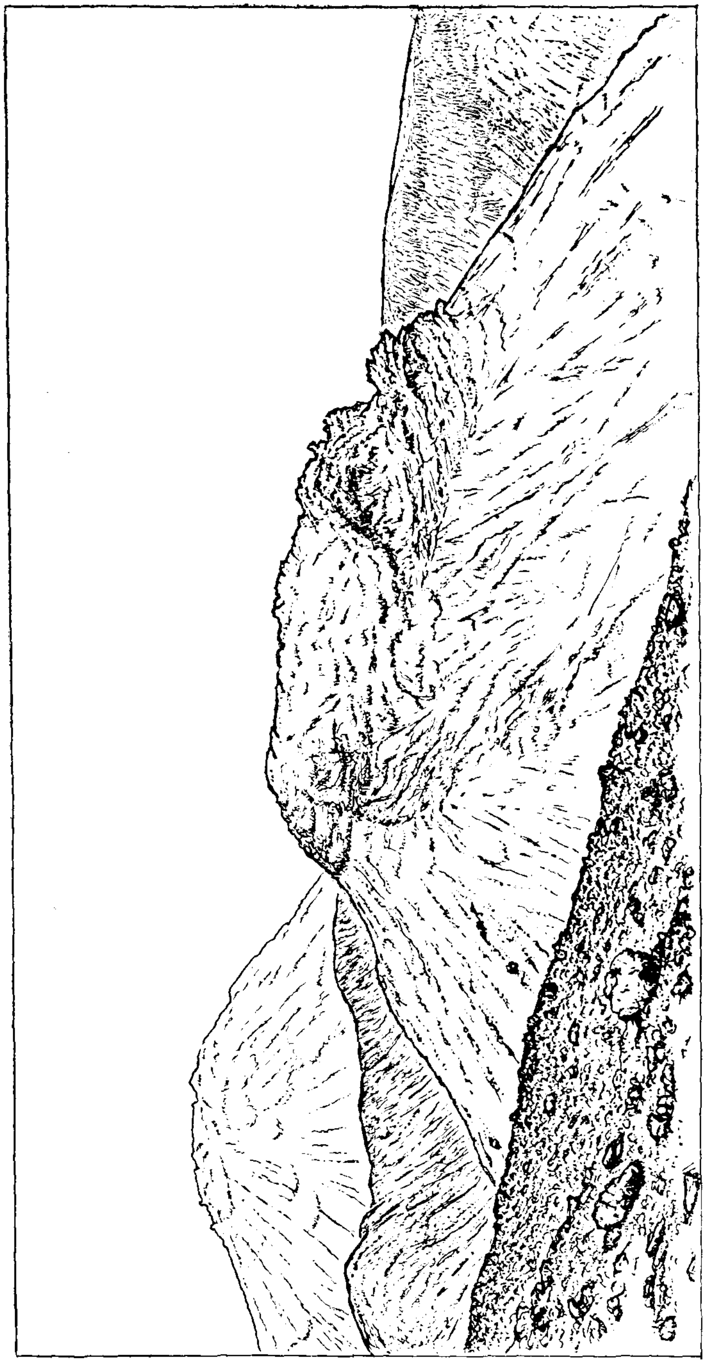


PLATE VIII.

Looking east from near Cricket Valley to: (left) White Hill dome of trachyte with overflow; (midground) Wig Hill dome of trachyte mantled with the basaltic "wig"; (background) Southeast Head dome of trachyte flooded with thin flow of trachyandesite; débris of Cricket Valley explosion in foreground. Drawn from a photograph.

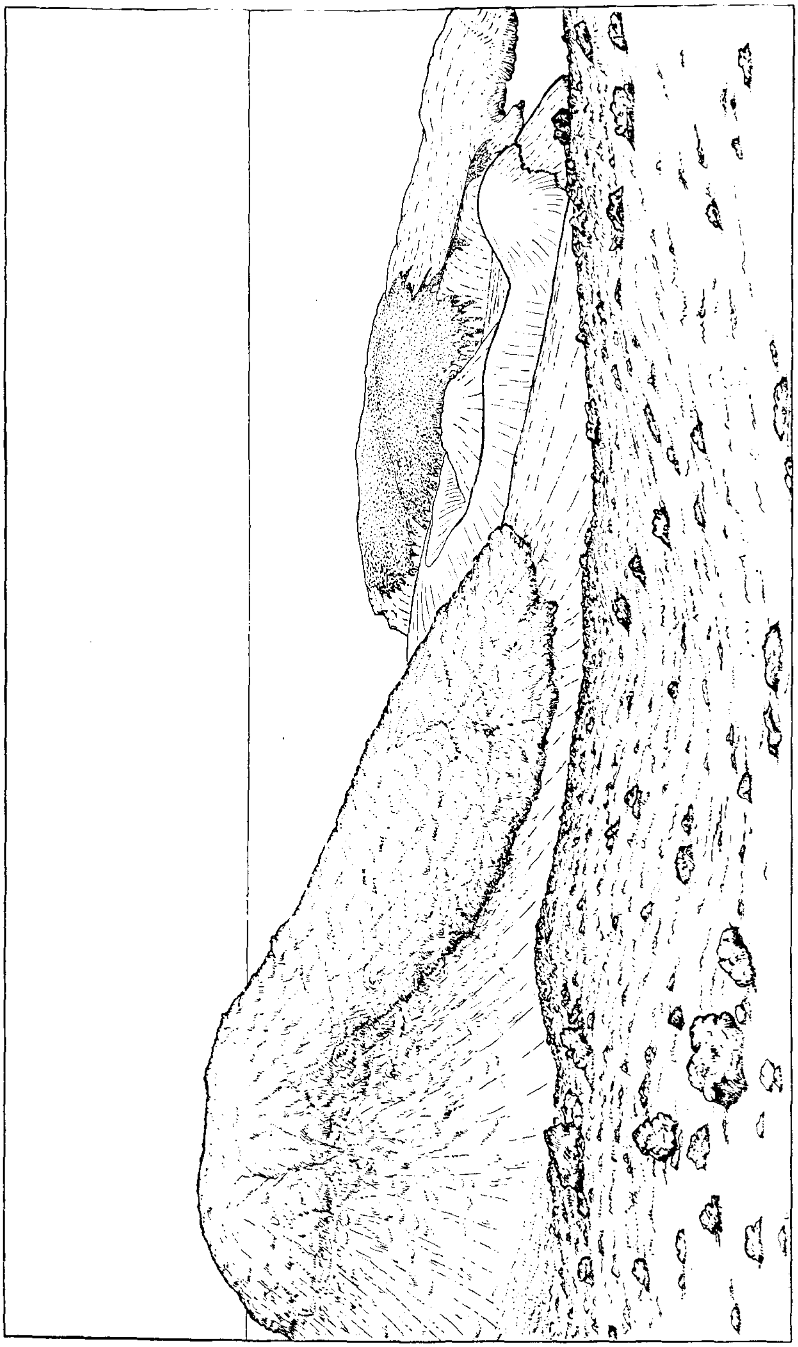


PLATE IX.

FIGURE *A*. Wig Hill seen from the west; Southeast Head in the background.

FIGURE *B*. Closer view of the "wig" of basaltic scoria overlying the Wig Hill dome of trachyte.

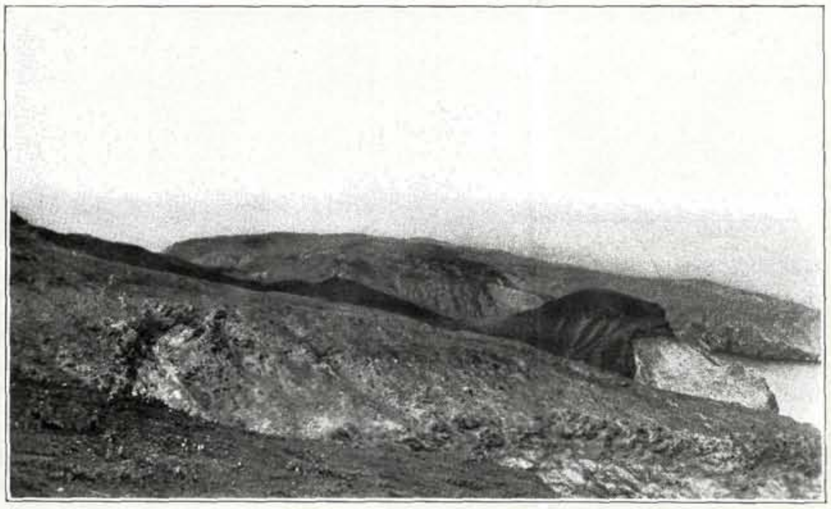


FIG. A

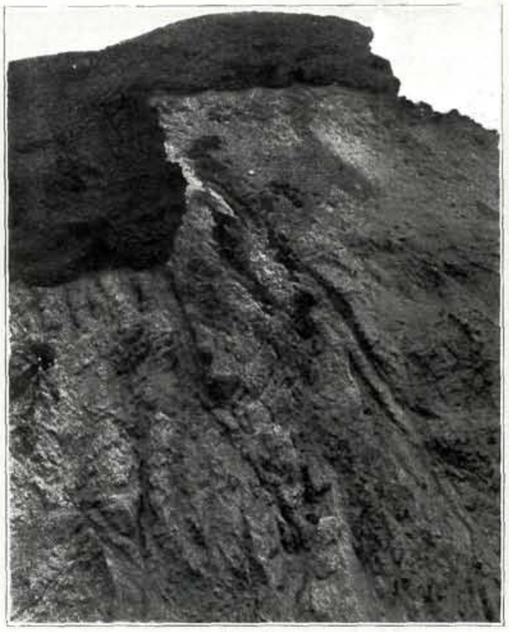
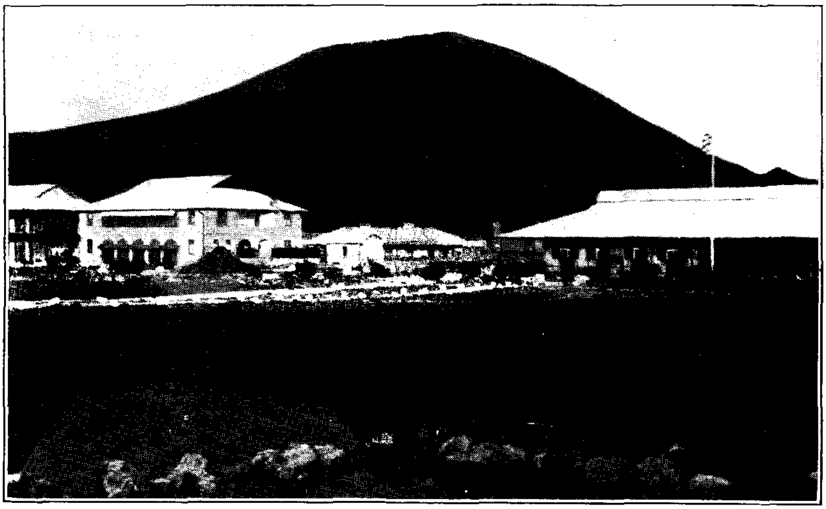


FIG. B

PLATE X.

Figure *A*. Cross Hill from Georgetown, showing basaltic tuff-ash deposit which overlies a core of trachyte (not visible in this view).

Figure *B*. Looking across Riding School crater to Green Mountain (left) and Mountain Red Hill (right); in the crater a thick layer of basaltic tuff overlying well-bedded deposits of dust-like, silicious ash, said to contain silicious "infusoria."



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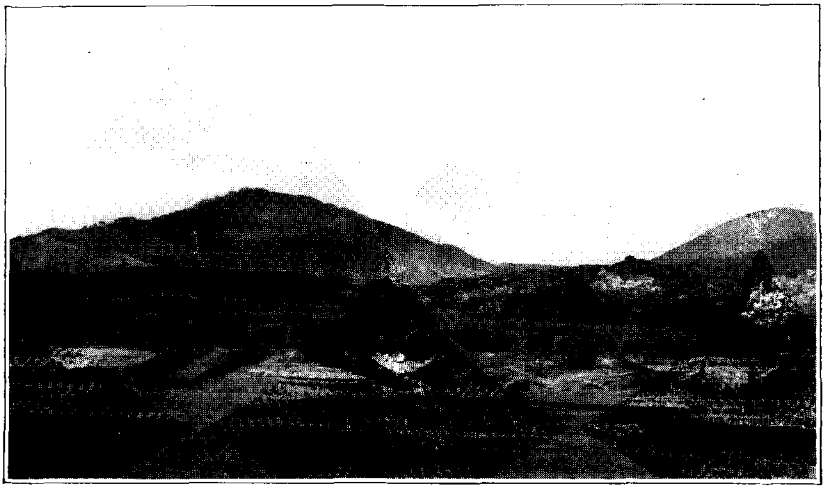


Fig. B

PLATE XI.

Panorama of Riding School crater, looking west from a point on the trachytic rim, across the centripetally-dipping tuff beds and water-laid beds, to the basaltic part of the rim. Near the middle, background, is Dark Slope crater. Structure brought out by the use of ink on the photograph.

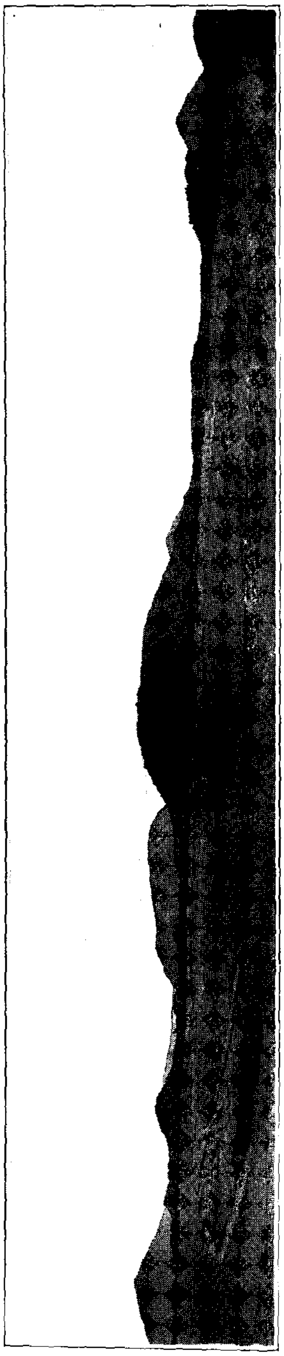
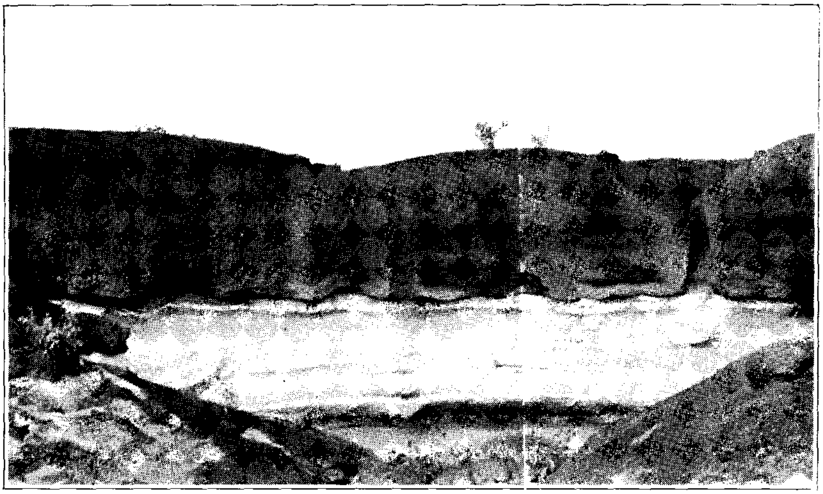


PLATE XII.

FIGURE *A*. Basaltic tuff overlying silicious beds, in Riding School crater; hammer 30 cm. long.

FIGURE *B*. Dipping tuff and "lake" beds in Riding School crater.



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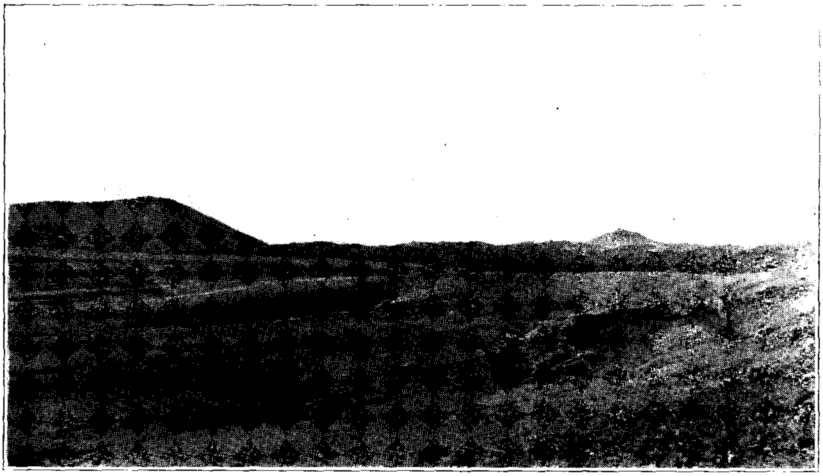


Fig. B

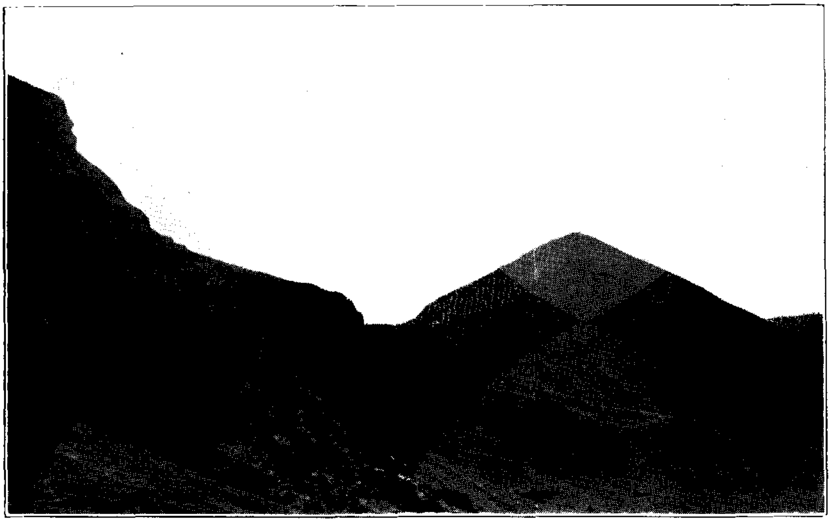
PLATE XIII.

Carious weathering of Riding School trachyte, south side of this dome.

PLATE XIV.

FIGURE *A*. Looking south from road to edge of thick overflow from trachytic dome of Green Mountain (left); Mountain Red Hill, basaltic cone, in background.

FIGURE *B*. Steeply dipping, eroded beds of tuff composing The Peak, which stands in a caldera rimmed with older trachyte of Green Mountain. The black, basaltic tuff of The Peak is beyond the windmill; the crags with paler tint on right are trachyte. Young erosion-valley opened along the contact of trachyte and tuff.



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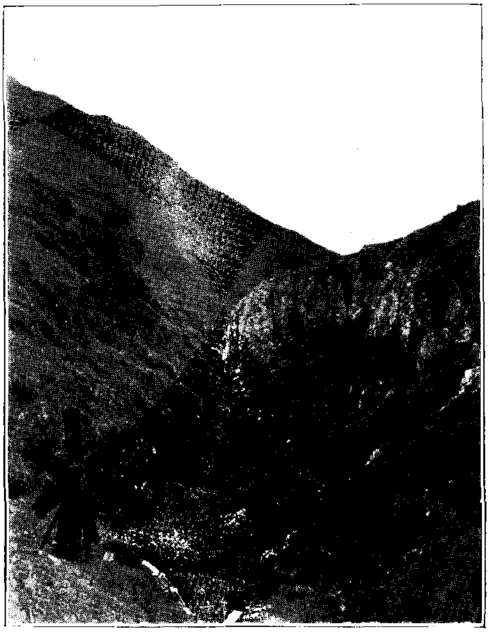

FIG. B

PLATE XV.

FIGURE *A*. Looking across the basaltic tuff-ash slope of Green Mountain to The Weather Post (left) and White Hill (right) domes of trachyte.

FIGURE *B*. Flow-structure and characteristic rough surface of an Ascension Island flow of trachyte; end of flow, southwest side of White Hill.

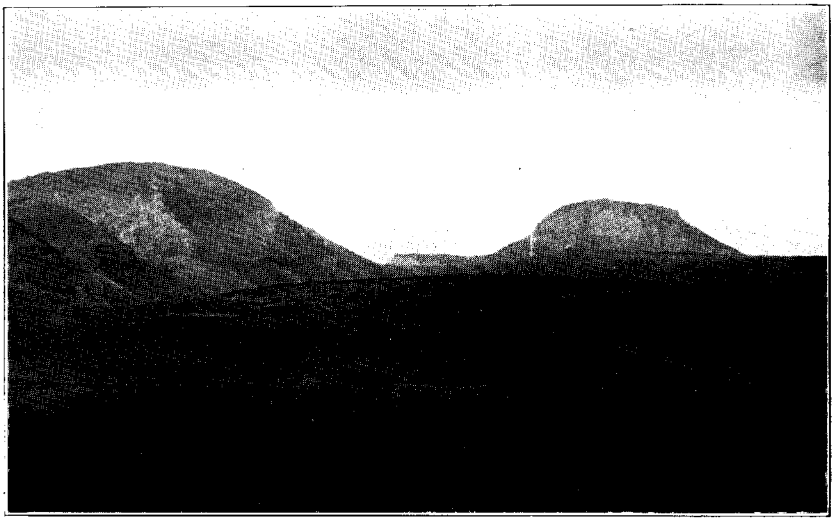

FIG. A

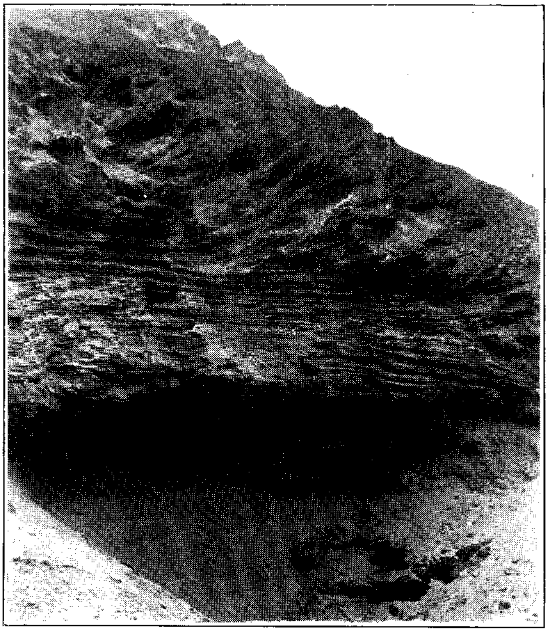

FIG. B

PLATE XVI.

The Devils Cauldron (Punchbowl), seen from the summit of The Peak; the Cauldron is 2.3 kilometers distant. Photograph by G. H. Wilkins, naturalist to the Shackleton-Rowett Expedition.

PLATE XVII.

Lower end of great trachyte flow from White Hill dome, southwest side. Wig Hill appears behind the flow. Southeast Head trachyte dome in the distance.

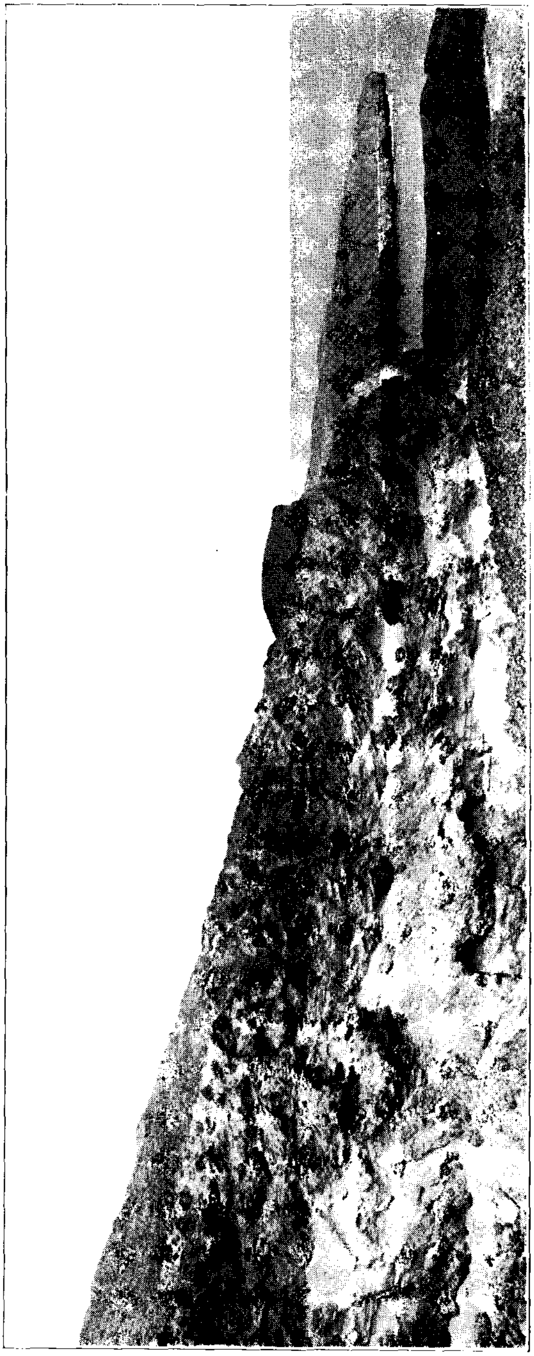

PLATE XVIII.

Figure *A*. Southeast Head dome-plateau of trachyte, veneered with a younger flood of trachyandesite, shown in darker tint.

Figure *B*. Continuation of view *A*. Sea-cliff of Southeast Bay on the right.

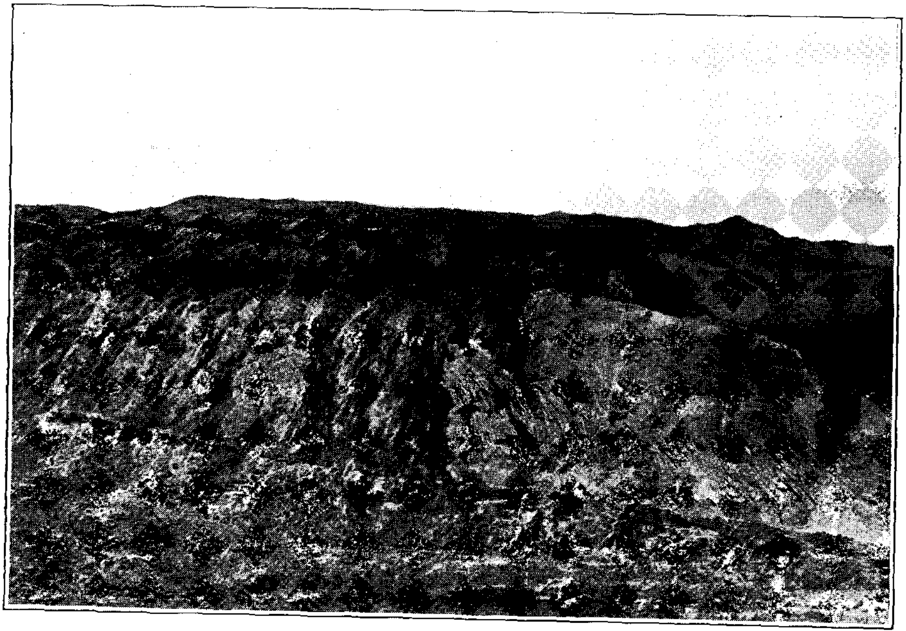

FIG. A

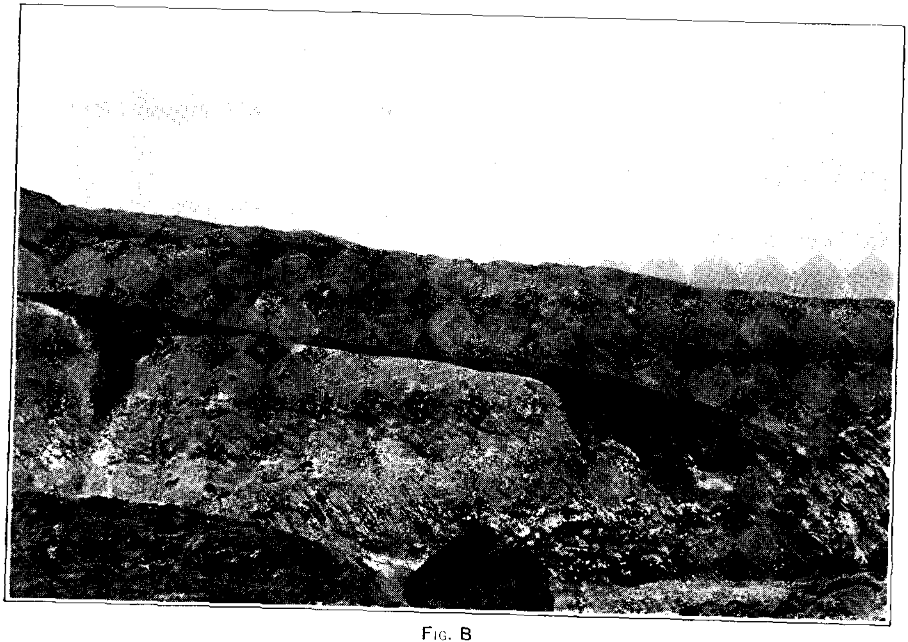

FIG. B

PLATE XIX.

FIGURE *A*. Boatswain Bird islet, seen from high sea-cliff east of the Devils Cauldron.

FIGURE *B*. Sea-cliff, 90 meters high, cut in monolithic trachyte of Boatswain Bird islet, northwest side.

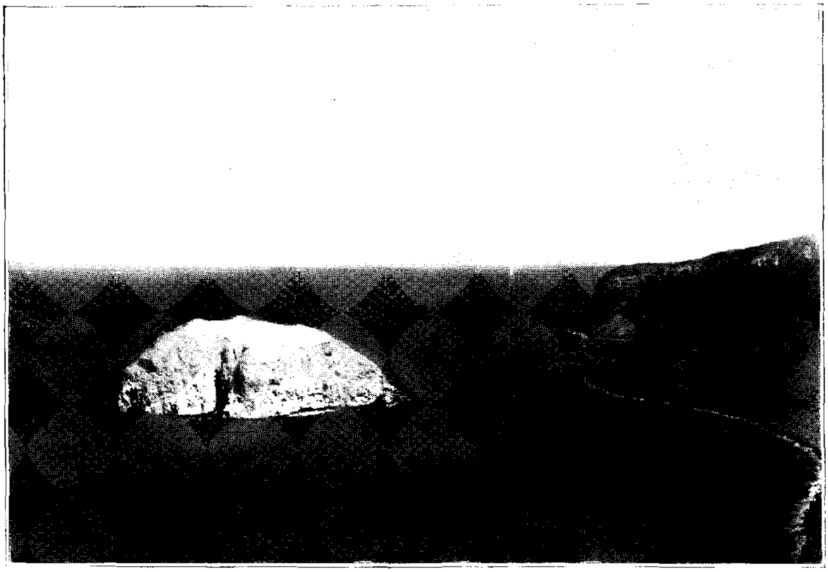

FIG. A

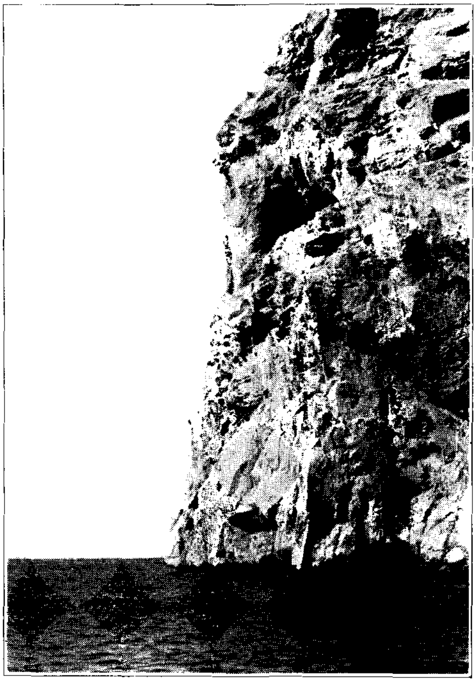

FIG. B

PLATE XX.

Sea-cave cut in trachyte of Boatswain Bird islet, southwest side.

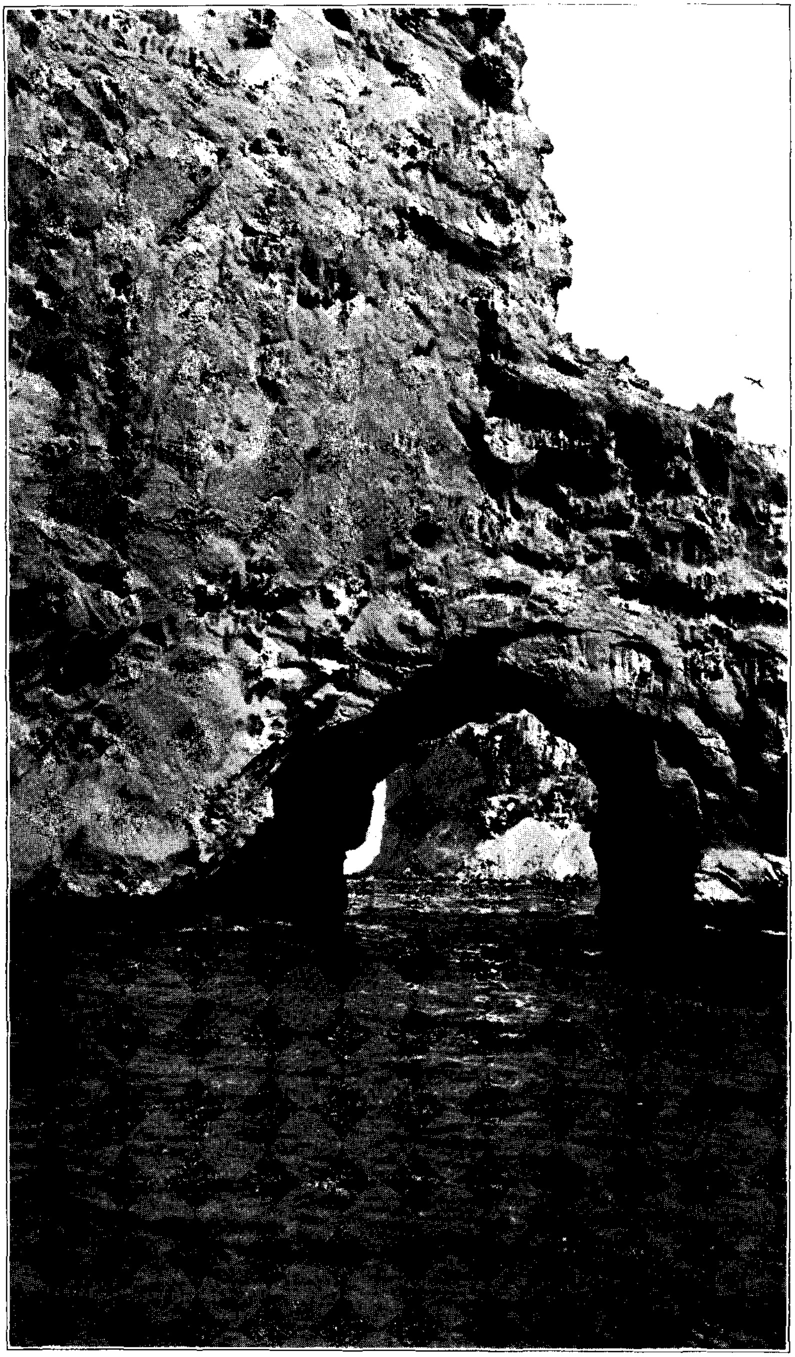

PLATE XXI.

Figure *A.* Georgetown; Hayes Hill, a scoriaceous cone of trachydolerite, in background.

Figure *B.* View from Cross Hill, over Georgetown, to a low scoriaceous cone of trachydolerite at the Landing Pier.

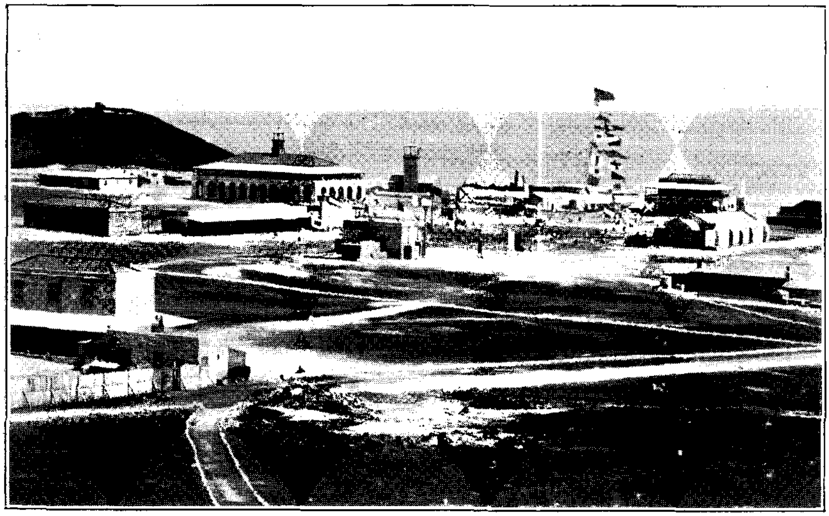

FIG. A

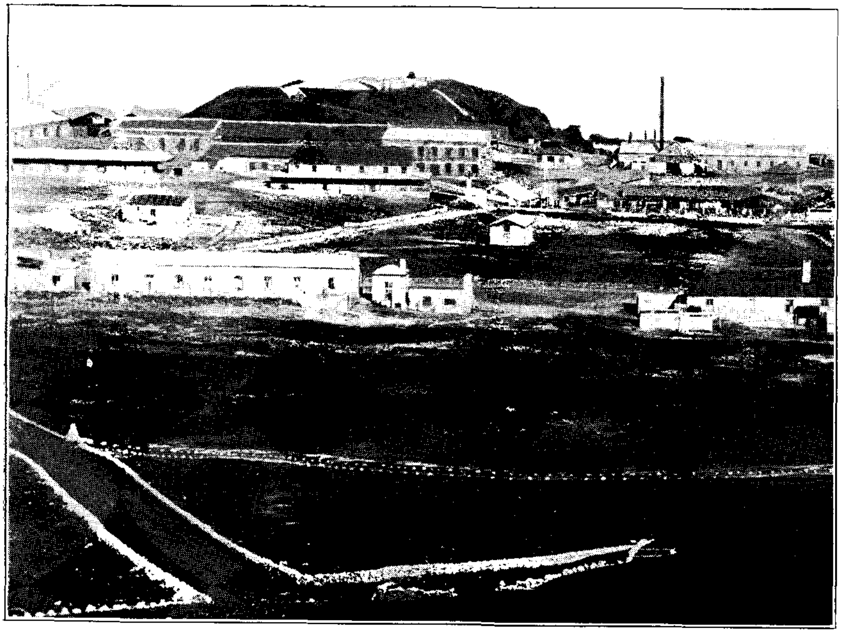

FIG. B

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NEW RESEARCHES ON THE MAGNETIZATION OF FERRO-MAGNETIC SUBSTANCES BY ROTATION AND THE NATURE OF THE ELEMENTARY MAGNET.

By S. J. Barnett and L. J. H. Barnett.

NEW RESEARCHES ON THE MAGNETIZATION OF FERROMAGNETIC SUBSTANCES BY ROTATION AND THE NATURE OF THE ELEMENTARY MAGNET.[1]

By S. J. Barnett and L. J. H. Barnett.

Received May 14, 1924. Presented May 14, 1924.

1. **Theoretical relations.** In earlier papers [2] it has been shown that the rotation of iron, cobalt or nickel about a fixed axis in a magnetically neutral region at an angular velocity N revolutions per second magnetizes the substance as it would be magnetized, at rest, by the application of an axial magnetic field with intensity

$$H_r = \lambda N \tag{1}$$

where λ, within the limits of the experimental error, is constant for any one substance and identical for all the substances investigated. The quantity λN may be called the *intrinsic magnetic intensity of rotation.*

Furthermore, on the assumption that only one kind of elementary magnet, or magneton, is responsible for the magnetism of a substance, classical dynamical theory has been shown to require that λ be equal to R, the ratio of the angular momentum of the magneton to its magnetic moment, multiplied by 2π. Thus we have, on this assumption,

$$H_r = \lambda N = 2\pi R N \tag{2}$$

For a magneton such as has ordinarily been held responsible for magnetism, and consisting of an electron or a ring of electrons, each with mass m and charge e, in revolution about a fixed positive nucleus, classical theory gives

$$2\pi R = 2\pi \times 2\, m/e \tag{3}$$

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[1] Read before the American Physical Society, April 25, 1924, and before the American Academy of Arts and Sciences, May 14, 1924. Revised, June, 1925.

[2] S. J. Barnett, *Phys. Rev.*, **6**, 239, 1915, and **10**, 7, 1917. The fundamental idea on which these experiments are based occurred to me in 1909 (*Science*, **30**, 413, 1909), while thinking about the origin of solar and terrestrial magnetism. In 1918 I found that John Perry, with the same fundamental idea in mind (*Spinning Tops*, October, 1890, footnote on p. 112, a book which I had read more than twenty years before), had tried, but without success, to detect a change in the magnetization of an iron rod produced by its rotation. If, as is very probable, I had read Perry's footnote, it had long been forgotten.—S. J. B.

The quantity m/e for an electron has been determined with considerable precision in well-known experiments. Substituting the experimental value for slow motion in (3) we obtain

$$2\pi R = -7.10 \times 10^{-7} \text{ gauss/rev. per second} \tag{4}$$

The values of λ which we have obtained from experiment agree in sign with this value of $2\pi R$, but they are much smaller in magnitude. They therefore prove either (1) that in producing ferromagnetic phenomena positive magnetons are associated with negative magnetons, whose effect is preponderant; or (2) that the negative magneton is not an orbital electron or a ring of electrons, but a magneton with a much smaller value of R; or else (3), that the classical theory according to which equation (2) is derived is not valid, or (4) that equation (3) is not valid, or both.

Our phenomenon is undoubtedly connected closely with the Zeeman effect, as our magnetons may be considered to be executing regular precession with frequency N under the action of the torques upon them brought about by the rotation. This consideration leads at once to equation (2), which is equivalent to the classical equation for the normal Zeeman effect.[3] As Landé has suggested, the anomaly in the Zeeman effect, which Sommerfeld and Debye had partially explained by the ideas of spatial quantization (now supported in the field of magnetism by the work of Pauli,[4] Sommerfeld,[5] Epstein,[6] Gerlach,[7] and Gerlach and Stern[8]), is probably related closely to the anomaly in our phenomenon. This anomaly Landé [9] and Sommerfeld [10] have attempted to explain by a process which appears to be equivalent to identifying our magneton with the atom in the s-state and attributing to this a value of R equal to m/e, which is approximately the value of R given by our experiments.

2. **General experimental methods.** In the experimental study of magnetization by rotation two general types of methods may be

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[3] S. J. Barnett, N. R. C. Bulletin on *Theories of Magnetism*, German translation, 1925.

[4] W. Pauli, Jr., *Phys. Zeit.*, **21**, 615, 1920.

[5] A. Sommerfeld, *Ann. d. Phys.*, **70**, 32, 1923; *Zeit. f. Phys.*, **19**, 221, 1923.

[6] P. S. Epstein, *Science*, **57**, 532, 1923.

[7] W. Gerlach, *Phys. Zeit.*, **24**, 275, 1923.

[8] Gerlach and Stern, *Ann. d. Phys.*, **74**, 673, 1924; W. Gerlach and A. C. Cilliers, *Z. f. Phys.*, **26**, 106, 1924; W. Gerlach, *Ann. d. Phys.*, **76**, 160, 1925.

[9] A. Landé, *Phys. Zeit.*, **24**, 44, 1923.

[10] A. Sommerfeld, *Ann. d. Phys.*, **73**, 209, 1924; Atombau, p. 635, 1924.

used, viz., methods of electromagnetic induction and magnetometer methods.

In all methods the substance under investigation is preferably in the form of an approximately cylindrical rod, or *rotor*, which is mounted with its axis horizontal and in the magnetic prime vertical in a region in which the earth's magnetic field has been neutralized as nearly as practicable.

In the methods of electromagnetic induction the intrinsic magnetic intensity of rotation λN is determined by comparing the change of flux through the rotor, due to rotation about its axis at measured speed, with the change of flux produced through the same rotor by the application, parallel to the axis of rotation, of a uniform magnetic field of known intensity. The changes of flux are proportional to the intensities, if small.

In the only induction method which we have used with success the changes of flux were measured ballistically, with a galvanometer of the type which has come to be known as a fluxmeter. Two nearly similar rods *A* and *B* of steel (Fig. 1) were mounted parallel, and two similar coils *C* and *D* of insulated wire were mounted about their centers. The coils, at rest, were connected in series with one another and with the fluxmeter *F*, and in opposition, so that the variations in the earth's field might produce no effect on the fluxmeter. One of the two rods served as rotor; the other was called the *compensator*.

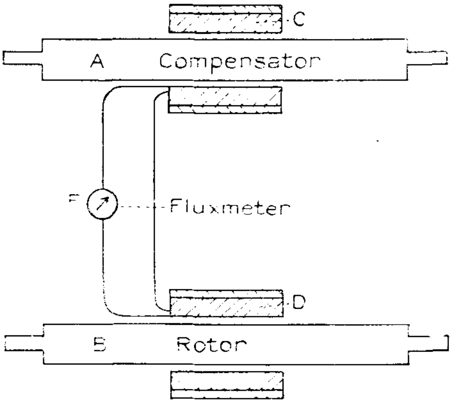

FIG. 1.

In another method,[11] which has not been used, and which we believe would involve still more serious difficulties, the rod may be oscillated about its axis instead of being set into continuous unidirectional rotation; and the alternating electromotive force thus developed in the surrounding coil of wire may be compared with the electromotive force produced by an alternating field of known intensity, an amplifier being used to increase the sensibility if necessary.

Earlier induction methods were not successful. In one series of experiments the magnetic circuit was almost wholly of iron. The cylinder under investigation rotated between the pole faces of a large U-shaped electromagnet, with minute air gaps; and similar coils on the two legs of the iron core, connected in series in the usual manner, were in series with the fluxmeter. But relatively large extraneous reactions, too complicated to interpret,[12] masked the small effect under investigation. In a still earlier series, the method was essentially similar to that which was later successful, but here also large extraneous disturbances,[13] too complicated to interpret, masked the effect looked for.

In the magnetometer method an astatic magnetometer is mounted with the center of its lower magnet system in the polar axis (magnetometer in *axial position*), or, preferably, in the equatorial plane (magnetometer in *equatorial position*) of the rod, or *rotor*, under investigation. A second rod, the *compensator*, is so mounted as to have as nearly as practicable the same relation to the upper magnet system as the rotor has to the lower, so as to eliminate in a measure disturbances due to variations in the earth's field. The magnetometer deflection produced by reversing the direction of rotation of the rotor, driven at a measured speed, is compared with the deflection produced by reversing a uniform magnetic field of known intensity in the rotor parallel to its axis. The deflections are proportional to the changes in the magnetic moments in the two cases, and these are proportional to the intrinsic intensity of rotation and the standardizing field intensity.

3. **Early successful experiments.** Our first conclusive experiments were made in 1914 at the Ohio State University on cold-rolled

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[11] Suggested by an investigation of Tolman's in a different field, now published. See. R. C. Tolman, S. Karrer, and E. W. Guernsey, *Phys. Rev.*, **21**, 525, 1923.

[12] One of these reactions, as suggested by Dr. A. E. Flowers, was doubtless *due to the nonuniformity of the residual axial field through the rotor, or,* rather, to its departure from axial symmetry.

[13] Professor C. E. Mendenhall suggested torsion, but was inclined to doubt the applicability of the suggestion when he learned that the rod was 7 cm. in diameter.

steel by the fluxmeter method of electromagnetic induction, and were described before the Ohio Academy of Sciences and the American Physical Society in November and December of that year.[14]

They gave for λ the value -3.6×10^{-7} gauss/r.p.s. Another and better investigation by the same general method in 1915[14] gave $\lambda = -(3.1+) \times 10^{-7}$ gauss/r.p.s.; or -3.4×10^{-7} gauss/r.p.s. if we consider only observations at the highest speed, which there is much reason to consider the most trustworthy. All these values are equal to $\frac{1}{2} \times 4\pi m/e$ within the experimental errors.

These experiments were followed by an investigation[15] made in the same laboratory and completed in 1917 on cold-rolled steel, nickel, and cobalt. The values of λ obtained were negative as before, but were intermediate between $\frac{1}{2} \times 4\pi m/e$ and $4\pi m/e$. We also made, a little later, preliminary experiments, consistent with the others, but much rougher, on soft iron and Heusler's alloy.[16]

In the meantime, early in 1915, but *after our first conclusive experiments were described to the American Physical Society*, Einstein and de Haas[17] published experiments on the converse effect, viz., rotation by magnetization, in iron, which yielded, when interpreted on the simplest hypothesis, $\lambda = 4\pi m/e$, though *the sign of the effect was not determined with certainty*.[18] This was done later in the same year, and in 1916, by de Haas[19] and Einstein[20] separately. In 1918 a much more thorough investigation of this converse effect, in both iron and nickel, was published by J. Q. Stewart,[21] who obtained values of λ similar to those we had obtained in 1914 and again in 1915, and equal to one half the value obtained by Einstein and de Haas.

We here call special attention to the priority of our 1914 work over all related work by others, because this work has frequently and incorrectly been referred to[22] as if it had followed, or had been presented to the scientific world later than, or was a (later) confirmation of the work of Einstein and de Haas.

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[14] S. J. Barnett, *Phys. Rev.*, **6**, 239, 1915.

[15] S. J. Barnett, *Phys. Rev.*, **10**, 7, 1917.

[16] S. J. Barnett, *Science*, August 7, 1918.

[17] A. Einstein and W. J. de Haas, *Verh. d. D. Phys. Ges.*, **17**, 152, 1915.

[18] As pointed out by Lorentz. See A. Einstein, *Verh. d. D. Phys. Ges.*, **17**, 203, 1915; also W. J. de Haas, *K. Akad. Amsterdam Proc.*, **18**, 1280, 1916.

[19] W. J. de Haas, *loc. cit.*

[20] A. Einstein, *Verh. d. D. Phys. Ges.*, **18**, 173, 1916.

[21] J. Q. Stewart, *Phys. Rev.*, **11**, 100, 1918.

[22] H. S. Uhler, *Am. J. Sci.*, **41**, 558, 1916; B. Cabrera, *J. de chim. Phys.*, **16**, 455, 1918; R. A. Millikan, *The Electron*, 2d ed., p. 212, 1924 (also 1st ed.); A. Sommerfeld, *Atombau*, 4th ed., p. 635, 1924; *and numerous others.*

4. **New experiments.** The disagreement between our values of λ and the commonly accepted value of $2\pi R$ for electron orbits, and the discrepancies between the values obtained in the different investigations, together with the great importance which must be attached to a more precise knowledge of the magnitude of the constant λ for the substances already investigated and to its determination for other ferromagnetic substances, led us to begin in 1918 a more elaborate investigation of magnetization by rotation by the magnetometer method. In that year we removed to Washington, and were able to do but little more with the work before the summer of 1920. Since that time, as opportunity has offered, and especially in the years 1922–23, we have made an extensive investigation of many ferromagnetic substances. All this work is in agreement, within the limits of the accidental errors, with the fluxmeter observations of 1914 and 1915, and shows that the systematic errors were not completely removed in the earlier magnetometer work.[23]

The rotors, compensating frame, magnetometer, and other appliances constructed for this work in Ohio, and for the most part used in the earlier investigations, were kindly loaned to us by the Ohio State University. With such modifications as were found desirable most of them were used, and rendered important services, in the new work.

5. **The laboratory and its magnetic field.** Nearly all of the new rotation experiments were carried on in a small concrete laboratory,[24] forming a part of the plant of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, and located at a considerable distance from the main building, from which power was obtained. This laboratory is constructed of practically nonmagnetic material throughout. It consists of a single room with floor space 25 × 50 feet, the longer sides being nearly parallel to the magnetic meridian. The concrete floor, on which the piers carrying the rotation apparatus were mounted, is twelve inches thick.

A survey of the undisturbed magnetic field of the laboratory was made in 1920 by Messrs. W. F. Wallis and A. Sterling, and showed that the field is very nearly uniform, the departure from uniformity being entirely negligible for the purposes of this investigation. For the sake of the experimental work it was, of course, necessary to introduce into the building many instruments with considerable magnetic moments, but they were so placed, or so oriented, sometimes in conjunction with permanent magnets introduced for the purpose, as to produce negligible effects in the region occupied by the rotating body.

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[23] For brief accounts of this work up to April, 1922, see S. J. Barnett and L. J. H. Barnett, *Proc. American Phys. Soc.* for December, 1920 (*Phys. Rev.*, **17**, 404, 1921) and for April, 1922 (*Phys. Rev.*, **20**, 90, 1922).

[24] See J. A. Fleming, *Researches of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington*, **4**, 351, 1921.

Also, to keep the field in the region about the rotor as nearly uniform as possible, the greatest care was taken in the selection and testing of the materials used for the construction of the adjacent apparatus and piers, with the result that they were certainly so free from iron as to have no appreciable effects. We are greatly indebted to the Bureau of Standards for supplying us with a considerable quantity of particularly pure sand for our piers, and for the use of their magnetic separator in improving the rest of the sand which was used. Many of the non-magnetic castings were made in the foundry belonging to our laboratory in Washington.

The fluctuations of the magnetic field. Night Work. In this work, as in the earlier magnetometer work, it was necessary to make the principal experiments after one o'clock at night in order to avoid as far as possible disturbances due to fluctuations of the earth's magnetic field.

The disturbances due to the electric railways seemed to us to become worse as the time progressed, and we finally found it advisable to restrict the rotation observations almost entirely to the interval between half past two and half past four, or even a quarter after four, in the morning. Shortly after we began the next to the last series (Series A) of observations, the Capital Traction Co. started to operate cars on its Chevy Chase line, five-eighths of a mile from our laboratory, throughout the interval in which we worked. When, however, Mr. John Hanna, Vice-President, in charge of operations, learned of the serious interference this service made with our work, the electric cars were most kindly taken off and a bus service substituted. For this generous interest in our work we are greatly indebted to Mr. Hanna and the Company.

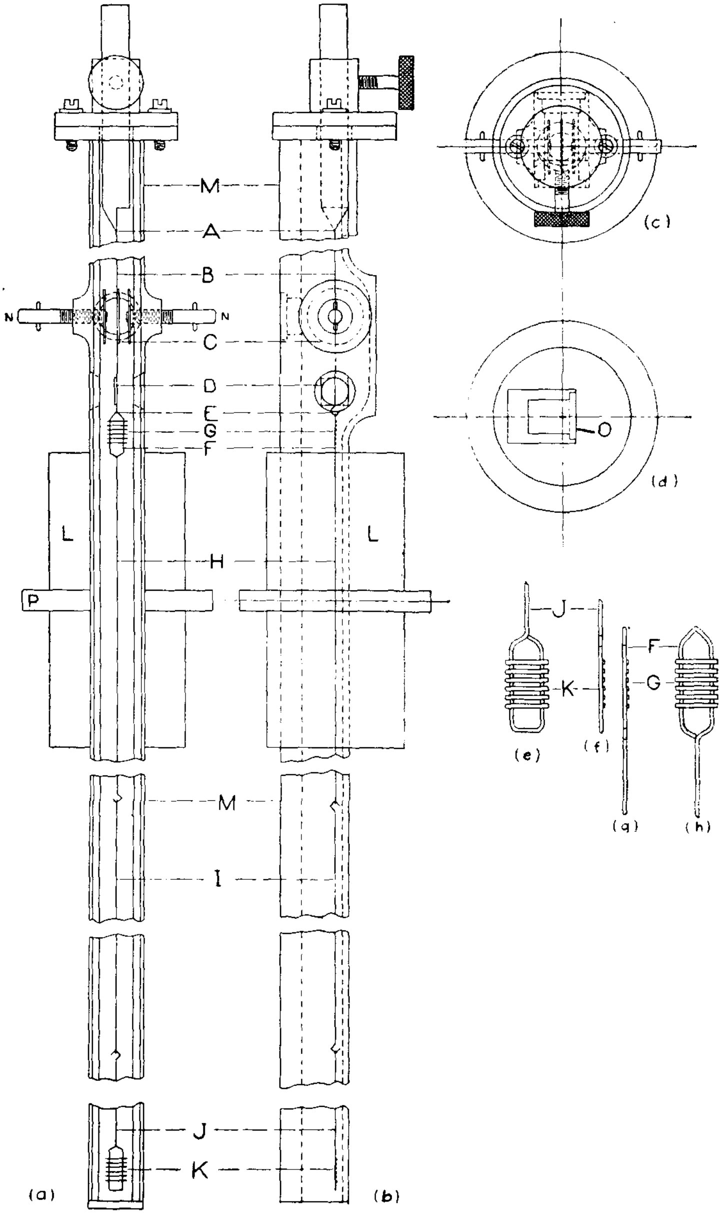
Solar disturbances, as would be expected, ordinarily gave little trouble at night. Moreover, the last and most important series of observations (Series B) was made at the time of the sun-spot minimum. For this series the average extreme changes in a single night, from two o'clock to five o'clock A.M., in the hourly means of the horizontal and vertical intensities, as calculated from the records of the Cheltenham Observatory, kindly furnished us by the Coast and Geodetic Survey, were only about 1 part in 5500 and 1 part in 35,000, respectively — quantities beyond our compensation requirements.

6. **The magnetometers.** Two magnetometer cases, one of them modified from the Ohio instrument and both constructed in much the same way, together with several suspension systems, were used in the course of the work.

The suspension, illustrated in Figure 2, was in four parts. The uppermost part, *A*, was a fine quartz fiber, whose substitution for silk considerably improved the behavior of the instrument. The fiber was attached by shellac to the torsion head above and to a geometric hook below. This engaged with a second geometric hook at the top of an aluminum rod *B*, which carried an aluminum damping vane *C* and the mirror *D* and ended below in a small geometric hook *E*. This hook carried the support *F* of the upper magnet system *G*. The support *F* was an aluminum rod fashioned into a geometric hook at its lower end and bent into a small loop with angular top and vertical sides at the upper end. The geometric hook at the bottom of *F* carried a third aluminum rod *I*, terminating above and below in geometric hooks. The lower hook carried the support *J* of the lower magnet system *K*, which was another aluminum rod fashioned into a geometric hook at the top and a loop with two sides vertical at the bottom. The distance between the centers of the two systems of magnets was about 48 cm.

There are three reasons for dividing the aluminum rod carrying the mirror and magnets into three pieces: In the first place, the shorter pieces could be made much more nearly straight and coaxial when mounted together than a single piece of the same total length; in the second place, the intermediate piece, without magnets, could be handled more safely than the others when a twist had to be produced to bring the two magnet systems into parallelism; and in the third place it was desirable for some other experimental work to have the magnet systems closer together than they were in the rotation experiments. This can be accomplished by inverting the central piece.

In all but the earlier part of the work each magnet system consisted of six or three rods about 5 mm. long, 0.17 mm. thick, and 0.35 mm. wide. They were of hardened steel and were kindly provided by Mr. Olof Ohlson, Technical Director of the Waltham Watch Company, at our request for steel treated in such a way as to make the mechanical properties as nearly permanent as possible (according to the recommendations of the Bureau of Standards). By means of a special frame the rods of each system were mounted parallel and central on the appropriate aluminum loop normal to its vertical sides and equidistant. All but one of the rods in each system were fastened per-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(f)
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(h)

Fig. 2.

manently in place with shellac dissolved in absolute alcohol. The remaining rod was attached with a minute quantity of soft wax. The two systems were then magnetized in an intense magnetic field, and their moments were compared by means of a sensitive astatic magnetometer.

It was our practice in earlier work, in case the moments did not agree within a sufficiently small fraction of one per cent, to remove the waxed-on magnet from the system having the greater moment and reduce its length on an oil stone, then to replace it, remagnetize both systems, and test again, this process being kept up as long as necessary. The two waxed-on magnets were then fastened permanently with shellac, which was allowed to dry without the application of artificial heat. The mechanical treatment, however, changed the properties of the ground magnet, and was abandoned as unsatisfactory. In the case of one suspension it happened that the two systems, whose moments on the first comparison differed by more than 3 per cent, acquired the same moment within about 0.3 per cent when the two removable magnets were interchanged and remagnetized. This system became better with time, and then retained its equality of moments within a small fraction of one per cent for more than two years. This magnetometer system we continued to use for several weeks in the beginning of the last series of observations. Then it was replaced by a system with three-magnet magnets, whose moments also were equal, and remained equal as long as tested in Washington, within a small fraction of one per cent.

Another method, much better than that of simple trial and error, was used in constructing some systems near the end of the investigation. One reason for mounting the magnets equidistant was to make the demagnetizing factors the same for the two systems. It finally occurred to Mr. Kotterman, our assistant, who made most of these systems for us, to use this principle to adjust the moments of the systems to equality. This is very easily done by simply moving one or both of the loose magnets to or from the others to decrease or increase the moment.

To adjust the two magnets to parallelism, the final method was first to hang the magnet system, in three sections, from a proper support, with a long straight aluminum wire fastened horizontally to one side of each loop. Then, if necessary, the central section (or one of the other sections) was twisted until the two horizontal wires became parallel within a fraction of a degree, the sections being separated, of course, while the twisting was done.

When the magnets are geometrically parallel, however, the magnetic axes are not necessarily parallel, and the angle between their axes had to be tested electrically, by processes discussed below in connection with other methods of testing the magnetometer, without removing the suspension from the case.

After the system was properly magnetized and astaticized the three sections together were hung from the mirror support and (usually) cemented together with minute quantities of shellac.

The moment of each of the six-magnet systems was about 0.4 c.g.s. electromagnetic unit.

The magnetometer mirrors were plane, some round and some square, and about 6 mm. along a diameter or edge. One of the mirror supports had two mirrors, facing in opposite directions, so that the magnetometer could be reversed without altering the position of the reading appliances.

The magnetometer case, in which the suspension was mounted by means of a simple torsion head, is also shown in Figure 2. The main part of the case was of soft nonmagnetic brass. It was cast in two (or three) pieces, the short thick portion L being soldered on after the long thin part (or parts), M, had been grooved to hold the suspension. In the original apparatus the supports NN of the damping plates were made to slide in tubes for adjustment, but in the magnetometers used in the present work the plate supports and the small protrusions (or castings) by which they were mounted on the case were threaded, so that excellent adjustment could be made much more readily. The case was made entirely open on one side, but could be closed by long narrow brass strips O pressed against narrow ledges milled for the purpose. Holes cut for the mirror were covered with pieces of spectacle lenses, and holes cut opposite the ends of a horizontal diameter of the damping vane covered with plane glass or mica. A small copper disc covered with a small quantity of radioactive salt was, in the later work, placed at the bottom of the case, which was sealed with soft wax.

The part L of the case was designed to be cylindrical (except for a protruding flange P) and coaxial with the suspension. The lower part of the cylinder was intended to fit accurately a cylindrical hole in the magnetometer support, A, Figure 3, to which the case could be secured by means of two (and later four) small clamps acting on the flange, P (Figures 2 and 4). These conditions were approximately fulfilled, and the magnetometer could be turned about the vertical and clamped in any azimuth with little if any readjustment.

To read the magnetometer deflections, the image of the filament of a

General Electric 10-volt 35-watt nitrogen-filled galvanometer lamp formed by the mirror and suitable lenses was focussed on a translucent scale about 6.2 m. from the instrument. The position of one edge was read to 0.1 mm.

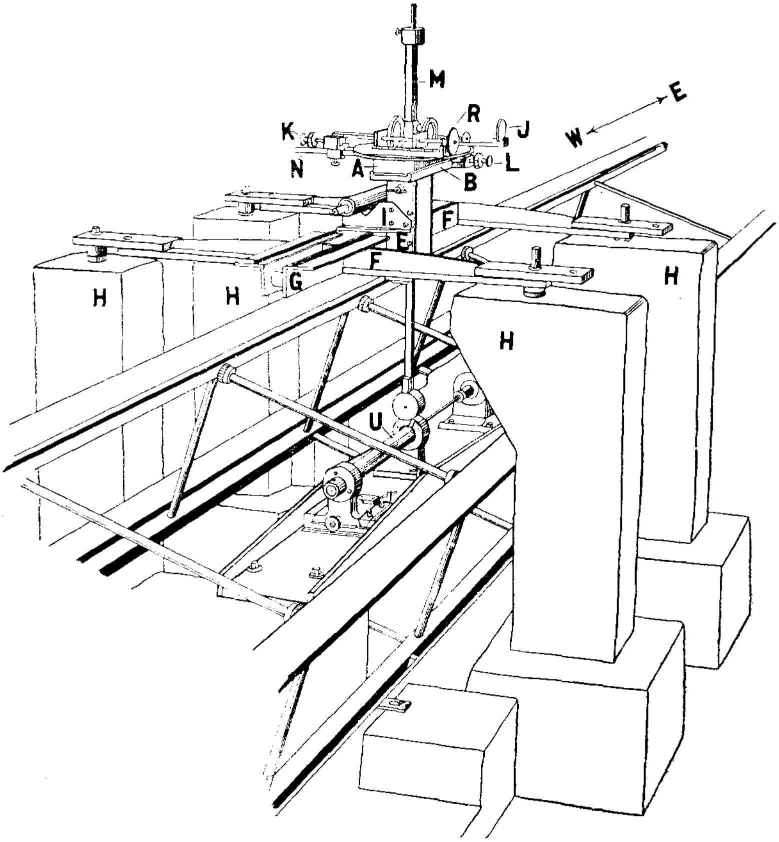

FIG. 3.

7. **The magnetometer supports.** The magnetometer supports, of brass, bronze, and concrete, as arranged in much of the work, are illustrated in Figures 3 and 4. The immediate support, *A*, was milled beneath to fit either of two slides *B*, *G*, which permitted motion east and west, parallel to the axis of the rotor. Slide *B*, by means of a projection from beneath, was movable vertically over a range of a

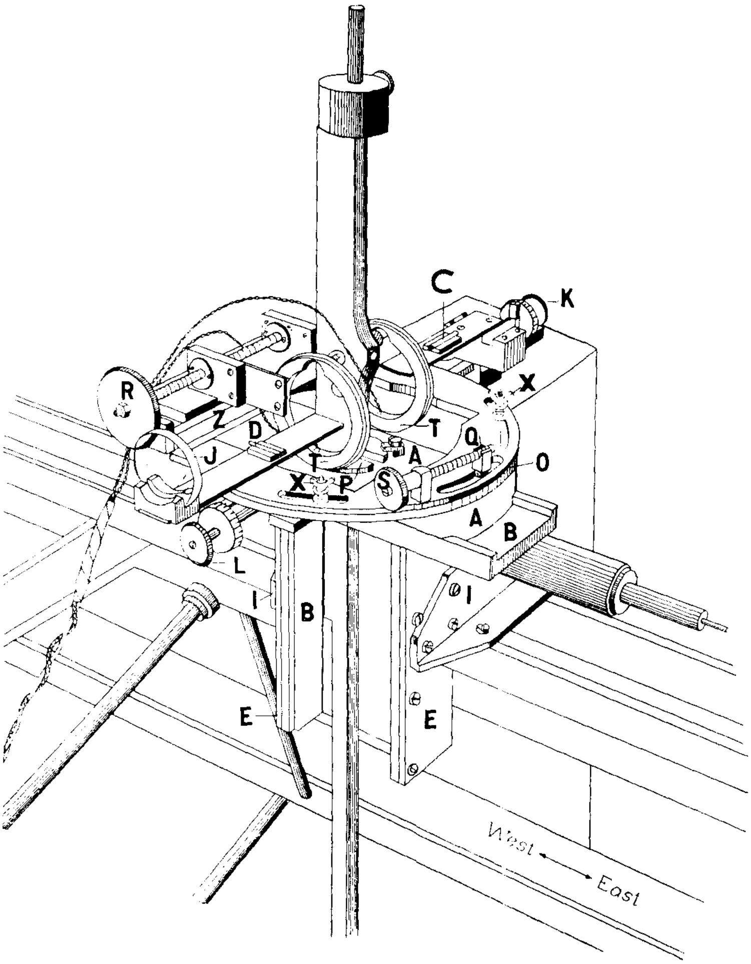

Fig. 4.

number of centimeters in the slide *E*. Clamps were provided for all the slides to prevent accidental displacement.

The slide *E* was screwed to angle pieces *I*, which were screwed to the H-form casting *F*, used in the earlier work; and the slide *G* was screwed and soldered to the same casting.

When *A* was mounted in the slide *B*, in its lowest position, the center of the lower magnet was very nearly 10 cm. above the axis of the rotor and 5.8 cm. south of the vertical plane through the axis. When *A* was mounted in slide G, the center of the lower magnet was very nearly on the geometrical axis of the rotor. In the second case, the magnetometer was said to have an *axial* position; in the first case, provided the vertical plane ϕ through the center of the rotor and normal to its axis passed very nearly through the centers of the magnets, it was said to have an *equatorial* position.

In most of the early work, strong horizontal boards were screwed between the north and south projecting arms of the casting *F* in order to reduce vibration.

The casting *F* was firmly supported by four concrete piers *H*.

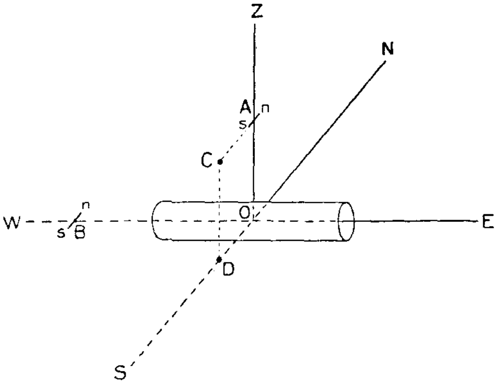

FIG. 5.

For the latter part of the work the angle pieces *I* were removed and the slide *E* was moved north about 5.8 cm. and was strongly secured by heavy angle pieces to the casting *F* in such a way that the center of the suspension lay in the vertical plane containing the rotor's axis. When, with this arrangement, the support *A* occupied such a position that the center of the suspension lay also in the plane ϕ, the magnetometer was said to occupy a *vertical-equatorial* position.

Figure 5 shows diagrammatically relative positions of rotor and lower magnetometer magnet in an axial position (*B*), and in equatorial positions (*A*, *C*, *D*), *A* being a vertical equatorial position and *D* (not used) being a horizontal equatorial position. The plane ϕ is the plane *ACD*. In the principal magnetometer positions which we have used the approximate distances were as follows: (1) Lower equatorial

(*LEQ*), $AC = 5.8$, $AO = 11.3$; higher equatorial (*HEQ*), $AC = 5.8$, $AO = 16.0$; lowest vertical equatorial (*LEQV*), $AC = 0$, $AO = 11.4$; standard vertical equatorial (*EQV*), $AC = 0$, $AO = 12.9$; near axial (*AN*), $OB = 12$; far axial (*AF*), $OB = 17$.

For the final work, in order to reduce possible errors from vibration, the whole framework *F* was covered with concrete, and the piers *H* were reinforced with concrete. The appearance of the magnetometer and its supports in this case is shown in Figure 6.

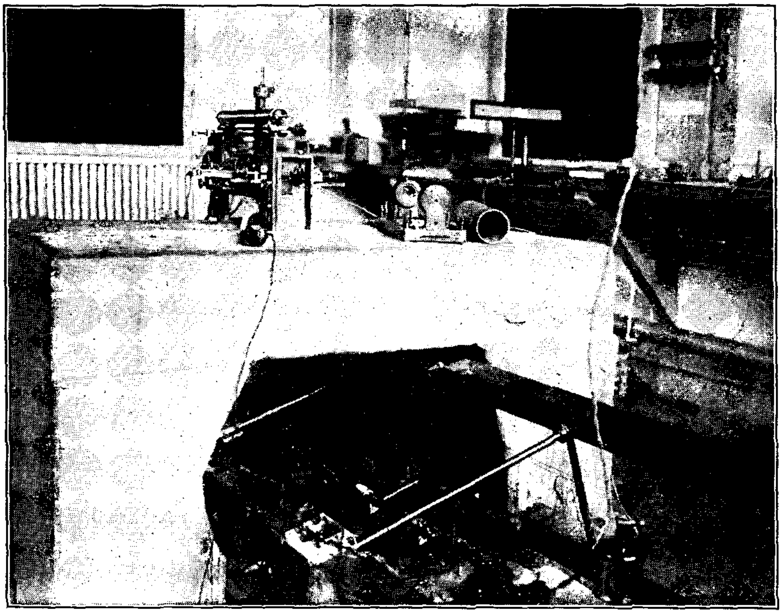

FIG. 6.

8. **The magnetometer controls.** In place of a single control magnet, as used in the earlier work, two minute magnets, *C* and *D*, Figure 4, have been used, both symmetrically arranged near the upper system. Motion of one of the magnets, *C*, with axis parallel to that of the system, alters the sensibility without altering the zero (with perfect adjustment); motion of the other, *D*, with axis normal to that of the system, alters the zero without altering the sensitivity (with perfect adjustment). Each magnet is provided with a slide for rough adjustment and a micrometer screw (*K*, *L*) for fine adjustment.

The adoption of this control system, which we have used for about seven years, has greatly lessened our labors in this and other work. Indeed the work described here would hardly have been possible without it. This system of control, in spite of its simplicity, is new, so far as we have been able to learn.

For this investigation it was necessary that the moments of the magnets be small to avoid the possibility of very objectionable eddy-current effects in the rotor. They were made from short pieces of hardened tungsten steel wire and had very small moments. The moment of the zero control changed from about 1.0 to about 0.7 c.g.s. unit, and that of the sensibility control from about 0.5 to about 0.4 c.g.s. unit from August, 1920 to December, 1923.

9. **The magnetometer sensibility coils.** In order to make it possible to measure the sensibility of the magnetometer at any time, two arrangements have been used. In one, a small pair of Helmholtz coils, easily removable, was attached to the lower end of the magnetometer case with its center approximately coincident with that of the lower magnet system and its axis normal to the axis of the system. Each coil, in the case of one magnetometer, had four turns; the radius was about 3.33 cm., and the mean distance between corresponding turns was 3.5 cm., so that the constant of the coils was about 1.05 gauss per ampere.

In the other arrangement, the two coils were mounted on the magnetometer support *A*, Figures 3 and 4, and their axes passed normally through the upper magnet on which the coil system was centered. These coils were intended to be, and were, except at first, placed so far apart that magnetometers could be exchanged without interfering with them. This arrangement has some advantages over the other, but has the disadvantage that the coils and the rotor act chiefly on different magnets, necessitating greater care in calibration. The constants of the various coil systems used did not differ greatly from one another.

10. **The main electrical compensating system.** To neutralize the earth's magnetic field in the early part of the work we used the continuously wound Columbus compensating frame. The wires on the frame were restretched, extra distance pieces were inserted, end coils were added to increase the breadth of the nearly uniform part of the field, and to make it more nearly symmetrical about the center, and the frame was mounted with precision with its longest sides normal to the magnetic meridian and its axis parallel to the earth's intensity.

This coil produced a field which was uniform along the axis of the magnetic part of the rotors within less than 1 part in 1200, and uniform over the central circular section of the largest rotor within about 1 part in 1000. We came to distrust this coil, however, partly because its dip and compensating current underwent changes which we could explain only, if at all, by assuming that warping had occurred; and we finally abandoned the coil for another of larger dimensions and better design.[25]

The new coil was designed to produce a uniform cylindrical field on the principle on which the Helmholtz double coil produces a uniform spherical field. A system of two infinite parallel wires distant a apart and traversed by the same current in opposite directions, together with an exactly similar system symmetrically placed and distant d from the first, produces a field the central portion of which is very nearly uniform, provided that $\frac{a}{d} = \sqrt{3}$.[26] If $d = 30$ cm. and $a = 52$ cm. this relation is very nearly satisfied, and calculation shows that the intensities at points distant 4 cm. from the central line in the direction of the field, and normal to this direction and to the central line, differ only by about 1 part in 10,000 and 1 part in 3000, respectively, from that at the central line.

For the coils wound on the new compensating frame a was 52 cm. and the mean value of d was 30 cm., the width (parallel to d) of each coil being about 1.7 cm. The length of the coil was 702 cm.

The construction of the frame is illustrated in Figures 3 and 6–10. There are four beams A of three-ply hardwood, 2.6 cm. square, running the full length of the frame. These beams were originally held in position by twelve sets of four thick-walled brass tubes B and C, all the tubes in each group having accurately the same length. A few of the tubes had to be removed later. The sets of tubes were nearly uniformly spaced along the frame. The beams were counterbored everywhere to the same depth to fit the tubes, which were threaded at the ends and fastened in place by brass screws.

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[25] Experience has convinced us that it would be much better to use, in place of a single coil system, two coils, traversed by different currents, one to compensate the horizontal intensity and the other to compensate the vertical intensity; also, that the wires should be stretched, without intermediate supports, between terminal machined plates. Suitable tension can be provided either by using sufficiently elastic wires or by using springs. In our last experiments on eddy currents we have used a frame built somewhat in this way for other experiments. We should prefer uninsulated wires, one or two diameters apart.

[26] See J. J. Thomson's *Electricity and Magnetism* (5th edition), p. 257.

The frame was fastened to low piers *D* on the floor with its length as nearly normal to the magnetic meridian as practicable and the brass tubes as nearly parallel and perpendicular to the earth's intensity as practicable. It was secured to brass lugs in the piers by brass angle pieces about a meter apart and brass screws, provision being made for both horizontal and vertical adjustment. An accurately machined piece of pyralin *F* was then fastened to each end of one beam and a single wire was stretched tightly along the beam. This wire was made level and the frame adjusted until the wire was normal to the meridian. With this wire as a guide, pyralin blocks, *F*, machined to hold an exact number of wires and to bring them accurately into the required positions, were placed at intervals along each beam and permanently fastened with brass screws. These pyralin blocks were intended to be placed opposite the ends of the tubes *BB*; but we discovered too late that this precaution had not been taken, and some of the blocks were not even close to the tubes.

The upper and lower coils were at first wound continuously with enameled wire, and later with silk-covered wire; but the insulation, injured in the winding, broke down and introduced uncertainties as to the exact compensation of the earth's field.

Each coil was therefore finally replaced by two coils, each consisting of nine turns of No. 20 d.s.c. wire, the strands of one coil being alternate with those of the other; and such terminal arrangements were made that by means of three double throw switches and a voltmeter the voltages over the alternate strands, as well as the insulation resistance between them, could be readily measured. When the arrangements were completed, the resistance was always far higher than necessary.

Sagging of the wires between the pyralin blocks, against which they were kept pressed, occurred twice after the coils were wound, but was done away with by means of a stretching device, illustrated in Figure 10, attached to each corner of the frame at the west end. This device did not act continuously and permanently, however, until stiff springs *SS* were inserted, and kept under strong pressure, between the frame *A* and the nut *B*, after which there was no more trouble.

To keep the cross-section of the coil system rectangular, four diagonal braces of brass, one of them with a ring at the center (Figure 7) to admit the driving shaft, were originally inserted. One brace near the end was later removed, to extend the shaft through the frame, and another was modified as in Figure 7. Later still, when the heavy concrete was cast about the magnetometer, four adjustable braces, acting between the upper part of the frame and the concrete piers *H*,

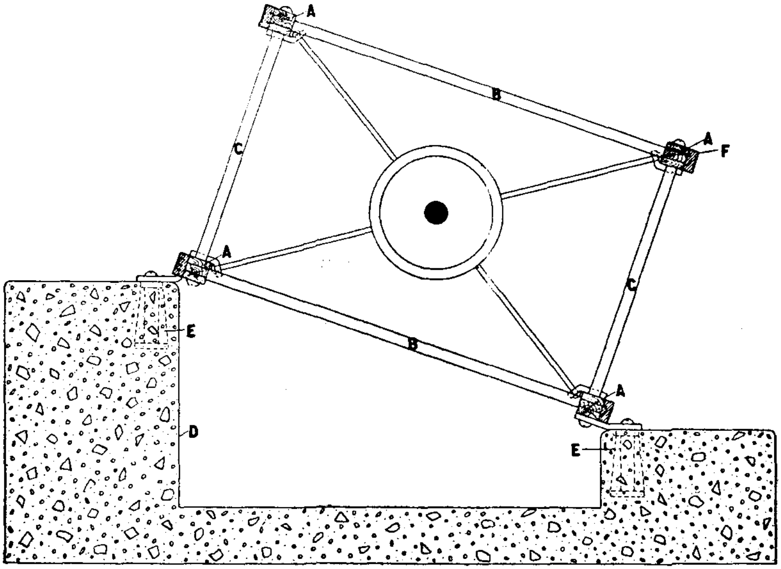
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FIG. 7.

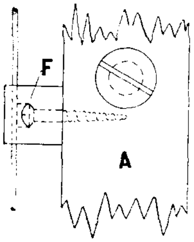


FIG. 9.

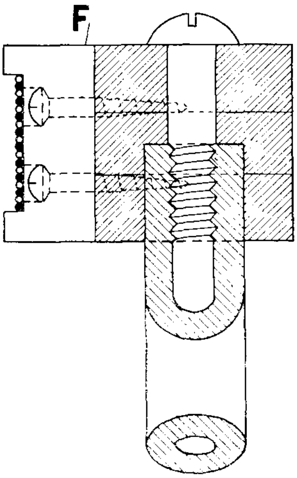


FIG. 8.

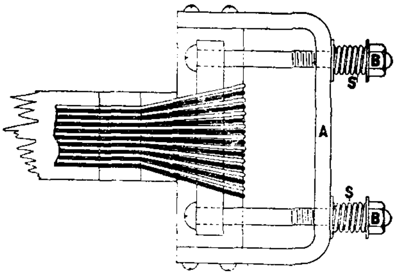


FIG. 10.

were added to fix more certainly the position of the central part of the coil system.

Several years ago the axial component of the intensity produced by the frame in a central horizontal cylindrical space about 2 m. long and 10 cm. in diameter was measured with an induction balance and was found uniform to 1 part in 2500. Later on, in August, 1923, the axial intensity (approximately parallel to the earth's intensity) along a central horizontal line about 40 cm. long was found uniform to about 1 part in 4000. The horizontal intensity over the same length was found to vary about 1 part in 400, the frame having become distorted. By means of the braces the upper part of the frame was therefore readjusted until the upper coil, whose west end we found to have moved a few millimeters south, was parallel throughout its length to the lower; and gages with fixed points of reference were set along the frame so that any future displacements could be accurately read. Many readings by means of these gages during the remainder of the work showed that no displacements beyond a fraction, usually a small fraction, of a millimeter occurred. Induction balance tests just after the frame was readjusted and at two later dates, one of them near the end of the work, showed that the horizontal component over the central line referred to was uniform to about 1 part in 14,000, 1 part in 7,000 and 1 part in 14,000 or better.

11. **The subsidiary electrical compensating system.** Changes in the elements of the earth's magnetic field, secular, diurnal, etc., together with originally imperfect construction and adjustment and later possible alterations in the compensating frame itself, and the necessity of testing for the effects of small variations from correct compensation, made it necessary to install a subsidiary compensating system. If we suppose that the *mean vertical* intensity acting upon the rotor is annulled by the current in the main compensating coil, there will, in general, be left acting on the rotor: (1) a small axial intensity, which may or may not be uniform; (2) a small uniform horizontal intensity H_0 (Fig. 11); (3) a small nonuniform north and south horizontal intensity h (Fig. 11) which may be considered (except so far as very close sources are involved) proportional to the axial distance from the center of the rotor, and thus opposite on opposite sides of the center; and (4) a small nonuniform vertical intensity, v, proportional, like h, to the axial distance from the center of the rotor.

We at first used no subsidiary system, and we made several changes before adopting the final arrangement, which alone will be described here.

To compensate the residual intensities (1), (2), and (3) we used three pairs *A*, *B*, *C* of circular cylindrical coils of enameled copper wire carefully wound on bakelite bobbins. The coils of pair *A*, used to compensate (1), were about 6 cm. thick, of No. 16 wire; the others about 7½ cm. thick, of No. 14 wire. All had internal and external diameters of about 12 cm. and 22 cm. The coils of each pair had practically equal constants and were connected in series. All the coils were mounted with their centers in the equatorial plane of the rotor, one coil of each pair being as far north of the axis as the other was south.

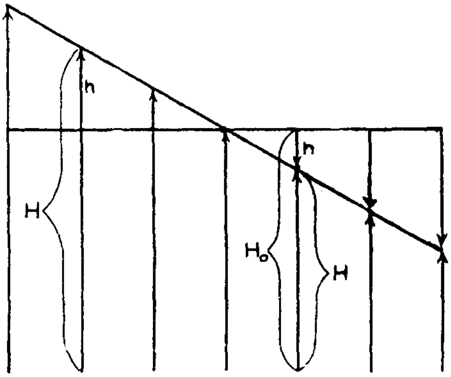


FIG. 11.

Coils *A* were mounted with their axes parallel to that of the rotor, very nearly on a straight line passing through its center, and about 433 cm. north and south of its axis. They were traversed by currents in the same direction and produced a very nearly uniform axial field throughout the rotor.

Coils *B* were mounted 400 cm. north and south of the rotor's axis, with their axes in its equatorial plane, and (finally) in the horizontal plane passing halfway between the magnets of the magnetometer when in the equatorial position used in the last experiments (earlier they were mounted lower). They were traversed by currents in the same direction and produced a field which was very nearly uniform over the magnet system, the rotor, and the compensator.

Coils *C* were mounted about 416 cm. from the vertical plane through the rotor's axis, with their axes parallel to this axis and in the same horizontal plane which contained the axes of *B*. They were traversed by currents in *opposite* directions, when used normally, to produce a nonuniform horizontal intensity at the rotor, normal to it, and proportional to the axial distance from its center; but the currents could be given the *same* direction for purposes of magnetometer testing (see below, § 27).

The south coils of the pairs B and C, and the uppermost part of the south coil of the pair A, are visible in the lower left part of Figure 12, which shows much of the experimental apparatus in its final state.

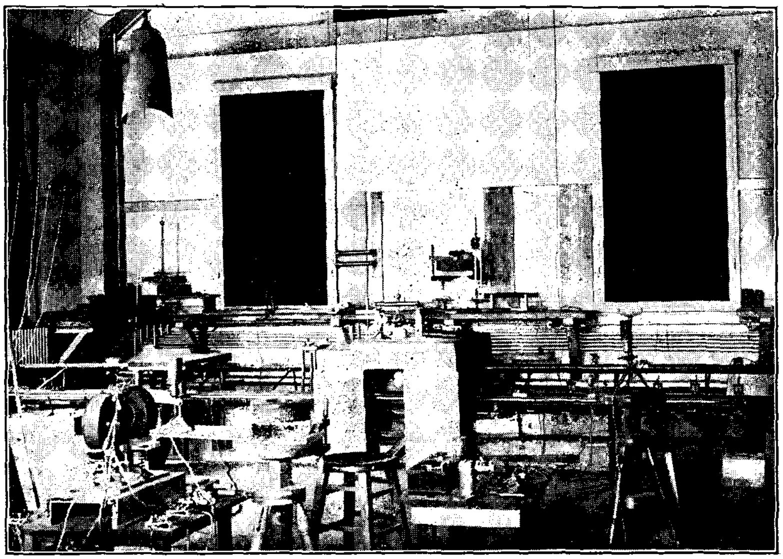


FIG. 12.

The three coils north of the magnetometer were mounted over a brass plate on a concrete block in a partially double-walled box outside the laboratory, and were connected to the interior by lead-covered twisted wires. One coil of each pair was mounted on a tripod with adjustments for altitude and azimuth, which, however, proved unnecessary.

The field of each of the coils in the region occupied by the rotor was, of course, not strictly uniform. The maximum variation for any coil, however, was less than 1% of its value at the center of the rotor. Hence, especially as that central value was itself never greater than about 1% of the earth's horizontal intensity, the degree of uniformity of the field produced by the combined compensating system in the region occupied by the rotor was very high.

The horizontal intensity, normal to the rotor and directed south, produced by system *C* along the east half of the rotor, when the axis of the inner coil was directed west and that of the outer coil east, for a current of 1 ampere in the coils, is given in Figure 13. For the other half of the rotor the intensity was equal and opposite. When the current in the outer coil was reversed, the (axial) intensity acting on the rotor was about 0.0044 *H*, where *H* denotes the horizontal component of the earth's intensity.

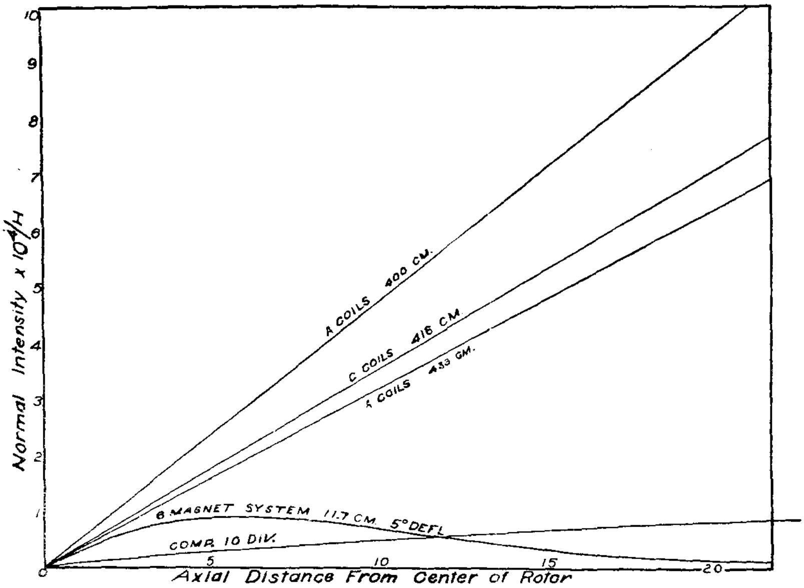


FIG. 13.

Currents of 1 ampere in coil systems *A* and *B* produced at the rotor intensities about 0.0063 *H* and 0.011 *H*.

In order to annul any nonuniformity in the magnetic field acting on the rotor in the direction of the axis of the main compensating frame, an extra coil or loop consisting at first of one turn and later of two turns, parallel and near together, was provided.

In the east-west direction the wires of this loop were of the same length as those of the frame. The median plane of the loop coincided with that of the frame, and its axis, when its wires were parallel, coincided with that of the frame. By means of two slides at the west end of the loop, and two similar slides at the east end, each with a range of 4 cm., the west ends of the wires could be moved apart and the east ends closer together, or vice versa, while the distance between them at the center remained constant (60 cm.).

When the slides of the double loop are each displaced, outward at one end and inward at the other, the maximum amount 4 cm., and the current flows through all the wires in the same direction, calculation shows that over the central 40 cm. containing the rotor the variation of the total intensity thus produced is about 0.0004 of its value at the center. With this arrangement the double deflections produced by rotating copper with the field through the rotor parallel to the axis of the frame annulled could be made zero. But under these conditions the deflection produced by rotating permalloy and annealed steel rotors did not differ from those for the same mean compensating field when the wires were all parallel. The use of the loop was soon abandoned in favor of the extra coil system whose final arrangement has already been described.

12. **The variometers.** In order to be certain of the limits within which the earth's magnetic field in the region occupied by the rotor was compensated, it became necessary either to make very frequent observations with inductors, a course consuming more time and energy than were available in the best hours of the night, when the rotation work had to be done, or else to use three variometers, whose readings could be obtained quickly at any time and could be used in connection with less frequent inductor observations. This latter course was followed in all of the latest work, and in some of that which preceded.

Horizontal intensity and vertical intensity variometers and a declination variometer were installed in the laboratory. The first was a Wild-Edelmann instrument with a bifilar suspension; the second a Lloyd balance of the Wild type by Edelmann; the third a C. I. W. magnetometer (No. 4). As the instruments were finally arranged, 1 mm. deflection was produced on the first by a change of about 1 part in 2244 in the horizontal intensity; 1 mm. on the second by a change of about 1 part in 3400 in the vertical intensity; and one scale division on the third by a change of 1.5 minutes in the declination. The Lloyd balance, as is usually the case, could not be relied on over great lengths of time, the base line being subject to unavoidable changes; but it could be used over short intervals with advantage.

The declination instrument was quite reliable, the horizontal intensity variometer usually so.

The readings of all three instruments were, of course, affected by the currents in the compensating coils, but these were readily allowed for. Most of the observations, indeed, were made when these currents were zero; or when all were zero except that of the main compensating frame, whose two coil systems, exceedingly close together, were then connected in opposition. They were sometimes connected in this way in order to let the current have more time to become steady while the variometers were being read.

The temperature coefficients of the horizontal and vertical variometers were obtained by means of values of the intensities furnished by the Coast and Geodetic Survey from observations at the Cheltenham Observatory. These coefficients for the horizontal and vertical intensity instruments, as used in the more recent work, were −2.5 mm. per degree C. and −1.05 mm. per degree C., respectively.

Inasmuch as the observations show that the compensating frame itself underwent small changes with the time, the use of variometers was not so reliable as frequent tests with inductors; but the observations show also that the small changes occurring were of little importance when the magnetometer adjustments were approximately correct.

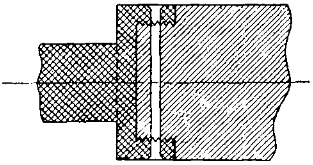


FIG. 14.

13. **The types of rotors.** After some magnetic tests made for us by Mr. D. M. Wise with iron cylinders of different diameters and lengths, it was decided to adhere to rotors of about the dimensions used in the Columbus magnetometer experiments. Indeed, many of the rotors we have used are Columbus rotors, usually modified.

The type of rotor, with half-inch journals, used in Columbus, was also used in the earliest part of the more recent work. The terminal parts, including the journals, were of bell-metal, and were soldered to the magnetic material. Two of the rotors, of Heusler's alloy, were both originally of this type, except that the journals were cast integral with the main cylinder, and the whole ground into final form. One of these, known as Heusler's alloy II, was left in the original form; the other, Heusler's alloy I, was later modified as shown in Figure 14, which gives a longitudinal central section of one end. The end of the main cylinder was threaded and a brass cap integral with a half-inch bell-metal journal was tightly screwed onto it and pinned in position. At the other end of the bell-metal journal, and shrunk into it, was an eighth-inch journal of Stellite II, as in Figure 15. All the journals were accurately coaxial cylinders, the corresponding journals at the two ends of each rotor had very nearly equal diameters, and the stellites were carefully lapped and polished.

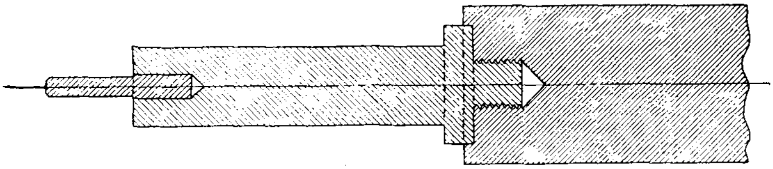
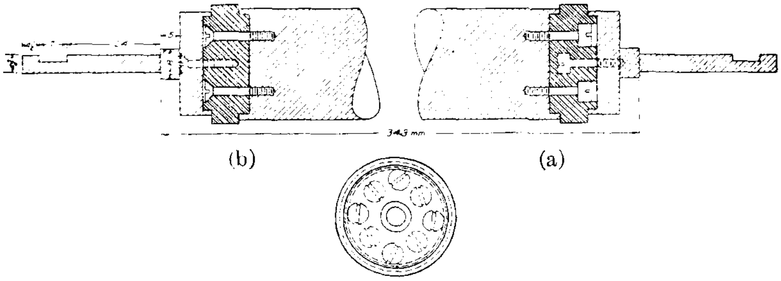


Fig. 15.

Four other styles of magnetic rotor were used, improvements being made as the work progressed. The first was similar to that of the old rotors, but with stellite journals added as in the case of Heusler alloy I. The second is illustrated in Figure 15. The bell-metal journals were screwed as well as soldered (with certain exceptions, as permalloy) or pinned, to the magnetic material; and the stellite journals were added as in the other rotors.



(b) (a)

(c)

Fig. 16.

In the third type, illustrated in Figure 16 (a), each journal piece was cut out of one piece of drawn phosphor bronze. A disc of bakelite dilecto about 1 cm. thick was screwed to the flange, and the disc then also screwed to the rotor, both sets of screws being deeply countersunk in the bakelite. In the fourth, Figure 16 (b) and (c), the construction was somewhat similar, but mechanically much stronger. The bakelite disc was screwed to the rotor, then the bronze flange was screwed to the disc, both sets of screws passing through nearly the whole thickness of the bakelite.

The journals of these two types of rotors were carefully worked to true and nearly equal cylinders and to fit with minimum practicable play the three-sixteenth inch agate bearings described below. The lengths of these rotors, from journal proper to journal proper, were made almost precisely the same, 343 mm., and the two ends were almost exactly alike, including the location of the notches. The approximate dimensions of the journals are given in millimeters in Figure 16 (a) and (b). All parts of the figure are drawn to the same scale.

14. **The magnetic parts of the rotors.** Steel I and Steel III were cut from the same rod of commercial cold-rolled steel, which was not analyzed; Steel II and Steel IV were similarly cut from a smaller rod of the same material. Steel II and Steel IV were annealed, the other two were not, except as some annealing occurred while the journals were being attached.

Cobalt I and II were machined from a single rod cast for us by the Deloro Mining and Reduction Company, through the kindness of Mr. S. B. Wright. The material from which the Cobalt II was made we annealed, but the process had no appreciable effect on its properties. The complete rod was probably annealed in Deloro. No special analysis of the material was made, but from analysis of other specimens from the same company it probably contained about 95 or 96% cobalt, 3% nickel, 1% iron and smaller quantities of carbon, silicon and sulphur. Cobalt, as is well known, is difficult to cast and to work and our rotors were not homogeneous. Cobalt II was better than Cobalt I, which was rotated in Ohio, and was statically balanced.

The Armco Iron rotor and compensator were cut from a single rod, for which we are indebted to the Page Steel and Wire Company.

The Norway Iron rotor and compensator were cut from a single rod of the commercial material.

Electrolytic Iron I and its compensator were cast from the same heat at the Bureau of Standards, and contained only minute quantities of impurities, carbon (0.03%) being much the most prominent. The rotor was far from homogeneous, but was statically balanced, though only after the chief experiments with it were made. For this rotor and compensator, as well as for several described below, we are indebted to Dr. Burgess, Mr. Jordan and Mr. Neville.

Electrolytic Iron II was forged by the Westinghouse Electric & Mfg. Company from an ingot of Yensen's pure electrolytic iron, melted in an induction vacuum furnace. We are indebted to the company and Mr. Yensen for the material. The rotor is apparently very nearly homogeneous and only a slight improvement was made by static balancing.

The nickel rotors were all machined from Boker's soft, pure nickel.

For the Iron-Cobalt, Cobalt-Nickel, and Iron-Nickel (Hopkinson's alloy) rotors and compensators, we are indebted chiefly to Dr. Burgess and Messrs. Jordan and Neville of the Bureau of Standards. Melting was done, as was the case with Electrolytic Iron I and its compensator, in crucibles of zirconium silicate, or lined with zirconium silicate, in an induction furnace.

The Iron-Cobalt (Preuss's alloy) rods contained about 34% cobalt, 0.2 and 1.5% silicon, 0.04 and 0.06% carbon.

The Cobalt-Nickel (Bloch's alloy) rods contained about 45% nickel, 54% cobalt, nearly 0.9% silicon and 0.3% carbon, and minute quantities of manganese, phosphorus and sulphur.

The Iron-Nickel (Hopkinson's alloy) rods contained about 24.5% nickel, 75.0% iron, and small quantities of silicon, etc. After being cast these rods were cooled for some time to −15° C. (rotor) and −18° C. (compensator), through the kindness of Dr. C. W. Kanolt.

All these rods were cooled very slowly in their moulds in the furnaces after casting, and thus came out annealed. All the rotors required balancing, the Preuss's alloy least, Hopkinson's alloy most.

For the permalloy[27] rotor and compensator we are indebted to Dr. Arnold and the Western Electric Company who were so kind as to supply us with the most nearly homogeneous rods they could produce and at a time when the development of the material was still in progress. The rods were "dead soft annealed," and the rotor required little (static) balancing, although its density was about 1% less than it would have been if it could have been thoroughly forged.

Permalloy, as is now known, contains about 80% nickel and 20% iron, and is extremely susceptible in weak fields. We could not, of course, take full advantage of this susceptibility on account of the considerable demagnetizing factor entailed by the shape of our rotors.

Heusler's alloy I and II were cast in an extremely cold room from "extra pure" aluminum, pure manganese and electrolytic copper, by

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[27] H. D. Arnold and G. W. Elmen, *Journal of the Franklin Institute,* **195,** 621, 1923.

Professor W. J. McCaughey of the Ohio State University, and one of us. The ingredients were inserted in atomic proportions, melted in a graphite crucible, and cast in a heavy, specially prepared, graphite mould, with projections for journals greater than half an inch in diameter. The two rods were cast from separate heats, and had considerably different moments in weak fields. They were cast with axes vertical and are somewhat porous at the upper ends. Heusler's alloy II was not balanced; Heusler's alloy I was statically balanced for the final work.

TABLE I.

The Rotors and Some of Their Constants.

1	2	3	4	5	6
Rotor	Diameter	Weight	Moment*	K (end correction)	Conductance†
	cm.	kg.			
Steel I (cold-rolled)	3.10	1.88	0.76	2.1%	0.122
Steel II "	2.31	—	0.62	3.2	——
Steel III (annealed)	3.09	1.86	1.00	0.8	0.122
				(1.5 AF)	
Steel IV "	2.32	1.07	0.85	1.0	0.064
Electrolytic iron I	3.18	1.95	1.03	0.6	0.170
Electrolytic iron II	3.32	2.15	1.00	0.8	0.197
Norway iron I	2.79	1.52	0.79	1.8	0.127
Armco iron	2.56	1.27	0.71	1.9	0.110
Cobalt I	3.20±	2.21	0.82	1.7	0.052
Cobalt II	3.08	2.02	0.82	1.9	0.051
				(2.0 AF)	
Nickel I	2.26	1.14	0.75	1.7	0.066
Nickel II	3.04	2.07	0.94	0.9	0.157
Nickel III	3.01	2.01	0.93	1.0	0.153
Preuss's alloy (Fe-Co)	3.28	2.08	1.09	0.6	0.148
				(0.4 AF)	
Bloch's alloy (Co-Ni)	3.20	2.04	0.70	2.5	0.044
Hopkinson's alloy (Fe-Ni)	3.25	2.08	0.44	5.1	0.021
Permalloy (Ni-Fe)	3.34	2.32	1.25	0.0	0.130
				(−0.25 AF)	
Heusler's alloy I	3.14	1.57	0.48	4.6	0.043
Heusler's alloy II	3.13	1.60	0.37	8.1	0.045
Copper	3.07	2.07	—	—	——
Wood	3.6	1.27	—	—	——

* Relative to Steel III (approximate).
† Relative to copper rotor.

The rotors made from drawn or forged rods of nickel, iron and steel all appeared to be very nearly homogeneous, and little or no improvement could be made in attempts to balance some of them statically.

The Vibration Specialty Co. of Philadelphia endeavored to balance a number of the rotors both dynamically and statically. They were able in some cases to improve slightly on the static balancing done in our own laboratory, but their machines were not sufficiently delicate to balance our rotors dynamically. The balancing was done on the advice of Mr. Gano Dunn.

The diameters of the rotors in their final states, their weights, and some of their other constants are given in Table I.

15. **The bearings, countershafts, bedplates, etc.** The bearing piece in which the rotor moved was a great improvement over that used in the earlier investigation. It is illustrated in Figure 17. The bearing block *A*, at the east end, was bolted to the base *C*. At the west end the base was provided with an accurately made slide and jib *G* in which the heavy block *B* could be moved longitudinally either directly or by the screw *D*. It could be clamped in any position by the jib and its screws. Eight projections (four at first) on the bottom of the base were planed for attachment to a heavy bedplate, Figure 3, by thumb screws, some of which are shown in Figure 3. The base was provided with pins (removable in the latest part of the work) which fitted precisely into holes in the bedplate, making it easy to place it always in the same position.

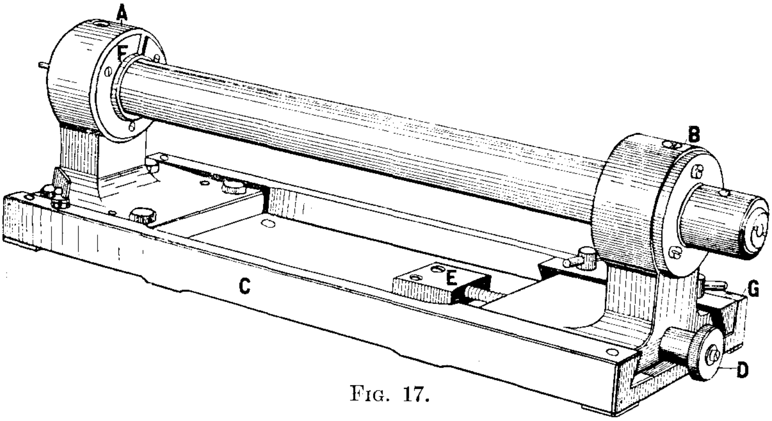

FIG. 17.

In some of the earliest work the original half-inch lumen metal rotor bearings were used. Later, new lumen bearings, made with greater precision, were substituted for them, as were also, sometimes, half-inch babbit bearings in bronze castings shaped externally like the original lumen bearings, so as to fit the blocks *A* and *B*. Half-inch bearings of lignum vitae, bakelite, and San Domingo mahogany were also used on some occasions.

In much of the work a half-inch bearing was used at the driven end of the rotor, and a small bearing of bronze, or later, in most cases, agate, fitting the eighth-inch stellite journals, was used at the other end. The construction is shown in Figure 18, where *D* is the bronze or agate. A lock nut, which is not shown in the figure, was later provided, as on one occasion the agate became loose.

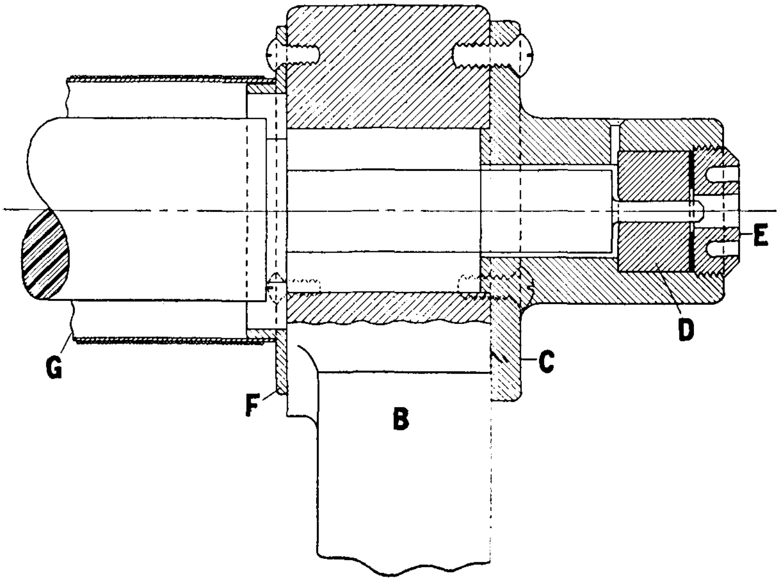

Fig. 18.

For the last series of experiments, beginning in August, 1923, three-sixteenths inch bakelite bearings, and, a little later, three-sixteenths inch agate bearings, were provided for both bearing blocks, the principles of construction and mounting being similar to that of Figure 18, with the details different, as indicated in Figure 19. The bearings had to be moved inward, as the journals were closer together, and the blocks had to be bored out to admit machined castings heavy enough to hold the agates or bakelites with sufficient firmness.

For the excellent work done in making the agate bearings we are indebted chiefly to Mr. A. F. Erickson, whose services were secured for us through the kindness of Mr. Olof Ohlson of the Waltham Watch Company.

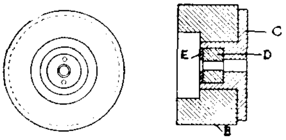

Fig. 19.

The bearing piece was for a long time mounted on the bronze bedplate, Figure 3, used in Columbus. In the later work this was replaced by a considerably heavier bedplate, Figure 6. This plate was much freer from magnetic material than the older plate; instead of the sloping ribs it was provided with machined horizontal ribs, which were exceedingly useful in several ways; and better workmanship on its mounting surfaces made more precise adjustments practicable.

At the east end of the plate was mounted the small countershaft used in Columbus (including, later, a new rod freer from iron), Figures 3 and 6. In all the later work each of the bearing blocks for this countershaft was electrically insulated from the bedplate.

When the rotor was driven by a half-inch journal at the east end, the countershaft was connected with the journal by a small rod (finally of phosphor bronze and one-fourth inch in diameter) and a chuck (also finally of phosphor bronze) arranged as in Figure 20. The rod, in the later work, passed through two guides of lignum vitae, Figure 6, which served to protect it from accident and which were originally intended as bearings to prevent vibrations — an arrangement which proved impracticable.

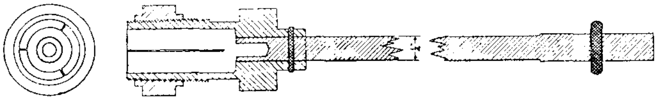

Fig. 20.

The rod and chuck, carefully balanced to prevent vibration, used in the last series of observations, on rotors with three-sixteenths inch journals, are illustrated in Figure 21.

The small countershaft on the bedplate was connected by a somewhat thicker shaft, five-sixteenths inch in diameter, about 42 cm. long, and provided with suitable chucks, to a third and larger shaft. In the earlier part of the work the third shaft was provided with a pulley and driven by a belt, or by a gear box, connected with the source of power. In the later work it was directly connected by flexible couplings (steam hose) and one or more additional shafts, connected by means of steam hose, to the gear box. In all of the work of the last two years, there were six of these larger shafts between the small shafts and the gear box.

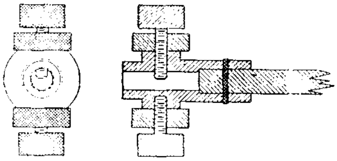

FIG. 21.

The bedplate and countershaft bearing blocks were bolted to brass cones or rods embedded in concrete piers cast on the concrete floor, with the axis of the rotor bearings (approximately) level and parallel with the wires of the compensating frame; and with the center of the rotor close to the center of the frame.

The later (and heavier) bedplate was bolted to the piers directly at twelve points; and at four points, under the central portion, to heavy brass rods ending below in concrete.

16. **The source of power.** The ideal source of power for this investigation would have been compressed air; but suitable compressed air appliances for producing and controlling with precision the speeds of our rotor were not available. The next best source of power is a polyphase electric system, but this also was not available when our work in Washington began. Hence we had again recourse to single-phase alternating current motors.

A large repulsion motor of this kind was used in the early part of our work in 1914, but it was abandoned for compressed air, because we suspected it of introducing a systematic error. Of this, however, there was no experimental evidence; and we used a small motor with repulsion start in the work of 1916–17, with an error which was undoubtedly small. As a possible source of systematic error with repulsion motors we had suspected the commutator, whose function is rectification; and Mr. Gano Dunn, who was not aware of our suspicion, later suggested the same idea to us and informed us that it was correct. The only commutator motor which we had then tested (in its proper position and by means of the astatic magnetometer) had shown no certain systematic effect; but larger motors of the same type, by which it had to be replaced to get sufficient power for the higher speeds, all showed distinct effects on reversal of direction of rotation.[28] One of these larger motors was mounted inside the building just inside the east end of the frame; the others which were tested, in a concrete pit east of the building and connecting with the interior through a small tunnel. The motor, when mounted in the pit, was about 6.7 m. from the magnetometer. Two of these motors were used in the rotation experiments, and the effects referred to were determined and allowed for; but we thought it desirable to eliminate them entirely. Hence we concluded to use a three-phase system when that became practicable, and finally mounted in the pit a General Electric 5 h.p. three-phase slip-ring motor, which was used in all the later work without appreciable systematic error.

17. **The speeds.** For this work it was important to obtain speeds as nearly uniform as practicable, and independent of the direction of rotation. Belts did not prove satisfactory, and we soon adopted spiral gears. Two boxes were made in the shop to hold bronze gears cut by the Philadelphia Gear Works, both with plain cylindrical bearings. Later, one box had thrust ball bearings inserted, and was substituted for the other, used first. One shaft of the box was joined to the motor shaft by a flexible coupling, the other to the countershaft most remote from the rotor. The speeds obtainable were 1/1, 2/1, 1/2, and 2/7 times the speed of the motor, which was a four-pole machine operating at about 30 r.p.s. at the frequency 60 cycles per second and with the secondaries short-circuited. The motor was started with a standard controller, and had also inserted in the secondaries three adjustable extra rheostats with resistances much greater than those of the controller. Thus the speed could be varied, between wide limits, either by considerable steps or by small gradations.

In much of the work a Weston magneto, repeatedly calibrated, was connected by a flexible coupling to the gear shaft operated directly by the motor, and speeds were determined from the voltmeter readings (and, in the later work, the magneto temperature). In nearly all of the recent work, however, the magneto was used only as an adjunct, and the speeds directly determined with a standard chronometer and a standard chronograph. For the loan of this chronograph we are indebted to the Coast and Geodetic Survey. The substitution of chronograph for magneto considerably increased the labor involved, but until it was done we were often uncertain whether apparent speed troubles were real or were due to magneto imperfections.

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[28] For the opportunity of testing various motors without their purchase, we are indebted to Mr. Eastman of the National Electric Supply Co.

The motor ordinarily required from 5 to 7 seconds to reach approximately full speed. It was operated ordinarily for 30^s (or sometimes 35^s). Speeds were determined from the time of 500 revolutions (approximately 16^s, except during special tests) of the motor near the end of that interval.

The average behavior of the motor as finally used when developing the maximum power, with the gear ratio 2—1, is illustrated in Table II, which gives a mean time of each hundred revolutions of the motor between successive chronograph breaks for the 700 revolutions ending with the last break before the magnetometer was read. Each mean is obtained from thirty-five runs from four sets of observations taken at random from the last series (Series B). Group 1 is the earliest in time, group 7, the latest.

TABLE II.

MEAN TIMES OF SUCCESSIVE HUNDREDS OF REVOLUTIONS OF MOTOR.
GEAR RATIO 2—1.

Order of group	1	2	3	4	5	6	7
Mean time in seconds	3.34_2	3.28_2	3.27_7	3.27_1	3.27_3	3.27_4	3.27_4

18. **The upper compensating system.** In order to make the sensibility of the magnetometer and its zero approximately independent of the current in the main compensating frame, it was necessary, as in the earlier experiments, to provide special coils, Figure 4, *T*, *T*, to compensate approximately the horizontal intensity of the earth's magnetic field in the region occupied by the upper magnet system. As in the earlier experiments, two coils forming an approximate Helmholtz pair, centered on the upper magnet, were used. In the final apparatus each coil consisted of a single turn of wire about 6.8 cm. in diameter, wound on a vertical ring of bakelite dilecto, the rings being screwed to brass bases movable for axial adjustment in a brass slide

by a right-and-left-handed screw *R* in such a way that the magnet was always approximately at the center of the system. As before, the coils were connected in series with the main compensating coil and in such a way as to oppose the earth's horizontal intensity.

The brass slide *Z* was screwed to a horizontal circle coaxial with the suspension and moving in a second coaxial circle *O* screwed to the magnetometer support *A*. A tangent screw *S* provided slow motion in azimuth.

By the axial motion of the coils the sensibility was easily made identical, or nearly identical, when the current was on or off; and by the motion in azimuth the zero was made identical, or nearly identical, without the necessity of adjusting the control magnets. These arrangements were very great improvements over those of the earlier experiments.

The control magnet slides, already referred to, were mounted on the fixed circle *O*.

19. **The method of calibration. Slow rotation during calibration.** The method of calibration used in nearly all the work was much superior to that used in the first magnetometer investigation, and involved three processes *A*, *B* and *C*, including the use of three coils as primary, secondary and reduction standards.

In process *A* it is necessary for convenience and precision that the rotor be in rotation, so that a mean effect is obtained; otherwise readings must be taken for several equally spaced position angles of the rotor, as was done in the earlier work. When the earth's field is just compensated this necessity arises chiefly from the fact that the field about the magnetometer magnets due to the residual magnetism of the rotor, in both direction and magnitude, is in general a function of the position angle. When the earth's field is not entirely compensated there is a similar effect on the magnetometer due to the fact that the rotor is not in general magnetically symmetrical about its axis.

In processes *B* and *C*, where only *ratios* of sensibilities are involved, slow rotation, when necessary, is required only on account of a rotor's lack of axial symmetry.

Even with unsymmetrical rotors no appreciable error is made in process *C*, and only minute errors in process *B*, if the rotor is kept at rest.

Throughout nearly all of the investigation described here the rotor, in processes *A* and *B*, was rotated slowly and uniformly either by a small and remote induction motor, or by the motor used in the principal experiments, the speed in this case being reduced either by gears or by suitable impedances in the circuit or circuits. In some cases, for special reasons, the calibrations were made at the speeds of the principal experiments.

20. **Process A. The magnetometer or absolute sensibility.** In process *A*, the sensibility of the magnetometer itself, largely independent of the particular rotor and compensator under investigation, was determined by reversing a minute known current *I* through the small *sensibility coils* (the *secondary standard*) centered on the upper or lower magnet, and adjusting the sensibility control magnet until the desired deflection, *A*, was obtained. This is the *magnetometer*, or *absolute*, sensibility.

In process *A*, as also in processes *B* and *C*, observations were made on a regular time schedule, reversals, alternately in opposite directions, being made at equal intervals. In process *A* the readings (and reversals) were usually 30^s apart.

This sensibility was ordinarily determined at the beginning and at the end of each series of observations. Large deflections, of the order of 20 cm., read to tenths of a millimeter, were ordinarily used in order to reduce the accidental error, with the result that quite sufficient precision was obtained, although the determinations were usually made just before the beginning and just after the end of the best part of the night. Usually the sensibility changed but little during a long series of observations, so that the mean of the two sets was applicable with negligible error to all of the series. In the last series of observations the average absolute error in the sensibility made in this way was about one-eighth per cent. Interpolation was used when necessary. For some (shorter) series the sensibility was obtained only at the beginning or at the end.

As several absolute sensibility coils were used in the course of this work, sensibilities obtained with other coils are all reduced to those which would have been obtained with the coil used exclusively in the last series of observations.

21. **Process B. The rotor sensibility and the ratio of rotor sensibility to magnetometer sensibility.** In process B the ratio of the *rotor sensibility* to the magnetometer sensibility was obtained. To form the *primary standard*, a brass tube, *G*, Figure 18, with length and internal diameter somewhat greater than those of the magnetic part of the largest rotor, was uniformly wound on a lathe with a single-layer solenoid of insulated copper wire. One end of this solenoid was

soldered to the adjacent end of the tube, and the other end of the solenoid and the adjacent end of the tube were soldered to two twisted leads. The tube was slipped over the rotor and then held centrally and coaxially in position by two properly fitting rings *FF* attached to the bearing blocks *A*, *B*, Figures 17, 18. By obtaining the deflection *B* which would be produced by the reversal through this solenoid of the same current *I* as that used in process *A* the *rotor sensibility B* could be determined.

If *A* denotes the deflection for a given current produced on reversal in the first case, and *B* that in the second case, the ratio $Q = B/A$ of the two sensibilities is fixed for a given rotor in slow rotation and a fixed position and condition of the magnetometer.

In general the magnetometer sensibility was determined, as already stated, immediately in connection with each group of rotation observations at night. The rotor sensibility in the latest work was not ordinarily determined with the magnetometer sensibility equal to its value in the principal experiment; but in this work the ratio $Q = B/A$ was usually determined in the daytime with magnetometer sensibilities of the order of one-tenth those used in the principal experiments. In some cases, for the sake of additional security, determinations were made with the greater sensibilities, but no appreciable differences were found. The ratio B/A was always determined from long series of large deflections, which were usually made approximately equal by making one of the currents an exactly known multiple of the other, and multiplying by the proper factor.

In determining the quantity $Q = B/A$ it is important, for precise work, especially when the upper absolute sensibility coil is used, to have the field in which the rotor moves approximately neutralized. This is necessary because *Q* depends on the ratio of the moments of the magnetometer magnets, and this ratio is different with and without compensation. The effects produced by compensation, however, are very slight, *Q* with and without compensation differing by only about one-third per cent.

22. **Process C. Reduction to uniform field. The reduction standards.** If the calibrating solenoid used in obtaining the rotor sensibility *B* were infinite in length, it would, for equatorial positions of the magnetometer, have *no effect* on the magnetometer and would produce a uniform axial field throughout the rotor, like that of the intrinsic intensity of rotation. The shortness of the solenoid actually used, as well as the work with axial positions of the magnetometer,

necessitated the third process C, in which for each rotor and position, so far as necessary, a determination was made of the ratio of C, the rotor sensibility as it would be if the solenoid were very long, to B.

For this purpose two other solenoids, the *reduction standards*, were constructed—one for equatorial positions and the other for axial positions. The axial standard was similar to the equatorial except that it was shorter, because it could reach only to the magnetometer. The equatorial reduction standard was more than 40 cm. longer than the primary, and had very closely the same diameter. Like the primary it was very uniformly wound with 36 turns per inch. The main terminals were arranged as in the primary solenoid. In addition, leads were soldered to the ends of a central section equal in length to the primary standard, and twisted together in the usual way.[29]

In the determination of C/B, or of $K = (B - C)/B$, a quantity more convenient in computation, two alternative processes were used. In the first, and ordinarily used process, in order to facilitate the determination, one of the astatic magnetometers was mounted on a separate stand with the same orientation as in the principal experiment, but with greatly reduced sensibility, and currents were used which were much larger than the calibrating currents, but not too large to prevent the magnetic moments from being strictly proportional thereto. As the solenoids were wound with the same pitch, and identical currents were used, the moments produced were nearly the same and therefore, in the equatorial positions, the deflections.

An adjustable stand was arranged to bring the rotor under investigation, centrally and axially mounted in the primary standard, and its compensator, into essentially the same positions with respect to the magnetometer system as those used in the principal experiments.

For each of the chosen positions, identical currents were passed in turn through the complete solenoid and its central part, and the corresponding magnetometer deflections were obtained on repeated reversals.[30] The absolute sensibility was determined, and the whole process was, in most cases, repeated (though this was found to be unnecessary) when the rotor had been turned through one-third of a revolution about its axis, and again when it had been turned through two-thirds of a revolution (or through one, two, and three quarters of a revolution). Then the rotor was removed, the magnetometer sensibility again determined, and deflections obtained on repeated reversals through the complete solenoid of a current much larger than that used when the rotor was present, but bearing a known ratio to it.

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[29] Equality of the number of turns per unit length in primary and reduction standards is of course unnecessary. Minute differences in diameter and length, if present, produce only differences of a higher order in the resulting moments, and are thus of no consequence.

[30] In some later experiments the process was much improved as follows: The equatorial solenoid was replaced by one with the same length of central section and (as it happened) greater pitch, *the whole being insulated from the tube.* Deflections were obtained when the current traversed the *terminal sections only* and when it traversed the *central section only*, with rotor inside, and when it traversed the complete solenoid alone.

In the other process, the procedure was essentially the same, except that the determinations were made with the magnetometer and rotor in their standard experimental positions in the compensating frame and with the sensibility of the same order as those actually used in the rotation work.

For a given position of the magnetometer and a given rotor, let d_1 and d_2 denote the mean deflections, reduced to the same magnetometer sensibility, obtained on reversal when the current traverses the complete solenoid and its central portion only, respectively, with the rotor inside; and let d_3 denote the deflection, reduced to the same magnetometer sensibility, obtained when the same current traverses the complete solenoid with the rotor removed. Then the deflection d which would be produced, for the same sensibility, by the rotor alone when magnetized by the uniform field of the complete solenoid is $d = d_1 - d_3$. Hence, since the effect of the central portion of the reduction solenoid is identical with that of the primary standard, $C/B = d/d_2$, or $K = (d_2 - d)/d_2$.

The quantity K as determined for all of the rotors in equatorial positions is given in Table I, column 5. Most of the experiments were made in the nearer equatorial positions, but in the several cases (including extreme cases) tested, no difference could be detected when the rotor was nearer to or more remote from the magnetometer. Nor did the removal of the compensator make any difference, nor the substitution of weaker for stronger fields. The observations for K were made with great care, and no one of the results given for equatorial positions is probably in error by as much as one unit in the first decimal place. Accurate observations (indicated by *AF* in the table), but not so accurate as those for the equatorial positions, were made for one axial position and with but few rotors; but rough observations, made with a simple magnetometer, showed no differences for different axial positions. Also, observations made on some of the rotors near the end of the investigation gave practically identical results with those made much earlier.

23. **Insulation of the standard coils.** The three secondary standard coils used in the course of the principal rotation work were

wound on insulating cores. After the completion of the work, two of these coils, including that used throughout Series B, were completely unwound and their resistances found (practically) equal to the resistances when wound. In the case of the remaining coil, the only part in which there was any possibility of a short circuit was unwound and tested, with the same result as for the other coils.

Of the two reduction standards used in the principal experiments to determine the end effects, and wound, as already stated, on metal tubes, the insulation of one was perfect, that of the other, tested after the experiments were completed, so nearly perfect as to introduce no appreciable error. It gave, moreover, identical end effects at the end of the rotation work and very much earlier.

Before the beginning of Series A the insulation of the primary standard was tested and found to be perfect, the end of the coil soldered to the tube being temporarily freed for the test. The resistance of the coil and its leads was then measured at intervals until just after the completion of Series B, and never showed a diminution below its value when the insulation was known to be perfect. Eight months after the completion of Series B, however, and after transportation through Panama to California, with no precautions in packing to protect the coil, the resistance of the coil was found to have fallen 1.6 per cent, and the defect located just at the insulated end. Suspicion that the insulation might have become defective, at least at times, during the rotation work led us to apply additional tests as follows:

(1) With coils whose insulation was known to be (practically) perfect, we have made three independent determinations of the quantity Q for Steel III; one from observations made for other purposes near the end of Series A, with the magnetometer in its standard position, two from observations made especially for this purpose. When reduced to the conditions existing in Series A and Series B, these observations give values of Q two-thirds per cent *less*, one-half per cent *less*, and two per cent *less* than the values obtained with the coil which was suspected and whose insulation they would, if the discrepancies were not accounted for by the errors in observation and reduction, indicate to be *more than perfect*.

(2) As a further check on the values of the constants Q obtained for the various rotors in Series A and B (and therefore on the insulation of the primary standard) we have calculated for these series the ratio, ρ, of Q for each group of experiments with other rotors (so far as data were available) to Q for Steel III, whose behavior was exceedingly constant; and we have made four more series of direct determinations of this ratio with coils known to be (practically) perfect. All the values of ρ are given in Table III.

TABLE III.

Values of ρ = Ratio of Q for each of the Rotors to Q for Steel III.*

Rotor	Values of ρ					
	(1) Series A	(2) Series B	(3) Aug. 26, 1924	(4) Aug. 29–31, 1924	(5) Sept. 12, 1924	(6) Dec. 2–4 1924
El. iron II	——	0.986 1.004	0.982	0.982	1.016	0.985
Armco iron	0.712	0.711	0.714	0.710	——	0.715
Norway iron	——	0.795	0.777	0.775	0.790	0.778
Steel III	1.000	1.000	1.000	1.000	1.000	1.000
Steel IV	——	0.833 0.855	0.860	0.854	——	0.862
Steel I	——	0.760	0.767	0.761	——	0.764
Nickel III	——	0.928	0.935	0.938	——	0.942
Nickel I	——	0.739	0.763	0.757	——	0.761
Cobalt II	0.810	0.817 0.820	0.786	0.790	0.801	0.792
Heusler's alloy I	——	0.483	0.476	0.474	0.473	0.474
Heusler's alloy II	0.372	——	0.374	——	——	——
Permalloy	1.257	1.257 1.255AF 1.238	1.245	1.250	——	1.259
Hopkinson's alloy	——	0.442	0.440	0.440	——	0.444
Preuss's alloy	1.079	1.091	1.104	1.095	——	1.105
Bloch's alloy	——	0.697	0.689	0.692	——	0.696

* Magnetometer in equatorial positions except when otherwise stated.

All the values of ρ in columns (5) and (6) and most of those in columns (1) and (2) are probably correct to one-fifth per cent. The remaining values of ρ are probably correct to one-third per cent. Yet discrepancies much greater than these errors occur in the table; and the discrepancies between the individual values in columns (3)–(6) are in general as great as those between these values and those of columns (1) and (2), or between different values for the same rotor in column (2).

The only factors to which it seems possible to attribute the differences in columns (3)–(6) are the magnetization, which was small in (6), but not so small as in (1) and (2), and larger in (3)–(5), and the methods used for demagnetization. No systematic differences, however, are apparent, and the considerable differences which occur would not be expected to originate from these causes. It is apparent that some of the rotors are much more nearly constant in their behavior than others.

It is clear from the table that we could not, with any certainty, ascribe the differences occurring for the same rotors in columns (1)–(2), or the differences between the values in these columns and those in (3)–(5), to imperfection of insulation.

24. **Sets of observations.** In order to reduce the accidental errors due to the fluctuations of the earth's field and other causes it was of course necessary in the principal experiments to make many magnetometer readings and to make them on a regular time schedule, at equal intervals, as in the case of processes *A*, *B*, and *C*; and to eliminate various systematic errors (see below) it was necessary to spin the rotor alternately in opposite directions at the same speed.

In the later work observations were usually made in sets of twelve, sometimes twenty or more for low speeds. In the case of a set of twelve, the motor was started, alternately right handed (as seen from the east) and left-handed, at the beginnings of twelve successive minutes.[31] Thirty seconds[32] later the magnetometer scale was read and the motor switch then opened. The procedure was similar for sets of eight or ten rotations, as in some of the earlier work, and for the larger sets. To obtain the mean double deflection for the set, the second scale reading was subtracted from the mean of the first and third, the mean of the second and fourth from the third, etc., and the average taken. Examples are given in § 60.

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[31] In a few cases other equal intervals were used.

[32] In some cases 35^s intervals were used.

25. **The equation for experimental determination of λ.** Let H_r denote the intrinsic magnetic intensity of rotation of a rotor for the angular velocity N revolutions per second, and d the magnetometer deflection produced by reversal of the direction of rotation. Also let H_c denote the uniform magnetic intensity which would be produced throughout the rotor by a minute electric current I traversing the calibrating solenoid if infinite in length, and C the deflection produced by the reversal of the magnetization it would cause. And let B denote the deflection produced by reversal of the came current I through the actual calibrating solenoid with the rotor inside. Then we evidently have

$$\frac{H_r}{H_c} = \frac{d}{C} = \frac{d}{B\,(1-K)} \tag{5}$$

Let E denote the open circuit e.m.f. of the calibrating cell, in volts; R_o the resistance in the circuit, in ohms (that of the connections, battery, and solenoid being negligible); and n the number of turns per centimeter length of the primary solenoid. Then

$$H_c = 4\pi n\,E/10R_o \tag{6}$$

Also let A denote the deflection produced on reversal of the current I through the absolute sensibility coils, so that $B = AQ$.

Let G denote the ratio (gear ratio) of the rotor speed N to the motor speed S, and let T denote the time of 500 revolutions of the motor, so that

$$N = GS = 500\;G/T \tag{7}$$

Then, since we may write, with an error which is negligible in all of our experiments,

$$1/(1-K) = 1+K$$

we obtain in place of (1) the equation

$$\lambda = \frac{H_r}{N} = \frac{4\pi n}{5000\,R_o}\cdot\frac{EdT}{GQA}\,(1+K)\;\frac{\text{gauss}}{\text{r.p.s.}} \tag{8}$$

which gives the experimental value of the intrinsic intensity of rotation per unit angular velocity. In nearly all of our experiments n was 14.71 turns per cm. (36.00 turns per inch), R_0 was 100,000 ohms, and E somewhat greater than 1.5 volt.

When the calculation was made from magneto voltmeter readings instead of chronograph readings an equivalent formula was of course used.

The resistance R_o was furnished by a coil in a standard box by

Leeds and Northrup, and agreed closely with standard resistances from boxes by Wolff and the Pyrolectric Co. The electromotive force E was furnished by two dry cells in parallel and was measured nightly with a Wolff potentiometer, calibrated at the Bureau of Standards, and a Weston cell which agreed to 1 part in 50.000 with another similar cell, whose e.m.f. was determined at the same Bureau. The only appreciable errors accruing in connection with equation (8), as applied to any rotor, come from d, T, Q, and A, and the last three are ordinarily negligible, or very small in comparison with the error in d.

26. **The currents in the compensating coils.** The current through the coils of the main compensating frame was supplied by a 60-volt, 10-ampere storage battery used for no other purpose while the actual experiments were in progress. The compensating value of the current, in the final arrangement, was about 1.383 ampere and was adjusted and read to about 0.0005 ampere by a Weston semiportable voltmeter and shunt and suitable variable rheostats. A Wolff standard one-ohm coil was also in the circuit, and the currentmeter was frequently, in the later work nightly, checked with a Wolff potentiometer calibrated by a Weston cell. The copper resistance in the circuit was nearly one-half of the whole, so that the current required some time to become steady. It often did not change appreciably, but also often changed as much as 0.001 ampere, rarely more during a complete set of observations, though the circuit was always closed long before these observations began.

A single 60-volt, 30-ampere storage battery supplied the current for the three subsidiary compensating systems, an arrangement of switches permitting each current in turn to be read by a single Weston portable ammeter. This ammeter and suitable variable rheostats made it possible readily to adjust and read the three currents with much more than necessary precision.

The proper compensating currents in the coils of the main frame and in the A coils and B coils were determined with inductor coils and a ballistic galvanometer with very heavy electromagnetic damping. The inductor coil used in most of the work, for the main frame and the B coils, is shown in Figure 22. The length of its two coils was about 26 cm. and the breadth about 7 cm. A few minutes sufficed to clamp the coils over any of the rotors, or to remove them.

By setting this coil with the planes of its turns horizontal, and then turning it with the rotor, through 180°, first in one direction and then in the other, and reading the galvanometer throws for currents slightly above and slightly below compensation values, the frame current necessary to reduce the mean vertical intensity through the coil to zero was readily determined by interpolation with greater precision than necessary to set the ammeter to 0.0005 ampere.

After the frame current had been adjusted to the proper value the test coil was set with the planes of its turns vertical, and galvanometer throws obtained on its reversal through 180°. This was done first with the current in the *B* coils zero, then with the current 0.50 ampere, 1.00 ampere, or some other appropriate value; and the correct value of the current for compensation of the mean horizontal intensity found by interpolation. The proper *A* coil current was determined in the same way, but with a cylindrical inductor about 8 cm. in diameter and 6 cm. long mounted with its axis horizontal and so as to be movable about a vertical axis.

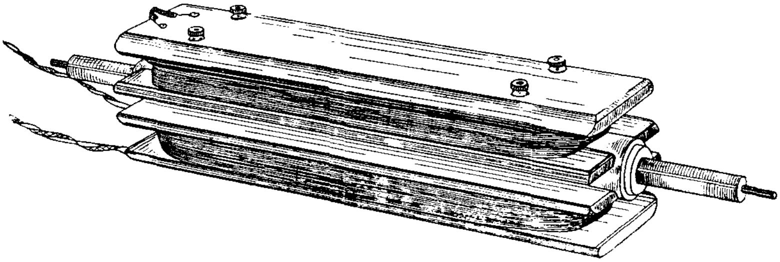

FIG. 22.

If we could assume the frame to remain entirely constant and the variometers to be entirely reliable, one set of determinations such as those just considered, together with the variometer readings, would suffice for the complete investigation. As two of the variometers, especially the vertical intensity variometer, were not trusted entirely, and as the frame (and for a time the windings) certainly underwent small changes, the compensation currents referred to (as well as the *C* compensation for nonuniform field) were determined on many occasions. In much of the work, in view of the insensitiveness of the magnetometer to eddy-current effects, the precautions taken were greater than necessary.

The proper setting of the current in the *C* coils was determined from observations on the rotation of a copper rotor, as described below.

27. **Position and adjustment of the compensator.** If the magnetometer is exactly astatic and the compensator and rotor magnetically exactly alike, it is clear that for the best compensation

they should occupy similar or symmetrical positions with respect to the upper and lower magnetometer magnets.

In much of the work the compensator was placed approximately as if these conditions were realized. Thus, in some of the work, it lay in *V* grooves, in the brass angle pieces *I*, Figure 3. Usually it was more exactly mounted, a closely fitting half-ring of brass, with 4 cm. external diameter, lying between compensator and *V*'s, which were so constructed that then the axis of the compensator was very nearly parallel, and in the same vertical plane, with that of the rotor, and the vertical distance between their axes very nearly equal to that between the centers of the magnet systems. The center of the compensator was placed vertically above the center of the rotor. Even when the rings were used, the arrangement was evidently not strictly correct for equatorial positions even on the assumption that rotor and compensator were just alike.

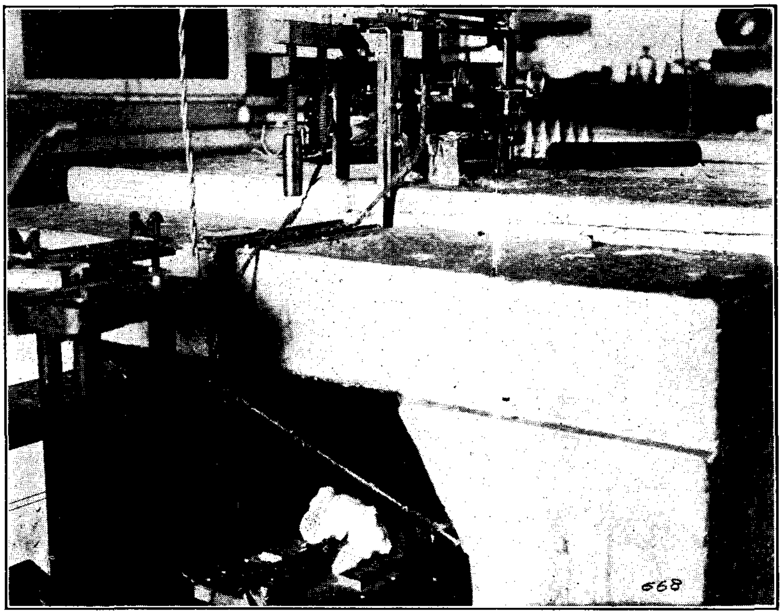

FIG. 23.

When the magnetometer was shifted on the casting *F*, Figure 3, in order to make the vertical equatorial positions possible, new arrangements had to be made for holding the compensator. The final arrangements are shown in Figures 6, 12, and 23. In the axial position the compensator (not shown in the figures) lay in V's about as far west of the upper magnet as the rotor was east of the lower. In the equatorial position the compensator was placed *above* the upper magnet and about as far from it as the rotor was from the lower magnet. Arrangements were provided for adjusting (equatorial position) in height, in axial distance from upper magnet (axial position), in altitude, and in azimuth.

These adjustments made it possible to correct very largely for (1) inequalities between rotor and compensator, for (2) inequality between the moments of the upper and lower magnetometer systems, and for (3) the horizontal angle between their magnetic axes. We shall describe the methods of adjustment for the equatorial position only. The same principles applied for the axial position.

Adjustments for (1) and (2) were made together. The frame holding the compensator, with its center in the equatorial plane passing through the suspension, and its axis approximately parallel to that of the rotor, was clamped in position. The magnetometer was then adjusted to the desired zero and sensibility.

A suitable current was then sent *in the same direction* through the two C coils, producing very nearly the same axial intensities in the rotor and compensator. If reversal of this current produced a deflection greater than the minimum desired, the height of the compensator was readjusted, and the process repeated until the deflection vanished or was reduced to the minimum sought. Fluctuations of the declination, or the east and west component of the horizontal intensity, then had little effect on the magnetometer.

In making the adjustment for (3) the B coils were used. With a suitable current traversing the coils and the magnetometer in adjustment, the deflection produced by reversing the current was determined. Then the azimuth of the compensator was altered, and the process repeated until the deflection on reversal vanished or was not greater than the minimum desired. The magnetometer was then affected but little by fluctuations in the north and south component of the horizontal intensity.

During the adjustments mentioned the rotor was ordinarily in slow motion, usually in a neutral field, so that the mean magnetic axis of the rotor coincided with its geometric axis, and the moments of the magnetometer magnets assumed the values they had in the rotation experiments. The adjustments for (2) naturally differed considerably when made in and out of the earth's field, while that for (3) was not affected in this way.

If the compensator is homogeneous and so leveled as to be parallel to the rotor, adjustments made as above are sufficient. If, however, the compensator is not homogeneous, or its magnetic axis (for induced magnetization) not coincident with its geometric axis, the adjustments here described are not sufficient to prevent fluctuations in the vertical intensity from affecting the magnetometer. Hence, for bad compensators, it was necessary to proceed as follows in adjustment (3): A divided scale was pasted around the compensator so that its position angle with reference to a fixed mark could be read. The position angle, sensibility, and deflection were then determined. Then the process was repeated for a number of angles through a complete revolution of the compensator, and the position angle at which the deflection (reduced) was a minimum or a maximum determined. The compensator was then set at one of these two angles, with its magnetic axis in the plane normal to the central line through rotor and compensator, and the proper azimuth found as above. The magnetometer was then insensitive to fluctuations of either horizontal or vertical intensity.

In making the adjustment (3) when the horizontal angle between the magnetometor magnetic axes was considerable, the compensator had sometimes to be turned out of parallelism with the rotor through a number of degrees. This, of course, magnetized the compensator and produced a magnetic field at the rotor. Crude experiments, however, were quite sufficient to prove that the maximum intensities produced in this way were entirely negligible in their eddy-current effects on the rotor.

In the actual practice of the adjustments we did not, of course, attempt to make one adjustment final before making the other approximately, but proceeded by successive approximations.

28. **Systematic errors.** Aside from insulation errors, discussed below (§ 23), and other errors in the standards, which were entirely negligible, the principal systematic errors which had to be investigated or overcome may be divided into two classes: (A) those with origin independent of the magnetization of the rotor; and (B) those with origin dependent on the magnetization of the rotor, either permanent or induced.

Errors in class A may arise from: (1) eddy currents in the rotor due to incomplete compensation of the uniform part of the residual magnetic field; (2) eddy currents in the rotor due to incomplete compensation of the nonuniform part of the residual field; (3) electric currents due to thermal effects at the bearings; (4) electric currents due to thermal effects produced by the air driven against the bedplate, etc., by the moving rotor; (5) eddy currents and other electrical or magnetic effects in the motor, starter and other driving apparatus; (6) thermal effects on the magnetometer due to air thrown against it by the motion of the rotor; (7) vibrations of the air inside the magnetometer case, affecting the suspension differently for different directions of rotation (see § 50).

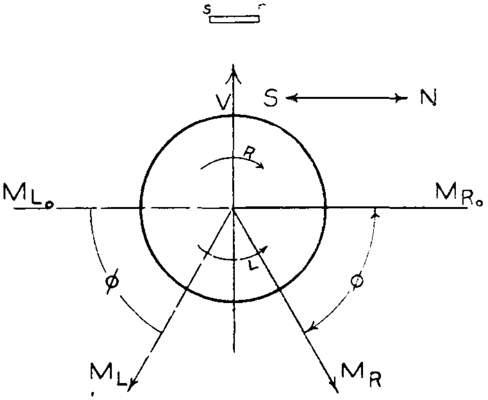

FIG. 24.

Errors in class B may arise from: (1) torsion of the rotor, which is driven from one end and moves with friction in a bearing at the other; (2) thermal effects on the rotor due to the bearings; (3) thermal effects on the rotor due to the air currents produced by its motion; (4) centrifugal expansion of the rotor or other strain, aside from torsion, produced by the rotation; (5) axial displacement of the rotor which may accompany its rotary motion; (6) displacement of the rotor in altitude or azimuth in the residual part of the field with intensity normal to its axis; (7) the Thomson repulsion effect on the magnetic system due to the irregularity of the rotor's magnetization; and (8) mechanical disturbances, different for right-and left-handed rotations.

29. **Eddy-current errors from residual uniform field.** Let us first suppose that the rotor is symmetrical and homogeneous and the residual field in which it moves uniform. The intensity of this field may be resolved into three rectangular components, L horizontal and parallel to the rotor's axis, H horizontal and normal to the axis, and V vertical and normal to the axis.

The component L produces no eddy currents, and only the horizontal components at the magnetometer magnets of the intensities h and v produced by H and V can affect the magnetometer. Let us denote the axial components of h and v by h_a and v_a, and their horizontal components parallel to the equatorial plane through the rotor by h_e and v_e.

The way in which the intensities h_a, h_e and v_a, v_e affect the magnetometer system, supposed always in the vertical plane through the axis of rotation, can be seen from Figures 24–27, which give diagrammatic equatorial sections of the rotor and lower magnet when the magnetometer is vertically above the center of the rotor and which show diagrammatically the moments of the eddy-current systems to which similar right- and left-handed rotations give rise. The moments M_{R_0} and M_{L_0} are the moments which the eddy-current systems would have if there were no self-induction, and are 90° in advance of V and H; the actual moments M_R and M_L lead these vectors by an angle increasing with the speed.

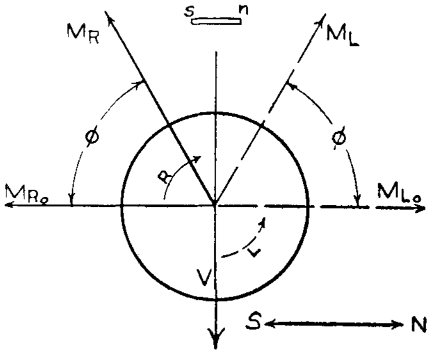

FIG. 25.

30. **Eddy-current effects from residual uniform vertical intensity.** When H = 0, as in Figures 24 and 25, the vectors M_R and M_L, which are equal in magnitude, make equal angles with V on opposite sides, and their fields are symmetrical about the lines which represent them. Thus at any point in the vertical plane through the axis of rotation $(v_a)_R = (v_a)_L$, and $(v_e)_R = -(v_e)_L$.

The components v_e will affect the sensibility of the magnetometer, and oppositely for opposite directions of rotation, but will produce no deflection on reversal or other change in the rotor's velocity *unless* the axes of the magnetometer systems are not parallel to the equatorial plane. For simplicity, we may neglect the effect (relatively minute in equatorial positions, and small in axial positions) upon the upper magnet system and consider only the lower, which is much closer to the rotor. If its axis makes an angle A (supposed small) with the equatorial plane, rotation will produce a deflection proportional to AV, changing sign with the velocity, so that it is doubled by reversal (*effect* α).

When the magnetometer is strictly in the equatorial plane, $v_a = 0$, so that it can produce no deflection. But if the magnetometer is given an axial displacement (supposed small) there will be, on changing the speed, a deflection (effect β) proportional to V and increasing with the axial displacement, but slowly on account of the small ratio of the diameter of the rotor to its length and the nearness of the magnet. Since $(v_a)_R = (v_a)_L$, there is no deflection produced by *reversing* the motion.

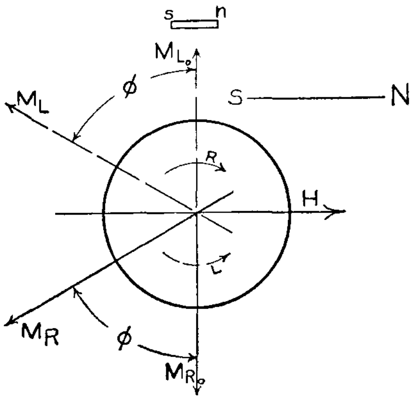

FIG. 26.

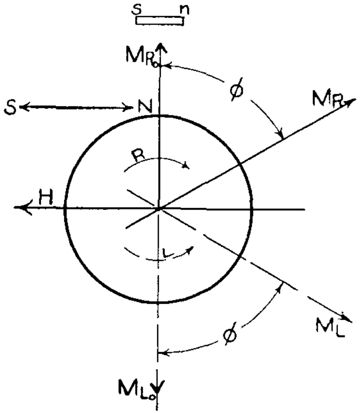

FIG. 27.

When the magnetometer is *axially* mounted, $(v_a)_R = (v_a)_L = 0$, and the effect β vanishes for *each* direction of rotation.

31. **Eddy-current effects from residual uniform horizontal intensity.** When $V = 0$, as in Figures 26 and 27, we find, in a similar way, that $(h_e)_R = (h_e)_L$. This component of the induced intensity alters the sensibility, equally for opposite directions of rotation, the alteration changing sign with the residual field intensity H, but produces no deflection unless the magnet system makes an angle

(*A*) with the equatorial plane. In this case there results a deflection (*effect* γ) proportional to AH, but independent of the direction of rotation.

In the equatorial plane, $(h_a)_R = -(h_a)_L = 0$, so that the magnetometer is not deflected by this component. But if the magnetometer is given a small axial displacement, there will ensue a deflection (*effect* δ) increasing with the displacement and proportional to H, and changing sign, without alteration of magnitude, on reversal of the rotation.

If the magnetometer is in an axial position the effect δ vanishes.

32. **Eddy-current effects due to residual nonuniform intensity.** If the residual *horizontal intensity* H is not constant throughout the rotor, it may, with sufficiently close approximation to the truth, be resolved into two components, one, H_0, constant throughout the rotor, and equal to its value at the center, and the other, h, proportional to the axial distance from the equatorial plane, Figure 11.

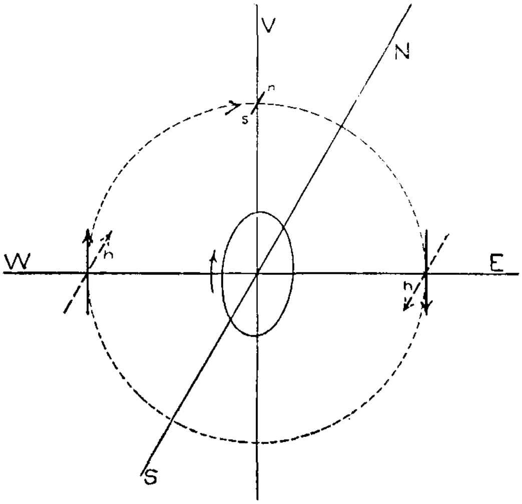

FIG. 28.

The component H_0 may be treated precisely like H in what precedes. For qualitative purposes the component h can be treated much in the same way, due attention being paid to the fact that its directions are opposite on the two sides of the equatorial plane. Figures 26–27 may then be applied roughly to each *half* of the rotor.

Figure 28 shows, for right-handed rotation of the rotor, and a given distribution of h, the vertical components of the resulting moments of the two halves of the rotor and the direction of the field they produce at the lower magnetometer magnet in a vertical equatorial position. The horizontal components of the moments have evidently no effect on the magnetometer. Reversal of the rotor produces a deflection (effect ϵ) proportional to h and opposite in sign for the two directions of reversal.

The *vertical intensity* V, when not uniform, may be treated precisely like H, and resolved into two components V_o and v. Figures 24–25 may be applied roughly to each half of the rotor, and the effect of v may be seen through diagrams analogous to those of Figures 11 and 28. The component v produces deflections (effect ζ) which are independent of the direction of rotation, so that the deflection on *reversal* of the rotor's speed vanishes.

Effects ϵ and ζ both vanish in the axial positions of the magnetometer for each direction of rotation if the angle A is zero, as does the effect ϵ in any case, but the horizontal components of the fields associated with them may affect the sensibility, which they do not do in equatorial positions.

33. **Nonhomogeneity of the rotor.** If the rotor is not homogeneous, the two halves being unlike, what precedes will apply to the mean results obtained by observing first with one end of the rotor east, then with the other east.

Second order effects. In all that precedes effects, such as $\delta\alpha$ and $\delta\gamma$, due to the slight change of A produced by the rotations themselves are neglected. In our experiments they vanish or are eliminated with α, β, etc.

34. **Elimination of the eddy-current errors.** The process of eliminating the effect α consisted in making the uniform part of the field in which the rotor moved so weak and the angle A so small as to render the effect negligible.

Two general methods of testing for and reducing the angle A were used. In one, a small and accurately made solenoid with horizontal axis was mounted with its center vertically beneath the magnetometer suspension. The solenoid was turned about a vertical axis until its (horizontal) axis lay east and west. A steady electric current was then sent through the solenoid and the deflection produced on reversing its direction was determined. The process was then repeated, but with the solenoid's axis north and south. From the two deflections the angle A could be determined, and then reduced as much as desired by the zero control.

In the other method, a copper rotor was sometimes used. With the main compensating current considerably increased or decreased from its normal value, right-handed and left-handed rotations were produced in the usual way in sets, and the mean deflection due to reversal obtained. The zero was then altered and the process repeated. From the two groups of observations the proper zero could then be obtained by interpolation. The same method was also applied by using magnetic rotors, the zero being adjusted until the deflections were nearly the same for under-compensation and over-compensation much greater than those occurring during the principal experiments.

The error due to the effect α is opposite for axial and equatorial positions, and can therefore be eliminated, or its limits determined, by using both.

The effect δ in the weak residual field was made negligible or very small by setting the magnetometer very nearly in the equatorial plane, or by setting it in an axial position. The effect ϵ was made negligible by compensating the nonuniform part of the residual horizontal field by means of the coil system *C*, described above. Both effects were tested for by rotating both copper and the magnetic rotors, the adjustments being such, in much of the recent work, that changes in the currents of the *B* and *C* coil systems corresponding to residual fields much stronger than could exist in the rotation experiments produced only negligible effects.

35. **Eddy-current observations on copper made with the compensating system of the principal experiments.** We have made many sets of observations on a rotor of copper about 3.1 cm. in diameter and length similar to that of the other rotors and similarly placed. If we assume the conductivity of copper to be eight times that of iron, and the rotors of the same diameter and infinite in length, in a uniform field normal to their axes, and if we neglect self-induction, it is easy to show that the effect α of the rotor eddy currents on the magnetometer in equatorial positions is about four times as great for copper as for iron. In general the eddy-current effects with copper are much greater than with the magnetic rotors.

In Table II, *A* is the absolute sensibility; *d* the deflection obtained when the copper is rotated, and the direction of rotation reversed, in a field whose uniform part, and that only, is supposed to be annulled (except for the second line for May 16, 1923, which refers to observations made with the *C* current properly adjusted to reduce the effect ϵ to zero, and for the observations of December 28, made with the *B* current too large by 13 divisions); $\Delta d/\Delta V$ is the increase in deflection produced by increasing V (the current in the main compensating frame) by one division (0.01 ampere), usually from half a division below the compensating value to half a division above; $100\Delta d/\Delta C$ is the increase of deflection produced by changing the C current 100 divisions (1.00 ampere) in the positive direction; and $50\Delta d/\Delta B$ is the increase of deflection produced by increasing the B current by 50 divisions (0.50 ampere).

TABLE IV.

Results of Observations on Copper, August 9, 1922 to December 28, 1923.*

All deflections in centimeters. Gear ratio 2—1.

Date	Mag. pos.	No. sets	Approx. A	d	$\Delta d/\Delta V$	$100\ \Delta d/\Delta C$	$50\ \Delta d/\Delta B$
1922 Aug. 9	*LEQ*	6	17.4	*minute*	+0.04	——	+0.04±0.02
1923 Mar. 6	*LEQ*	6	17.3	−0.28 ±0.03	+0.19	+0.27 0.02	——
Mar. 13	*AN*	4	17.6	−0.01 0.05	——	−0.03 0.05	——
†May 8	*LEQ*	7	18.2	−0.02 0.08	−0.10	——	——
‡May 16	*LEQV*	3	18.4	+0.16 0.04	——	+0.45 0.03	——
‡May 16	" (C=−31)	1	18.4	+0.03 0.06	——	——	——
Aug. 24	*LEQV*	4	17.3	+0.19 0.04	−0.58	+0.34 0.03	——
Oct. 19	*EQV*	4	18.4	+0.07 0.04	——	+0.26 0.04	——
Nov. 11, 14	"	10	18.0	+0.06 0.07	——	+0.29 0.04	——
Dec. 20	"	4	21.2	+0.03 0.06	——	+0.41 0.03	——
Dec. 28	"	2	16.1	+0.02 0.07	——	+0.17 0.04	——

* Some observations made on Nov. 19, 1922, with poor adjustments, are not included in the table.

† Single longitudinal loop in use.

‡ Double longitudinal loop in use.

From the value of A and that of $100\Delta d/\Delta C$ the proper value of C can be calculated on the assumption that d is wholly due to the nonuniform part of the horizontal intensity. Thus from $d = +0.16$ for May 16, 1923, and $100\Delta d/\Delta C = +0.45$, we find the proper value of C to be −31 divisions. When C on this date was made −31 divisions, d became very small, viz., $d = +0.03 \pm 0.06$.

The assumption that d is wholly due to the nonuniform part of the horizontal intensity is not strictly justified for the position *LEQ*, in which the nonuniform part of the vertical intensity is also effective, since the suspension is not in the vertical plane through the axis of the rotor.

36. **Further eddy-current experiments on copper and magnetic rotors.** In other special experiments on the rotors of copper and annealed steel we have confirmed the deductions of §§ 29 and 30 with reference to the effects of rotation on magnetometer sensibility, and have determined the effects β and δ in vertical and horizontal fields normal to the rotor and equal in magnitude to that produced by 100 divisions of current in the B coils. The experiments were made with the aid of a special compensating frame in relatively strong fields and at low sensibilities, and the results then reduced to those which would have been obtained with the intensity indicated above and the approximate absolute sensibility $A = 20$ cm.

In Table V β and δ are the increases in the deflection due to reversal of the rotation when the magnetometer is shifted through the central position 1 cm. toward the east, the first for a field in which the intensity is vertical and downward, the second for a field in which the intensity is horizontal and northward. The quantity $\Delta A/A$ in the last column is the change in the absolute sensibility produced by the rotation of the copper, either right-handed or left-handed, in the northward horizontal field (with intensity equal to that produced by a current of 100 divisions in the B coils). Changes of similar magnitude, but opposite in sign for the two directions of rotation, were produced by a vertical field, which produced effects not certainly appreciable on the steel rotors.

TABLE V.

EDDY-CURRENT EFFECTS IN COPPER AND STEEL. MAGNETOMETER POSITION *EQV*.

Rotor	Speed	β	δ	$\Delta A/A$
Copper	26 r.p.s.	−0.02 cm.	+0.92 cm.	—
	61	+0.04	+1.09	−⅓%
Steel III	26	+0.09	+0.18	—
	61	+0.02+	+0.22	—
Steel IV	61	−0.06	+0.09	—

The effect β, for reversals of rotation, should be zero, and the departures from zero in the table are doubtless accidental. The effect δ for copper, as would be expected, is much greater than for the other rotors.

The copper rotor (like the others) in these experiments was in its final state. Its homogeneity is proved by the fact that in a number of sets of observations reversal from mark east to mark west produced *(in the horizontal field indicated above and with $A = 20$ cm.) only the* deflections $+ 0.035 \pm 0.03$ cm.

The effect δ, as the table shows, is only slightly greater for the higher than for the lower speed, a fact which emphasizes the importance of working at the higher speeds in the principal experiments.

In the principal experiments in equatorial positions the axial displacement of the magnetometers center from the vertical plane passing normally through the center of the rotor was small, very minute in Series A, and never greater than about 2 or 3 mm. in Series B. The greatest value of ΔB, the excess of the B current over its correct value for exact compensation, viz., $+18$ divisions (see Table XIII), occurs with the Norway iron rotor, whose conductance (Table I) is about equal to that of Steel III. The double deflection for this rotor due to the effect δ at the highest speed would be, at the greatest, about $\frac{127}{122} \times \frac{3}{10} \times 0.22 \times \frac{17.4}{20} \times \frac{18}{100} = 0.01$ cm., which is much less than the accidental error. And the effects δ are smaller, in most cases much smaller, for the other rotors in Series B. The effect vanished for all the magnetic rotors in Series A.

Many additional observations on eddy-current effects in the magnetic rotors made with the compensating system used in the principal experiments are given in Table IX, § 56. We give here a few results, obtained with the same compensating system, on the variation of sensibility with speed on account of incorrect compensating currents.

With Permalloy, and the magnetometer in the position AN, the sensibility was determined at the gear ratio $2-1$, for right and left-handed rotations, the first giving $A = 20.65 \pm 0.08$, the second $A = 20.86 \pm 0.18$, when V was 0.5 division too small. The sign of the difference of 1% is in accord with the theory, the vertical components of the residual intensity in this case acting to diminish the sensibility for right-handed rotations, and to increase it for left-handed rotations.

With Steel III, magnetometer position $LEQV$, and the B current 40 divisions too small for correct compensation, very slow rotation gave $A = 20.18 \pm 0.09$; while right-handed rotation at the gear ratio $2-1$ gave $A = 20.30 \pm 0.16$. Again the sign of the small effect is in accord with theory. The result is larger than would be expected from the above results with the copper rotor, but a considerable part of the apparent effect may be due to accidental error.

37. **The effect of the lower magnetometer magnet on the compensation currents.** By means of simple induction experiments, involving the use of a magnet whose moment was a considerable and known multiple of that of the lower magnetometer system with six magnets, and which it is unnecessary to describe in detail, we determined for a number of different cases the flux Φ which this lower magnet sent through the coils of Figure 22 clamped over a rotor.

For the position *LEQ*, a copper rotor, and the planes of the turns *horizontal*, Φ (vertical) was equal to the flux produced by the frame current corresponding to 0.035 division on the ammeter, the vertical flux of the uncompensated field corresponding to about 138.5 divisions. With the planes of the turns *vertical*, Φ (horizontal) was equal to the flux produced by the frame current 0.011 division, the horizontal flux of the uncompensated field corresponding to about 138.5 divisions.

For the position *LEQV*, and a large nickel rotor, the vertical and horizontal fluxes were respectively 0 and that produced by an intensity 0.00088 *H*, corresponding to 0.077 ampere in the *B* coils.

For the position *EQV*, the vertical and horizontal fluxes were 0 and that produced by an intensity 0.00065 *H*, corresponding to 0.057 ampere in the *B* coils.

For the two axial positions used, the vertical fluxes were of course 0, and the horizontal fluxes for the positions *AF* and *AN* were about 0.1 and 0.2 that for *EQV*.

The above values are all for a lower system with six magnets. For a lower system with three magnets the fluxes are only about one-half as great.

38. **The effect of the control magnets and upper compensating coils on the compensation currents.** The small upper compensating coil, when traversed by the compensating current usually employed, viz., about 1.4 ampere, had a moment of about 10 c.g.s. units.

This coil thus produced at the axis of the rotor, about 60 cm. below, a normal horizontal intensity of about $H/4000$, symmetrical about its center.

The control magnets, whose moments are given in § 8, produced intensities which were very much less.

39. **Eddy-current effects of the lower magnetometer magnet.** When in the position *EQV*, a six-magnet system, 12.9 cm. above the rotor's axis, and normal to it produced at the center of the rotor a horizontal intensity about $0.00094 \times H$, a three-magnet system about $0.0005 \times H$. On account of the rapid falling-off of the intensity toward the ends of the rotor, this field, from symmetry, could give rise to only a minute effect δ even if its intensity were greater and the magnetometers were considerably displaced from the central position.

If the axis of the magnet is not strictly normal to the rotor's axis, the axial component of its moment produces along the axis of the rotor a nonuniform field whose vertical component for a special case is given in a curve (marked "6 magnet system," etc.) of Figure 13. If this field were *fixed*, and the magnet free to move, no change of magnetometer reading on reversal of the motion would occur. Inasmuch, however, as the angle changes, on account of the rotation effect under investigation, on reversal, there is a second order effect produced. From Figure 13 and the fact that the axis of the lower magnet points north, it is easy to see that effect is such as to *reduce* the deflection produced by reversal of rotation.

40. **Other effects of the lower magnet.** If the lower magnet, six-magnet system, distant 12.9 cm. from the rotor's axis, makes an angle θ with the equatorial plane, it produces at the center of the rotor an axial intensity $h = 0.00094 \times H \times \theta = 0.00018 \times \theta$ gauss; and if the angle changes by $\Delta\theta$, $\Delta h = 0.00018\Delta\theta$ gauss. For a 4-cm. deflection $\Delta\theta = \frac{1}{2} \times 4/600$ (approx.) $= 0.0033$; and $\Delta h = 6 \times 10^{-7}$ gauss.

If a 4 cm. deflection is produced by reversing the rotation at a speed 60 r.p.s., the change $\Delta h'$ in the intrinsic intensity of rotation is about $3.7 \times 10^{-7} \times 60$ gauss, or about 220×10^{-7} gauss. Thus Δh is less than 3% of $\Delta h'$.

Suspicion that Δh, as well as magnetometer eddy-current effects, might be sufficient to affect our results appreciably led to experiments in which the magnetometer sensibility was greatly reduced (ratio $\frac{1}{2}$), the six-magnet systems replaced by three-magnet systems, and the position of the magnetometer altered by increasing its height 4.7 cm. All were without appreciable effect. In theory Δh is fully taken account of in the calibration.

41. **Elimination of errors due to thermal currents from bearings.** When the bearings supporting the rotor and various countershafts are not electrically insulated from one another, electric

currents of thermal origin are in general produced by the rotation, and they produce a systematic error if the heating of the journals depends on the direction of rotation, as it may do even when the opposite angular velocities are equal. We have definitely observed the objectionable effects of such currents only in the case of the copper rotor, with which it is easy to recognize them as of thermal origin by their time characteristics. This effect has been avoided in all the recent work by sufficient insulation of all the bearings within a considerable distance of the magnetometer.

Furthermore, when all the bearings were insulated, and the copper was rotated in a weak residual field by a half-inch journal in a bearing of lignum vitae in the east bearing block, we obtained (January 11, 1923) from many rotation reversals at about 60 r.p.s. and with the magnetometer in the lower equatorial position the deflection $d = -0.02 \pm 0.08$ cm.; but when the wood was replaced by lumen metal, we obtained $d = +0.19 \pm 0.13$ cm.; with absolute sensibility A about 16.0 cm. The errors are large, but there is apparently evidence of a definite effect of failure to insulate one journal from its bearing. Any such effect was eliminated or greatly reduced by the type of journal and bearing finally adopted and already described.

42. **Effect of air currents on bedplate.** In seeking to account for some of the observed discrepancies, we were led to suspect that the air currents produced by the rotation of the rotor and impinging on the (bronze) bedplate differently for the two directions of rotation might produce a differential deflection on reversal. The rotations in Table VI on copper, which were obtained while investigating the effect ϵ, possibly show such an effect; while the somewhat better observations made with a still larger rotor of wood, for this purpose only, do not show the effect. In each case, some of the observations were made with a thick piece of cardboard between rotor and bedplate, and some with the cardboard removed.

TABLE VI.

(MAGNETOMETER POSITION *EQV.*)

Rotor	No. sets	Cardboard	Deflection	Sensibility
Copper	4	on	0.51 ± 0.06 cm.	18.1 cm.
"	6	off	0.65 0.09	18.1
Wood	3	on	−0.02 0.08	18.0
"	3	off	−0.02 0.06	18.0

Other experiments, with various rotors, in which rotations were made with and without the brass calibrating tube about the rotor, or with a semicylindrical piece of cardboard, designed to separate the rotor from the magnetometer but not from the bedplate, in place and removed, do not show any effect of the screening.

This effect will be referred to later in connection with the error *B* (3), § 28.

In most of the final work the rotor was screened from the bedplate by heavy cardboard or was completely surrounded by a thick cardboard tube about 6.2 cm. in diameter (internal). See Table XIII.

43. **Elimination of thermal effects on magnetometer.** It has long been known, or at least suspected, that small quantities of heat transferred to a sensitive magnetometer (at least a magnetometer with a metal case) may produce deflections which are far from negligible; and we have obtained much evidence for the existence of such an effect. We of course suspected the possibility of a systematic error from this source in this investigation, since the rotor, if it throws the air against the magnetometer, does so differently for the two directions of rotation. The error would evidently be much greater for equatorial than for axial positions of the magnetometer. The thorough wrapping of all but the upper part of the magnetometer with cotton and paper apparently sufficed (if indeed it was necessary) to prevent this effect, even without the tube or cardboard screen with which the instrument was separated from the rotor in nearly all but the early work.

44. **Errors due to the motor and other driving apparatus.** Table VII, *A*, gives the results of the tests which were made, with the field of the earth at the center of the frame annulled, on three single-phase motors which were used in work with the large compensating frame. The motor and gear box, when in the position marked "in," were inside the building, just inside the east end of the frame; in the position marked "out," they were in the pit east of the building and about twice as far away from the magnetometer. *A* is the magnetometer sensibility, *d* the mean magnetometer deflection $(R-L)$ produced on reversal of the motor, and $\Delta'd$ the average departure of this mean from the mean deflections for all the sets in the group.

The observations showed no certain differences for the gear ratios $2-1$, $1-1$, and 0 (motor and one gear shaft running alone); nor for different rotors (with their compensators), soft iron, nickel, and none. Hence the results for each motor and magnetometer position are all grouped together.

In these observations the rotor, when used, was clamped in position, and the small chuck and rod by which it was ordinarily connected to the first countershaft were removed. Many experiments have shown that in the compensated field this small element produces no appreciable deflection. When the compensating current was annulled, a deflection of + 0.84 cm. was obtained at the speed 2 − 1.

Table VII, *B*, gives the results with the three-phase motor in compensated fields. The first group was obtained from August, 1922 to January, 1923; the second in May-June, 1923; the third in October-November, 1923. In some cases there was no rotor (or compensator) present; in others there was a rotor of wood, either at rest or in rotation; in others, rotors of copper, nickel, permalloy, or cobalt, always at rest (with their compensators); in some cases the chuck and connecting rod were removed, while in all the later work they were present and in rotation. The observations show no differences depending on these conditions, as they should not if the motor and other very remote parts of the rotating system are alone responsible for any effect obtained. The quantity $\Delta'd$ is the average departure of the individual observations in a set from the mean for the set, averaged for all the sets in the group; while the quantity $\Delta''d$ is the average departure from the group mean of the means for the sets which constitute it. The effect is evidently *zero* within the limits of the experimented error for all speeds.

When the earth's field at the center of the frame is not annulled, and eddy currents are developed in the near parts of the system, the deflections are very different for different conditions — ranging, for the gear ratio 2−1, from + 1.4 cm. with a wooden rotor and the magnetometer position *LEQ* to −0.1 cm. for the permalloy rotor and the position *EQV*, with the connecting rod and chuck in rotation in both cases.

In obtaining the later results in compensated fields, in which the rotors were always clamped in position, it was of course possible that residual cross-magnetization of the (resting) rotor might induce currents in the near-by chuck and driving rod. Hence the experiments were made in pairs with the position angles of the rotor differing by 180°, and the means taken. No certain effects of the rotor's magnetization were noticed.

TABLE VII.

EFFECTS OF DRIVING APPARATUS, WITH SINGLE-PHASE AND THREE-PHASE MOTORS.

A. Repulsion Start Single-phase Motors.

Motor	Magnetometer position	Motor position	No. sets	*A*	*d*	Δ'*d*
1½ h.p.	*LEQ*	in	8	16.2	−0.18	±0.03
1½ h.p.	*AF*	"	8	18.4	−0.13	0.07
3 h.p. No. 1	*LEQ*	out	6	18.7	+0.06	0.02
3 h.p. No. 2	*LEQ*	"	8	18.3	+0.06	0.04

B. 5 h.p. Three-phase Slip-ring Motor.

Speed	Magnetometer position	Motor position	Group	No. days	No. sets	*A*	*d*	Δ'*d*	Δ''*d*
2–1	*LEQ*	out	1	4	12	18.9	−0.015	±0.02	±0.02
	LEQV	"	2	3	6	19.7	+0.02	0.02	0.03
	EQV	"	3	2	10	18.3	0.00	0.03	0.02
Means for 28 sets 2–1						19.0	0.00	±0.02	±0.02
1–1		"	2	2	4	19.7	+0.03	±0.04	±0.01
1–2	*LEQV*	"	2	2	3	19.7	−0.06	—	±0.06
	EQV	"	3	1	4	18.3	0.00	±0.02	—
Means for 7 sets 1–2						19.0	−0.03	—	—
2–7	*LEQ*	"	1	1	3	18.9	+0.01	±0.02	—
	LEQV	"	2	3	6	19.7	−0.04	0.02	±0.07
	EQV	"	3	1	4	18.3	+0.01	0.02	—
Means for 13 sets 2–7						19.0	−0.01	±0.02	—

45. **Demagnetization and reversal of rotors and compensators.** It is clear that all errors of class B would vanish with the magnetization of the rotor, and would be less the smaller the magnetization. They are eliminated more readily the more stable such magnetization as is residual.

In demagnetizing the rotors and compensators we have used the method of reversals and the method of mechanical shock. We have found that the stability of the residual magnetization is greatly increased if, in the application of the method of reversals, the current for each step is allowed to flow for a considerable interval. We have used, in all the recent work, intervals of about half a minute, with rather quick reversals between them, and very numerous steps. A great deal of time was saved by making the apparatus automatic. Sometimes we have succeeded in reducing the magnetization nearly or quite to the minimum readable with the testing magnetometer arrangement used by the method of reversals alone; but this was not usually the case, in spite of repeated attempts to perfect the apparatus. Most of the magnetization left after a long application of the method of reversals, with the current usually reduced to zero by hundreds of steps, was usually removed by striking the rotor, with a block of lead weighing about half a kilogram, more or less sharply and with its magnetic axis making a greater or less angle with the intensity of the earth's field, according to the rotor's characteristics.

In order to keep the magnetization as nearly constant as possible, both when the magnetization was being tested and when the rotor (or compensator) was being reversed in the principal experiments, reversals were always made, except in the earliest part of the work, with the axis of the rotor (or compensator) turning in a plane normal to the intensity of the earth's field.

The intensity of magnetization of the rotors as used in the last series of experiments is given in column 30 of Table XIII. For the preceding series, the rotors were in general still better demagnetized. The magnetization of the compensators was always so small as to have entirely negligible effects on the field in the region occupied by the rotor. A measure of the irregularity of the residual magnetization of the rotors as used in the last series is given in column 31 of Table XIII, where the "oscillation" is approximately equal to the range of motion of the spot of light on the magnetometer scale which would be produced by very slow rotation of the rotor in the compensated field.

46. **Elimination of errors due to torsion.** In the investigation of 1914–15 on large rods of cold-rolled steel the effects of torsion, due to journal friction at the end of the rotor remote from the motor, were found to be negligible. Also in the early magnetometer work with cobalt and nickel our tests did not reveal any certain torsional effect. In the recent work, however, in which the range and precision of the

observations have been greatly increased, torsional effects have often appeared.

Torsional effects are of two kinds, according as the magnetization is residual or induced. The first vanishes with the residual magnetization, the second with the residual axial field.

For very small twists, at least, the first is proportional to the torque, and thus changes sign with the direction of rotation; the second is proportional to the square of the torque and independent of the direction of rotation. The first effect may thus give rise to a systematic error. The second, provided the magnetization is stable, cannot do so if the principal observations are made only for equal speeds in opposite directions, unless the journal friction at the remote end of the rotor depends on the direction of rotation.

We have no evidence of the appearance of the second effect in our recent work, and many experiments have given the same results with and without an axial field.

If the two journals are exactly alike, the rotor balanced, and its magnetization stable, the first effect can be eliminated by observing first with the magnetic axis of the rotor in one direction, then with the axis turned through 180° by reversing the rotor in its bearings. Both effects can be eliminated by direct measurements upon the change of the magnetic moment of the rotor due to the application of a known torque, and the determination of the actual frictional torque acting upon the journal remote from the motor in the rotation experiments. When the journals and bearings are similar, this is just one-half the torque driving the rotor, which can be determined electrically. Both torsional effects can be eliminated simultaneously by driving the rotor alternately from each of the two ends, one end always being free except for journal friction, provided the journal torques are identical at the free ends.

The first method is the one which we have almost always used. We have tried the second in a number of cases, but since all other friction in addition to the friction of the rotor bearings introduces irregularities, the method is not satisfactory with the delicate rotors used in this investigation. The third method we have not tried.

It is of course difficult to be certain that the two journals are exactly alike. The frictional torque should be made as small as possible, and then the effect of residual irregularities will be reduced. It was for this reason that accurate eighth-inch stellite journals, well polished, and running in an agate bearing with clock oil as lubricant, were substituted, at the west end of the rotor, for our original half-inch journals. This effected a very great improvement. So far as torsion is concerned, the final three-sixteenths inch bronze journals and agate bearings appeared to perform equally well.

Inasmuch as the journal friction is nearly independent of the speed, the systematic error produced by torsion, when existent, is much less at high than at low speeds.

47. **Error due to thermal effects of journal friction on magnetization.** In addition to the thermal effect $A(3)$ which may produce a greater deflection with a magnetic rotor than with a non-magnetic one, the heat developed by journal friction may appreciably change the magnetic moment of the rotor, and differently for the two directions of rotation, and thus introduce systematic error.

So far as temporary magnetization is concerned, this effect vanished in our work, as no change was produced by the creation of even a considerable axial field.

As to permanent magnetization, if the journal torques for a given speed are identical at the two ends, the thermal effect of a systematic speed difference for the two directions of rotation can be eliminated by reversing the rotor in its bearings.

To reduce this error as far as possible, we first had the half-inch and eighth-inch rotor journals and bearings made as well as we could, and finally substituted the three-sixteenths inch bearings and insulated journals. This not only reduced the thermal effects at one end of the rotor, but greatly cut down the rate at which the heat produced was conducted to the rotor. Moreover, as the new journals were very precisely made, and as nearly alike as possible, the elimination of the error from any residual effect by reversal of the rotor was more nearly complete.

48. **Error due to thermal effect of air acting on rotor.** A very similar systematic error may be introduced by the air currents due to the motion, which, on account of asymmetry, may act differently on the rotor for the two directions of rotation. By reversing the rotor in its bearings this error also can be eliminated, provided the magnetization is sufficiently stable.

49. **Examples of torsion, journal friction, and air friction effects on magnetization.** In general these effects were made small by suitable demagnetization and by the other devices used. The following examples of the behavior of nickel show how troublesome they may be, also how they can be largely eliminated by reversal, even when of unusual magnitudes.

On October 16, 1922, a rod of nickel, through which the central residual flux density was about 0.04 line/cm.[2], was rotated at the gear ratio 2—1 with an eighth-inch bronze bearing west and a half-inch lumen bearing east. When the positive pole was *west*, right-handed and left-handed rotations *both* produced large deflections *up* the scale, the mean deflection being $\Delta = +21$ cm.±, but the differential deflection $d(= R - L) = +3.26$ cm. When the rotor was reversed, the positive pole being now *east*, the mean deflection was near $\Delta = -23$ cm.±, and the differential deflection $d = +3.60$ cm. The means for positive pole east and positive pole west are $\Delta = -1$ cm.± and $d = +3.43$ cm. (All deflections are reduced to the same magnetometer sensibility $A =$ about 17.8 cm.).

When half-inch bakelite bearings were substituted for the half-inch lumen bearing and the eighth-inch bronze bearing, and machine oil was used in place of clock oil (with which the bearings were lubricated in the experiments just described), the following results were obtained (for $A = 17.8$ cm., as above) with the same speed as before: With the positive pole *east*, $\Delta = -30$ cm., $d = -1.88$ cm.; with the positive pole *west*, $\Delta = +27$ cm. ±, $d = +8.63$ cm.±. The means are $\Delta = -1.5$ cm. ±, and $d = +3.38$ cm.

The observations for Δ were exceedingly rough.

With another rotor of nickel, of about the same dimensions, but provided with the latest type of insulated journals running in three-sixteenths inch agate bearings, and with residual induction about 0.07 line per cm.[2], we obtained, in November, 1923, for the gear ratio 2 — 1, the results $\Delta = -6.6$ cm., $d = +3.16$ cm. with the positive pole *west;* and $\Delta = +6.6$ cm., $d = +3.29$ cm. with the positive pole *east;* means, $\Delta = 0.0$ cm., $d = +3.22$ cm. All are reduced to the sensibility $A = 18.4$ cm. (These values are not comparable with those given above, as the magnetometers used and their positions were different.)

The effect of the air on the rotor's magnetization is well illustrated by observations made at the same speed on the same nickel rotor shortly after the observations just described. A stiff paper bag was put over the lower part of the magnetometer, in such a way as to lie quite unsymmetrically with the vertical plane through the rotor's axis. For positive pole west we now obtained $d_W = +3.04$ cm.; for positive pole east, $d_E = +3.42$ cm.; mean, $+3.23$ cm., all reduced to the same sensibility as above, viz., $A = 18.4$ cm. As the rotor's magnetization and the speed remained nearly the same, the difference between the new value of $d_E - d_W$, viz., $+0.38$ cm., and the old value, without the bag, viz., $+0.13$ cm., must be attributed almost entirely to the change in the symmetry of the air effects.

50. **The errors due to centrifugal expansion and other strains aside from torsion.** The possible systematic error due to the change in the magnetic moment of the rotor brought about by centrifugal expansion, and shown in 1915 to be proportional to the square of the speed, can be eliminated, as formerly, by observing for equal speeds in opposite directions. If there is a systematic difference between the speeds for the two directions of rotation, error due to the permanent part of the magnetization can be avoided by observing for both directions of the magnetic axis of the rotor.

If there is axial induced magnetization, however, any error arising from it and the speed differences cannot be eliminated in this way. As already stated, the application of considerable axial intensities in the work described here was without effect.

It is, of course, possible that a *part* of the deflections described in the last article had their origin in centrifugal expansion.

Possible errors due to strains arising from lack of axial symmetry are eliminated precisely like those due to centrifugal expansion.

51. **Errors from axial displacement of the rotor.** A certain amount of axial displacement is always possible when a rotor is set into motion, and if the displacement depends on the direction of rotation a systematic error may thus be introduced. This may happen in three ways: (1) eddy currents due to the residual field will affect the magnetometer differently for the two directions of rotation from the way they would if the rotor were axially fixed; (2) the effect of the permanent magnetization of the rotor on the magnetometer will depend on the direction of rotation; and (3) if the axial field does not vanish or is not uniform or symmetrical about the equatorial plane through the magnetometer, the temporary magnetization of the rotor, and its effect on the magnetometer, will depend on the direction of rotation.

In the final arrangement of our apparatus the maximum possible axial play of the rotor was 0.05 mm., but the actual displacement which occurred when the direction of rotation was reversed was only 0.01 mm., and was identical for all the gear ratios 2 — 7, 1—2, 1—1, and 2—1. During the steady rotations there was a minute axial vibration present, but the mean position of one end of a small journal, which protruded through the east agate bearing and was clamped in the chuck which drove the rotor, and on which the measurements were made, was 0.01 mm. farther east when the rotation was right-handed than when it was left-handed.

On account of the smallness of the displacement, effect (1) is not appreciable as shown by the measurements on the effect of axially displacing the magnetometer. Effect (2) will be much more pronounced in axial positions of the magnetometer than in equatorial positions, but can be eliminated by reversing the rotor. Effect (3) can be *made inappreciable only by making the axial field gradient* so small that the change in the rotor's moment due to the axial displacement does not affect the magnetometer appreciably.

In a series of experiments made before those referred to as final, effects (2) and (3) were looked for together by moving the Fe—Co rotor alternately through its maximum range east and west when the magnetometer was in an axial position, on a regular time schedule, as in the rotation experiments; but no appreciable effect was produced.

We may calculate the axial gradient necessary to produce an appreciable effect as follows: Let us assume that we have a rotor whose reversal of rotation at the speed 60 r.p.s., which is approximately equivalent to an axial field intensity of $60 \times 3.7 \times 10^{-7}$ gauss $=$ 2.2×10^{-5} gauss, produces a deflection of 4 cm. on the magnetometer scale. Suppose the reversal of rotation produces an axial motion y cm., that the intensity gradient in the axial direction is dF/dy gauss/cm., and that the resulting deflection is d cm. Then we evidently have $d/4 = y\, dF/dy/(2.2 \times 10^{-5})$ cm. $=$ (approx.) $0.5 \times 10^5\, y\, dF/dy$. Substituting $y = 0.001$ cm. and $d = 0.01$ cm., the minimum scale distance readable, we obtain $dF/dy = 2 \times 10^{-4}$ gauss/cm. $=$ (approx.) $0.001\ H$/cm., which is far greater than the possible axial gradient of the residual magnetic field in the rotor's vicinity.

52. The error from change in the rotor's azimuth or altitude. If the (small) angle between the normal to the axis of the rotor and the component ΔF of the residual magnetic intensity lying in the meridian (practically normal to the rotor) changes by an amount $\Delta\alpha$ when the direction of rotation is reversed, the effect will be equivalent to reversing in the rotor an axial intensity $f = \frac{1}{2} \times \Delta\alpha \times \Delta F$. If we assume that the difference between the diameters of the journals and bearings is as great as 0.05 mm., the maximum possible value of $\Delta\alpha$ will be about 1/3500 radian. Hence, if we assume ΔF as great as $H/500$, the maximum possible value of f will be about 0.05×10^{-7} gauss, an intensity equivalent to rotation at a speed of about 1/70 r.p.s., and therefore negligible.

The actual clearance between the journals and the bearings in the last series of experiments varied from about 0.035 mm. to about 0.05 mm., the latter value being reached in only a few cases. In the earlier experiments, with half-inch and eighth-inch journals, the clearance was about the same.

53. **Error from the Thomson repulsion effect.** Inasmuch as the magnetization of the rotor is never entirely symmetrical about its axis, the rotation produces an alternating magnetic field which induces currents of its own frequency in the magnetometer case and the suspension itself, and thus, on account of lack of perfect symmetry, may produce an appreciable torque on the suspension, different for different azimuths, and thus different for the two directions of rotation.

If this effect were appreciable, the values of λ in Table XIII would show a systematic dependence on the "oscillation" of column 31, which does not appear to be the case. We have, moreover, made special tests with negative results on the magnetometer with three-magnet systems. These tests were made with impressed alternating fields varying in frequency from 22 to 60 cycles per second, directed parallel, perpendicular, and at an angle of 45° to the magnetic axes, and much greater in intensity than the alternating parts of the field produced by any one of our rotors. With the intensities used, we obtained no certain permanent deflection except in the case of the most intense fields applied at 45° to the axes with the frequency 49 cycles per second. From the deflections obtained at different angles with both direct and alternating currents, from the ratio of these currents, and from the maximum angular deflection occurring on the reversal of a rotor in the principal experiments, it is easily shown that the deflection which would be produced by the Thomson effect in the principal experiments is less than about 1/300000 of the oscillation produced by very slow rotation of the rotor. It is thus in all cases entirely inappreciable.

54. **Errors from mechanical disturbances.** As the work progressed great improvements were made in the elimination of mechanical defects from the apparatus, especially within the year 1923, when we were fortunate enough to have the services of Mr. G. H. Jung on many occasions. We suspected that systematic errors might be introduced by (1) the failure of the motor to start with equal accelerations for right and left speeds, by (2) vibration and torsional shock of the rotor, and by (3) vibration of the magnetometer. Vibration of all sorts was made very small by such balancing of the rotors as was practicable and by such straightening and balancing of all but the most remote driving parts as were practicable; by bolting securely the rotor bedplate, the countershaft bearings, the motor, and the gear

box to heavy concrete piers; by keeping the motor and gear box remote from the rotor and magnetometer; and by surrounding the magnetometer supports with heavy concrete. By the use of thin countershafts near the rotor, and rubber hose connectors between the larger countershafts, torsional shock of the rotor was made very small, and there was no reason to fear a differential starting effect.

Even before the later precautions were taken the magnetometer image with well-demagnetized rotors showed no effect of the rotation (aside from displacements), except on one occasion, when it was discovered that the motor had become loose at one corner. (In some of the latest work with copper a new and poorly constructed magnetometer system had its image broadened by the rotation.)

The magnetic effect, if appreciable, of any mechanical disturbance *should become manifest as a variation of rotor sensibility with speed* in a compensated field. We have made experiments for the explicit purpose of testing this possibility with two rotors only. Electrolytic iron, of which one of them was made, is a substance whose residual magnetization is very easily affected by mechanical disturbances. Moreover, this particular rotor, Electrolytic Iron I, was made of a casting which had not been forged, and was quite badly unbalanced. In the experiments, which were made before the later improvements in the driving mechanism, it ran on a half-inch bronze journal and an eighth-inch stellite journal. For this rotor the rotor sensibility for the highest speed, gear ratio 2—1, was nearly 1.5% greater than for very low speed.

The other rotor, Cobalt-Nickel, had three-sixteenths inch journals of the earlier type, which were not so rigidly connected to the magnetic part of the rotor as were those of the later type. This rotor was also made of an unforged casting, but had been statically balanced. In a series of experiments at five different speeds, ranging from a few revolutions per second to about 60 r.p.s. (gear ratio 2 — 1) the rotor sensibility, as well as the absolute sensibility, which was determined with it, remained constant within the limits of the experimental error, about 1 part in 700.

55. **Errors due to inequality of right-handed and left-handed speeds.** As shown above, a number of systematic errors can be eliminated if the rotor can be driven at exactly equal speeds and with equal friction in opposite directions, conditions never exactly realized. It was therefore important to find out, for typical rotors, especially rotors for which the errors referred to are likely to be largest (those for which reversal of the magnetic axis produces the

greatest effects), how far a value of λ calculated from the mean speed for the two directions of rotation deviates from the true value when the speeds are not equal.

For this purpose resistance coils were inserted in the motor secondary for right-handed rotations, and cut out for left-handed rotations, making the right considerably slower than the left, and the value of λ calculated from the mean speed. Then the process was repeated, but with the resistances inserted for left-handed rotations.

Let T_R and T_L denote the times of 500 right- and left-handed revolutions of the motor, ΔT_1 the value of $T_R - T_L$ for the first set of observations just described, $\Delta T_2 = T_L - T_R$ for the second set, and $\Delta T = \Delta T_1 + \Delta T_2$; and let $\Delta\lambda$ denote the difference between the calculated values of λ obtained from the two sets. Then $\Delta\lambda$ is the error in λ produced by the difference of right-handed and left-handed speeds corresponding to the period difference ΔT, a quantity which was measured in nearly all of the recent experimental work.

Table VIII gives the results of all the observations which were made for the explicit purpose of studying the variations of λ and ΔT. The error $\Delta\lambda$, corresponding to the actual values of ΔT in the principal experiments, is in almost all cases entirely negligible. In the others, the proper corrections were applied.

TABLE VIII.

Errors Due to Inequality of Right-handed and Left-handed Speeds.
(Gear ratio 2–1.)

Rotor	No. sets	ΔT*	$10^7 \times \Delta\lambda$†	$10^7 \times \Delta\lambda / \Delta T$
Armco iron	4	+3.60	+0.10	+0.03
Nickel III	2	+3.12	−2.22	−0.71
	2	+2.17	−1.87	−0.86
	2	+1.30	−1.00	−0.77
		Mean for Nickel III		−0.78
Nickel I	4	+3.87	−0.08	−0.02
Preuss's alloy	2	+2.97	−0.14	−0.05

*ΔT is considered positive when right-handed rotations are slower.

†$\Delta\lambda$ is considered positive if for ΔT positive λ is greater when the positive pole of the rotor is east (and the negative pole west).

TABLE IX.

EFFECT ON λ OF ERRORS IN COMPENSATING CURRENTS.
(Gear ratio 2−1, except when otherwise stated.)

Date	Mag. pos.	Rotor	No. sets	$\Delta\lambda/\Delta V \times 10^7$	$100\Delta\lambda/\Delta C \times 10^7$	$40\Delta\lambda/\Delta B \times 10^7$
1923						
Jan. 25–26	*LEQ*	Cobalt II	27		+0.06	
Mar. 7–9	*LEQ*	Permalloy	16	+0.04 (1−1)	−0.02 (1−1 and 2−1)	
Mar. 16–17	*AN*	Steel III	18	+0.05	−0.04	
April 2, 3	*LEQ*	" "	6	−0.07	——	
May 22, 23, 28	*LEQV*	" "	12	+0.10	+0.01	−0.01
Aug. 31	*EQV*	Permalloy	2	−0.05±0.03		
Sept. 1	"	"	2	−0.19 0.04		
2	*AF*	"	2	+0.02 0.10		
10	*EQV*	Steel III	2	−0.42 0.06		
11	"	" "	2	−0.21 0.04		
13	*AF*	" "	2	+0.05 0.20		
Oct. 2	*EQV*	Steel IV	2	+0.03 0.08		
16	"	Armco Fe	4		+0.08±0.06	
20, 25	"	Yensen Fe (El. Fe II)	4	−0.07 0.06		
23	"	"	5		+0.08 0.06	
Nov. 4	"	Nickel III	4		+0.09 0.07	
8	"	" I	2		+0.04 0.05	
18	"	Hopk. alloy	2	−0.30 0.15		
18	"	" "	2		+0.26 0.06	
26–27	"	Cobalt II	4		+0.12 0.04	
Dec. 8	"	Yensen Fe	2	−0.20 0.07		
9	"	Heusler's alloy I	4	−0.24 0.19		
10	"	" "	2		+0.14 0.16	
13	"	Permalloy	2	+0.06 0.06		
16	"	Preuss's alloy	2		+0.01 0.05	
23–24	"	Steel IV	4		+0.03 0.02	
25	"	Steel I	3	−0.10 0.20		

56. **Errors in λ produced by errors in the compensating currents.** Table IX gives a summary of the observations made on the magnetic rotors in the course of the last two groups of experiments to determine the errors in λ due to errors in setting the compensation currents. $\Delta\lambda/\Delta V$ is the error in λ produced by increasing the current of the main compensating frame one division above the proper value; $40\Delta\lambda/\Delta B$ is the error produced by an excess of 40 divisions in the B current above the proper value; and $100\Delta\lambda/\Delta C$ is the effect on λ of a current of 100 divisions in the positive direction through the C coils. The absolute sensibilities are about the same as in the observations on copper for corresponding times (§ 35), but are not given here as they are implicit in λ.

Table X gives a summary of the errors made in setting the current V of the main compensating frame for the last series of experiments, also the values of the C current to compensate the nonuniform part of the horizontal intensity, calculated from the data of Table IV § 35, together with the actual settings of the C current for the rotation experiments. A comparison of Table X and Table IX will show that only minute, in nearly all cases negligible, errors in λ have been made by errors in setting the currents.

The actual errors in setting the V current are of course not quite so small as the table might indicate, since the current was set only to about 0.05 division.

57. **The observations for the interval September 24, 1922–August 7, 1923** (Series A). Table XI contains the results of nearly all the rotation observations made on the magnetic rotors between September 24, 1922 and August 7, 1923. At the beginning of this interval the apparatus was in much better condition than during the earlier work. The magnetometer was usually so well adjusted, and the inductors were so frequently used, that much greater errors than those actually made in setting the V and B currents could not affect the determination of λ; the rotors were, for the most part, better demagnetized and provided with new half-inch bronze and eighth-inch stellite journals very carefully worked by Mr. W. Steiner, and statically balanced; and improvements in the driving mechanism had been started by Mr. G. H. Jung. The three-phase slip-ring motor had been installed.

During the interval many improvements were made. The extra compensating coils were given almost their final disposition; the springs to keep the wires of the main compensating frame always taut

TABLE X.

Errors in Setting the Current V of the Main Compensating Frame and the Current C of the C Coils.

A. Magnetometer in position EQV. Gear ratio 2–1.

Date	Rotor	Error in current V	No. days	Current C	
				Calc.	Set
				−50 (Aug. 24)	
Aug. 31–Sept. 1	Permalloy	−0.02 ± 0.00	2		−50 div.
Sept. 10–12	Steel III	0.00 0.02	3		"
Sept. 30–Oct. 4	Steel IV	−0.01 0.01	5		"
Oct. 5–8	Co-Ni	+0.04 0.03	3		"
Oct. 14–17	Armco Fe	−0.01 0.02	4		"
				−27 (Oct. 19)	"
Oct. 20–26	Yensen Fe	−0.05 0.03	6		−27
Nov. 2–5	Nickel III	−0.03 0.06	4		"
Nov. 7–10	Nickel I	−0.02 0.02	4		"
				−18 (Nov. 11)	"
Nov. 16–18	Hopk. alloy	−0.03 0.03	3		−18
Nov. 19–27	Cobalt II	−0.05 0.02	6		"
Dec. 8	Yensen Fe	−0.06	1		"
Dec. 9–12	Heusler's alloy I	−0.06 0.04	4		"
Dec. 13–15	Permalloy	−0.04 0.02	3		"
Dec. 16–18	Preuss's alloy	+0.02 0.02	3		"
				−7 (Dec. 20)	"
Dec. 22–24	Steel IV	+0.09 0.04	3		"
Dec. 25–27	Steel I	−0.04 0.01	3		"
				−12 (Dec. 28)	
Dec. 29–30	Norway Fe	+0.03 0.05	2		"
Dec. 31–Jan. 1	Steel III	+0.10 0.03	2		"

Mean error in current V −0.01

B. Magnetometer in position AF. Gear ratio 2–1.

				−50 (Aug. 24)	
Sept. 2–6	Permalloy	−0.07 0.01	4		−50
Sept. 13–14	Steel III	−0.06 0.00	2		−50

Mean error in current V −0.06

were installed; the new, heavier, and less magnetic bedplate was instituted for the old; the driving mechanism was realigned, balanced, and in general greatly improved; the magnetometer and compensator supports were rearranged so as to make the vertical equatorial positions possible, and finally imbedded in heavy concrete; and the extra compensating loops which could be thrown in and out at will, were provided to modify the degree of uniformity of the total field produced by the main compensating frame. Most of the observations were made with magnetometers with six-magnet systems, and with high sensibility. Some were made with greatly reduced sensibility, and some with the three-magnet system substituted for the six-magnet system. Most of the observations were made with lumen or babbit bearings at the east end of the rotor; lignum vitae and mahogany were substituted in a few cases.

In the table the first column gives sequence of the observations, the second the magnetometer position, the third the effect on the calculated value of λ of reversing the rotor from mark west to mark east, the fourth the number of sets in the groups, the fifth the gear ratio or speed, the sixth the mean value of λ, the seventh the average departure of the individual values in the sets, the eighth the average departure of the set means from the group mean. The symbol $2-7ls$ means that the gear ratio was $2-7$ and that a rheostat was placed in the circuit reducing the speed to a few revolutions a second, about one-third of the usual speed corresponding to the ratio $2-7$. Most of the experiments were made with the magnetometer in lower equatorial positions, some with the height increased 4.7 cm. We are not satisfied that any of these changes produced any systematic effect, and have therefore, in most cases, presented only means and their average discrepancies. Attention has been called by notes to the changes in bearings.

The most reliable results are those obtained at high speed and under such other conditions that reversal of the rotor produced only a slight effect. Those obtained with equatorial positions of the magnetometer are given in Table XII.

In Table XI results are given also for two axial positions of the magnetometer. In the position *AN* the lower system was about 12 cm. from the end of the magnetic part of the rotor; in the position *AF*, about 17 cm. The most reliable observations for *AN* agree very nearly with those for the equatorial positions, while those for *AF* give much smaller values. A number of other sets of observations were taken without the reversal of the rotors and are not included in the table, except in one or two cases where the effect of reversal was

TABLE XI.

CHIEF OBSERVATIONS ON MAGNETIC ROTORS FROM SEPTEMBER 24, 1922 TO AUGUST 7, 1923.

(Series A.)

Steel III — September 24, 1922 to July 14, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E − W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*EQ**	+0.04	67	2–1	3.82	±0.06	±0.06
2		+0.05	31	1–1	4.02	0.09	0.13
3		+0.08	30	1–2	4.39	0.19	0.19
4		+0.20	20	2–7	4.49	02.9	0.14
5		+0.19	2	2–7*ls*	5.54	0.36	0.10
6	*AN*	−0.03	47	2–1	3.67	0.10	0.09
7	*AF*	−0.02	19	2–1	3.36	0.17	0.05
8	*AF*	−0.03	4	1–1	3.49	0.10	0.02
9	***EQ*	+0.27	3	2–1	3.61	0.10	—
10		+1.53	3	2–7	5.80	0.37	—

* Mahogany bearing E in a few cases.
** Lumen bearing W.

(B. of S.) Electrolytic Iron I — October 22, 1922.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E − W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*LEQ*	+0.22	4	2–1	4.07	±0.11	±0.05
2	*LEQ*	+0.20	3	1–1	3.40	0.16	0.06

Heusler's Alloy II — October 27–28, 1922.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E − W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*LEQ**	+0.09	9	2–1	3.92	±0.16	±0.11
2	*LEQ**	+0.15	7	1–1	3.81	0.32	0.12

* Babbit bearings E and W.

Western Electric Permalloy — December 26, 1922 to August 7, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E − W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*EQ*	+ 0.09	34	2–1	3.73	±0.05	±0.06
2		+ 0.10	21	1–1	3.66	0.09	0.08
3		+ 0.04	4	1–2	3.98	0.14	—
4		+ 0.20	4	2–7	4.64	0.29	0.10
5	*AN*	+0.28	14	2–1	3.72	0.06	0.05
6	*AF*	+0.25	8	2–1	3.33	0.14	0.08
7	*AF*	−1.23	3	2–7	4.23	0.64	—

Preuss's Alloy — January 16, 1923 to April 9, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E−W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	LEQ	* +0.04	16	2-1	3.63	±0.07	±0.08
2		+0.32	20	1-1	3.72	0.18	0.13
3		+0.29	3	2-1	3.79	0.07	—
4		+0.87	4	1-1	4.13	0.10	0.09
5		** +1.03	3	1-2	4.60	0.12	—
6		+0.51	3	2-7	5.32	0.29	—
7		−0.09	3	2-7*ls*	5.98	1.31	—

* Wood bearing E. ** Lumen bearing E.

Cobalt II — January 25-30, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E−W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*LEQ*	−0.005	16	2-1	3.79	±0.14	±0.08
2	*LEQ*	−0.16	17	1-1	3.68	0.22	0.20

Armco Iron — June 17, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E−W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*LEQ*	+0.11	4	2-1	3.99	±0.05	±0.02
2	*LEQ*	−0.07	2	2-7	3.20	0.23	—

Steel II — January 16-23, 1923.

1 Group	2 Mag. pos.	3 $10^7 \times \Delta\lambda$ (E−W)	4 No. sets	5 Gear ratio	6 $-\lambda \times 10^7$	7 $\Delta\lambda' \times 10^7$	8 $\Delta\lambda'' \times 10^7$
1	*LEQ*	+0.03	4	2-1	3.72	±0.07	±0.04
2	*LEQ*	+0.08	2	2-7	3.14	0.34	—
3	*LEQ*	−0.06	4	2-1	3.84	0.08	—
4	*LEQ*	* −0.26	4	1-1	3.64	0.20	—
5	*LEQ*	+0.35	2	1-2	3.18	0.29	—
6	*LEQ*	+1.10	2	2-7	3.51	0.51	—

* **Lumen half-inch and bronze eighth-inch bearings W.**

approximately known. These omitted observations agree well with the others.

For most of the rotors the values of λ for the gear ratio 1−1 do not differ greatly from those for 2−1. But in general for the lower speeds λ increases or decreases considerably, or even very greatly, as the speed goes down. For these speeds the accidental errors are relatively quite large, but the systematic effects are nevertheless undoubtedly authentic.

With the precautions taken and the tests made with reference to eddy-current effects we could not see how these discrepancies could be attributed to them. Also, some of the rotors were so well demagnetized and had journals at the two ends so nearly identical, and the right and left speeds were so nearly equal, that it seemed difficult to attribute the discrepancies to journal or bearing effects. We nevertheless became convinced that such effects were chiefly responsible, and therefore proceeded to modify the rotors and bearings in such a way as to cut down the heat developed at the bearings, at least the east bearing, and to insulate the journals from both the magnetic part of the rotor and from the bearing blocks. The devices used have already been described.

TABLE XII.

λ FROM THE BEST EQUATORIAL OBSERVATIONS OF TABLE XI.

A. Gear ratio 2–1.

Rotor	Group	$10^7 \times \Delta\lambda$(E–W)	$-\lambda \times 10^7$	$\Delta\lambda'' \times 10^7$	No. sets
Steel III	1	+0.04	3.82	±0.06	67
Heusler's alloy II	1	+0.09	3.92	0.11	9
Permalloy	1	+0.09	3.73	0.06	34
Preuss's alloy	1	+0.04	3.63	0.08	16
Cobalt II	1	−0.05	3.79	0.08	16
Armco iron	1	+0.11	3.99	0.02	4
Steel II	1 and 3	−0.02	3.78	0.04±	4

Weighted mean $\lambda \times 10^7$ for 150 sets $= -3.78 \pm 0.06$ Total 150

B. Gear ratio 1–1.

Rotor	Group	$10^7 \times \Delta\lambda$(E–W)	$-\lambda \times 10^7$	$\Delta\lambda'' \times 10^7$	No. sets
Steel III	2	+0.05	4.02	±0.13	31
Heusler's alloy II	2	+0.15	3.81	0.32	7
Permalloy	2	+0.10	3.66	0.08	21
Preuss's alloy	2	+0.32	3.72	0.13	20
Cobalt II	2	−0.16	3.68	0.20	17
Steel II	4	−0.26	3.64	—	4

Weighted mean $\lambda \times 10^7$ for 100 sets $= -3.80 \pm 0.15$ Total 100

58. **The last series of observations on λ** (Series B). The last series was begun near the end of August, 1923, and was completed on January 1, 1924.

A few observations, on Steel III and Permalloy, were made with the magnetometer in the position *AF*, the remainder with the magnetometer in the position *EQV*, as in the later observations of the series immediately preceding.

For these experiments all the rotors to be used were provided with three-sixteenths inch insulated journals of one of the two types already described in § 12 and illustrated in Figure 16, (a) and (b). The earlier experiments in the series were made with journals of type (a), the remainder with journals of type (b), which were mechanically much superior. The earlier experiments with Permalloy were made with bakelite bearings, all the others with agate bearings. Except in the case of the rotor of Bloch's alloy (Co-Ni) all of those rotors which were at first provided with journals of type (a) were later provided with, and rotated with, journals of type (b).

The substitution of these small, insulated, and almost perfectly made journals, together with the agate bearings, for those used before, the correction of the warp in the main compensating frame, the perfecting of the method of adjusting the compensator to correct for imperfections in the magnetometer, and the insertion of a cardboard screen between rotor and bedplate in part of the work or the surrounding of the rotor with a uniform cylindrical cardboard tube in much of the remainder of the work, constituted the chief sources of improvement over the conditions in which the immediately preceding series of observations was finished.

Unfortunately, the mirror of the excellent six-magnet magnetometer system with which the series was begun was not securely fastened, and became displaced while the magnetometer was being moved from one position to the other. In attempting to remount the mirror in better fashion the system was injured, and many attempts to put it in first-class condition again were failures. It was used again for observations on copper near the end of the series; but after the first month's work the three-magnet system already mentioned, and constructed only for special tests, was used exclusively with the magnetic rotors. Inasmuch as we were continually expecting to be able to use the six-magnet system again, this state of things led to the magnetometer's azimuth being usually not so well adjusted as in most of the preceding series; but, as the observations on V and $\Delta\lambda/\Delta V$, Tables IX and X, show, this introduced no error of any consequence. The three-magnet system was considerably slower than the six-magnet systems, but not too slow to avoid appreciable error, except possibly with those rotors (Heusler's alloy and Preuss's alloy) which exhibit considerable magnetic lag. We estimate that an error of one-fourth per cent is possible with these rotors.

During most of the immediately preceding series of observations induction measurements for setting the V and B currents were made almost every night, so that, even had the variometers been very unreliable, eddy-current errors from incorrect settings of these currents could have exceeded only very small quantities. In the last series, however, the severity of the work necessitated limiting greatly the number of induction measurements. This procedure was adopted with less reluctance because the earlier work had shown that, with good magnetometer adjustment, very considerable errors in the current settings could be made without affecting the values of λ appreciably. The observations in Tables IX and X show that only negligible errors in λ could have arisen from errors in setting the V and C currents, at least in nearly all cases. Had the C current been made zero, as the work with the induction balance seems to indicate was correct, the values of λ would have been changed (*increased*) by only minute amounts.

Throughout the last series the V and B current settings were made from the limited number of induction observations and the variometer readings. The change in the V current was so slight and so slow that it introduced no difficulty. Our observations have been confirmed by the vertical intensity observations of the Cheltenham Observatory, which show that this intensity decreased very slowly, by only a few parts in 5000, from the beginning to the end of our work. On one night when the work was in progress it was low by about 1 part in 1000.

Unfortunately, changes, slight as they were, in the compensating frame had greater effects on the proper value of the current B. As the exact effects of these changes were unknown, it was impossible, from the relatively few induction determinations and the variometer readings taken twice every night, to predetermine with exactness the proper value of B. No direct observations for $\Delta\lambda/\Delta B$ were made. However, from the Cheltenham horizontal intensity observations, which we have used generally in preference to our own less complete observations (although these appear to be reliable), and interpolation and extrapolation (after December 14) from our induction determinations, on the assumption of linear changes in the frame with time, it was possible to calculate for each night, or each hour of each night, the correct value of the current B. The amount ΔB by which our current B exceeded the correct value, obtained in the way indicated, is given for each group of observations in column 33 of Table XIII. Combined with the results of Table V, § 36, these values of ΔB show, as indicated in § 36, that no appreciable errors originated from the departure of ΔB from zero.

59. **Example of a night's work.** As an example of one night's work, neither as good as the best nor as bad as the worst, we shall give the results obtained on December 13, 1923, those for the first part of the night in detail, those for the latter part by means. The rotor was Permalloy, running in agate bearings and inclosed in the protecting cylinder of pasteboard, one side, X, being up during the first part of the work, and down the rest. The magnetometer was in the position EQV. The times are only approximately correct, and refer to the beginnings of the different groups.

$2^h\ 24^m$ A.M.— VARIOMETER READINGS AND CURRENT SETTINGS.

	Horizontal	Vertical	Declination
Temp.	19°.0 C.	18°.8 C.	——
Scale	+2.06 cm.	+0.46 cm.	56.95 div.
Comp. current	+0.79 amp. (B)	1.384+amp. (V)	1.49 amp. (A)

The C current was set at −0.18 amp.

$2^h\ 33^m$ A.M.— ABSOLUTE SENSIBILITY SCALE READINGS.

Switch R	Switch L	$A = \vert R-L \vert$
−1.48 cm.		
	+16.81 cm.	18.26 cm.
1.31		.17
	16.91	.15
1.16		.12
	17.00	.18
1.20		.16
	16.91	.01
0.99		.02
	17.19	

Mean $A = 18.13$ cm. ±0.06 cm.

ROTATIONS. MARK E. X UP.

$2^h\ 50^m$ A.M.— SCALE READINGS.

Set 1 Gear ratio 2−1 $V = 138.4+$

R	L	$R-L=d$	Δd	T_R (Time of 500 rev.)	T_L
+10.60 cm.				$16^s.90$	
	+6.30 cm.	4.35 cm.	−0.01 cm.		$16^s.90$
10.70		4.35	− .01	.63	
	6.40	4.35	− .01		.80
10.80		4.35	− .01	.52	
	6.50	4.41	+ .05		.67
11.02		4.47	+ .11	.52	
	6.60	4.46	+ .10		.58
11.10		4.32	− .04	.48	
	6.97	4.23	− .13		.52
11.30		4.26	− .10	.50	
	7.10				.54
	Means	4.36	±0.06		

Mean $T_R = 16.59$ $T_L = 16.67$

Mean $T = 16.63$

$T_R - T_L = -\ 0.08$

$3^h\ 8^m$A.M.— SCALE READINGS.

Set 2 Gear ratio 2−7 $V=138.4+$

R	L	R−L=d	Δd	T_R	T_L
+10.29 cm.				$16^s.08$	
	+9.47 cm.	0.675 cm.	+0.06 cm.		$16^s.02$
10.20		0.565	−0.05	.04	
	9.80	0.51	−0.10		.02
10.42		0.565	−0.05	.03	
	9.91	0.65	+0.04		.02
10.70−		0.74	+0.13	.05	
	10.01	0.79	+0.18		.04
10.90		0.76	+0.15	.02	
	10.27				.02
——		0.65	+0.04	.04	
	10.30				.02
10.97		0.51	−0.10	.00	
	10.62	0.42	−0.19		.07
11.11		0.59	−0.02	.02	
	10.41+	0.74	+0.13		.07
11.20−		0.665	+0.05	.05	
	10.66	0.645	+0.03		.04
11.41		0.62	+0.01	.05	
	10.92+	0.54	−0.07		.03
11.52		0.56	−0.05	——	
	11.00	0.56	−0.05		——
11.60		0.50	−0.11	——	
	11.20				
		Means 0.613	±0.08	Means 16.04	16.06

$V=138.3$

Mean T 16.05

$T_R - T_L$ −0.02

MARK E. X DOWN.

Time	Set	Gear ratio	d	Δd	T	T_R-T_L	V
3^h42^mA.M.	3	2−1	4.46	±0.07	16.52	−0.12	1.389−
58	4	2−1	4.41	0.08	16.46	−0.06	1.379−
4 16	5	2−7	0.623	0.10	16.00	+0.07	1.384

$4^h\ 37^m$A.M.— ABSOLUTE SENSIBILITY.

$A=18.24$

E.m.f. of battery = 1.546 volt

Ammeter — potentiometer = + 0.0015 amp.

$4^{h}\ 52^{m}$A.M.— VARIOMETER AND CURRENT READINGS.

Horizontal	Vertical	Declination
Temp. 19°.5C.	19°.1C.	——
Scale +1.91 cm.	+0.44 cm.	56.2+
Comp. current 0.785 amp. (B)	1.384 amp. (V)	1.43 amp. (A)

The C coil current read −0.18 amp.

From the above variometer scale and temperature readings, together with the variometer constants given in § 12, we find that the horizontal and vertical intensities at the end of the night differed from the intensities at the beginning by only about 1 part in 11,000 and 1 part in 34,000, respectively. The declination changed by only 1 minute.

The value of $Q = B/A$, determined on the preceding day, was $Q = 2.936$; the value of K was 0.0%.

From what precedes, and the formula (8) of § 25, we obtain the following values of λ:

Set	Gear ratio		$-\lambda \times 10^7$		$\Delta\lambda \times 10^7$
1	2−1		3.75		±0.05
2	2−7		3.56		0.46
3	2−1	over and under	3.79	3.76	0.06
4	2−1	compensated	3.73		0.07
5	2−7		3.59		0.58

which are to be combined with the observations of the next day, with the rotor reversed, to obtain values of λ more nearly free from error. The final results are given in Table XIII.

60. **The results of the last series** (Series B). This table (Table XIII) gives a summary of the results for the whole series. The values of λ are given in columns 6, 12, 18, and 24. Columns 4, 10, 16, 22 give the number of sets in each group of observations. Columns 7, 13, 19, and 25 give the average departure from the mean in the separate sets of which the groups are composed, while columns 8, 14, 20, and 26 give the average departure, from the mean for each group, of the means for the individual sets of which it is composed. Columns 3, 9, 15, 21 give ΔT, which is the excess of T for right-handed rotations over T for left-handed rotations when the marked end of the rotor is east *minus* the similar quantity when the marked end is west. Columns 5, 11, 17, 23 give the amounts by which the experimental value of λ with mark east exceeds that with mark west. It is twice the error

		Gear ratio: 2–7																			
1 Date	2 Rotor	3 ΔT	4 No. sets	5 $10^7 \times \Delta\lambda$ (E – W)	6 $-\lambda \times 10^7$	7 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	8 Av. dep. (sets) $10^7 \times \Delta''\lambda$	9 ΔT	10 No. sets	11 $10^7 \times \Delta\lambda$ (E – W)	12 $-\lambda \times 10^7$	13 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	14 Av. dep. (sets) $10^7 \times \Delta''\lambda$	15 ΔT	16 No. sets	17 $10^7 \times \Delta\lambda$ (E – W)	18 $-\lambda \times 10^7$	19 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	20 Av. dep. (sets) $10^7 \times \Delta''\lambda$	21 ΔT	I s
Oct. 20–21	El. iron II	$-0^{s}.02$	3	+1.06	3.95	±0.49	—													$-0^{s}.03$	
Oct. 23–26		+0.05	2	−0.05	3.84	0.48	—	+0.05	2	−0.06	4.01	±0.35	—	$-0^{s}.02$	4	−0.01	3.98	±0.17	±0.01	+0.03	
Dec. 8																				−0.05	
Oct. 14	Armco iron	—	2	+0.21	3.60	0.44	±0.16	+0.06	2	0.15	3.60	0.24	±0.08	−0.03	2	+0.03	3.60	0.11	0.02	−0.05	1
Dec. 29–30	Norway iron	−0.03	5	−0.37	2.63	0.86	0.36													−0.01	
Sept. 10–12	Steel III ¶	—	4	−0.31	4.07	0.35	0.32	—	3	+0.11	3.76	0.24	0.11	—	4	+0.01	3.86	0.14	0.24	—	
Sept. 13	*AF* ¶	—	2	east only	4.85	0.60	0.19	—	1	east only	3.93	0.26	—	—	2	east only	3.75	0.24	0.26	—	
Oct. 3, 4	¶	—	2	west only	3.86	0.32	0.08							−0.01	5	+0.06	3.83	0.07	0.04	+0.01	
Dec. 31–Jan. 1		+0.01	5	−0.15	3.71	0.37	0.11													+0.05	
Sept. 30–Oct. 2	Steel IV ¶	+0.08	2	−0.36	4.41	0.30	0.18	0.00	2	+0.04	4.10	0.28	0.02	+0.02	2	−0.04	4.11	0.08	0.02	+0.04	
Dec. 22–24		−0.02	6	−0.46	3.65	0.49	0.25													−0.02	13
Dec. 25–27	Steel I	−0.11	4	+0.28	3.60	0.57	0.19							0.00	4	+0.23	3.72	0.16	0.07	−0.02	9
Nov. 2–3	Nickel III							+0.02	2	−1.27	2.86	0.35	—	+0.02	4	−0.38	3.59	0.18	0.04	−0.06	
Nov. 4–5																				+0.03	
Nov. 8–10	Nickel I	+0.01	2	+0.28	3.49	0.70	0.14	+0.03	2	+0.30	3.74	0.36	0.15	+0.12	2	0.00	3.67	0.21	0.00	−0.03	
Nov. 19–21	Cobalt II																			−0.02	
Nov. 22–27								−0.01	6	−0.02	3.85	0.45	0.17	−0.04	6	0.00	3.73	0.17	0.14	−0.03	
Dec. 9–12	Heusler's alloy I	−0.01	4	+0.41	3.60	0.77	0.24	−0.07	4	+1.10	3.83	0.54	0.26	0.00	4	+0.01	3.48	0.24	0.12	−0.01	
Aug. 31–Sept. 1	Permalloy ¶	—	5	+0.08	3.97	0.26	0.15	—	3	+0.34	3.87	0.15	0.17	—	3	+0.04	3.76	0.08	0.05	—	
Sept. 2–6	*AF* ¶							+0.02	6	−0.08	3.90	0.58	0.03	+0.01	4	+0.03	3.91	0.26	0.04	−0.06	
Dec. 13–14		+0.02	3	−0.42	3.78	0.60	—													+0.05	
Nov. 16–18	Hopkinson's alloy													0.00	8	+0.01	3.87	0.36	0.22	−0.04	
Dec. 16–18	Preuss's alloy	+0.01	4	−0.72	4.48	0.55	0.36	−0.02	4	+0.16	4.06	0.22	0.10	+0.05	4	+0.01	3.83	0.17	0.06	−0.12	
Oct. 5–8	¶ Cobalt-nickel	+0.03	5	+0.07	3.77	0.60	0.43±	+0.07	4	+0.13	3.59	0.28	0.12	0.00	4	+0.09	3.81	0.12	0.05	+0.08	

¶ Rotor provided with earlier style three-sixteenths inch journals.

Means: 3.73 ± 0.51 ± 0.25.
Mean T ranged from $15^{s}.90$ *to* $16^{s}.11$.
Total number of sets = 60. In obtaining the mean value of λ only those sets (56) were used which occurred in groups in which the rotor was reversed.

Means: 3.81 ± 0.36 ± 0.12.
Mean T ranged from $16^{s}.02$ to $16^{s}.24$.
Total number of sets = 41. In obtaining the mean value of λ only those sets (34) were used which occurred in groups in which the rotor was reversed.

Means: 3.79 ± 0.19 ± 0.10.
Mean T ranged from $16^{s}.13$ to $16^{s}.36$.
Total number of sets = 62.

Means:
Mean T
Total n

2 o. ts	23 $10^7 \times \Delta\lambda$ (E−W)	24 $-\lambda \times 10^7$	25 Av. dep. (obs.) $10^7 \times \Delta'\lambda$	26 Av. dep. (sets) $10^7 \times \Delta''\lambda$	27 Approx. *d* (2−1)	28 Approx. *A*	29 *Q*	30 $I \times 10^3$ Begin.	 End	31 Osc.	32 ΔO (2−1)	33 ΔB	34 Remarks
					cm.	cm.				cm.	cm.	div.	
4	+0.03	3.97	±0.08	±0.05	3.6−3.8	18.4−19.2	2.337	*+12.5	+19.2	5.8	0.3	− 4.0	
9	−0.07	3.92	0.06	0.05	3.7−3.8	18.9−19.2	2.337	*+ 5.1	− 0.1	1.5	0.4	+ 3.4	
5	0.00	3.83	0.08	0.02	3.5−3.6	17.9−18.0	2.379	*+ 1.9	0.0	1.5	0.0	+ 5.5	Tube on. (See §42.)
0	0.00	3.67	0.06	0.04	2.3−2.5	18.0−18.8	1.684	*− 3.5	− 1.0	4.5	0.2	+ 2.6	
6	−0.08	3.69	0.12	0.04	2.6	17.2−17.4	1.885	*− 0.9	+ 0.1	26±	0.6	+18.0	Tube on.
6	+0.01	3.76	0.06	0.03	3.6−3.9	19.0−20.4	2.355	*− 2.1	− 4.0	4.5	0.9	− 0.2	Six-magnet systems.
2	east only	3.81	0.20	—	1.7	19.4	1.104	*− 4.0	− 5.2	1.2	0.1		Six-magnet systems.
5	0.00	3.77	0.05	0.05	3.5−3.6	17.9−18.7	2.371	*− 5.2	− 5.7	2.5	0.4	+ 1.0	
4	−0.16	3.72	0.06	0.02	3.1−3.2	16.1−16.7	2.371	*+ 1.3	+ 5.0	1.0	0.6	+ 9.3	Tube on. New journals.
3	+0.06	3.94	0.07	0.03	3.0−3.3	17.6−19.0	1.973	*+ 7.8	+ 2.0	8.7	0.6	− 1.7	
	−0.05	3.73	0.06	0.06	3.0−3.1	17.9−18.8	2.026	*− 8.2	−10.4	0.7	0.4	+10.8	Tube on. New journals.
	−0.11	3.75	0.10	0.07	2.3−2.6	15.4−18.1	1.802	− 1.7	− 0.1	2.4	0.7	+13.6	Tube on.
4	−0.14	3.67	0.09	0.01	3.2−3.3	18.4	2.199	− 5.8	− 4.4	0.9	6.6	+ 4.2	Card on.
6	−0.56	3.60	0.08	0.03	3.4−3.5	19.1	2.199	*− 4.4	− 4.9		—	+ 6.2	Bag on and card. (See §49.)
9	0.00	3.75	0.08	0.06	2.3−2.5	17.2−17.5	1.752	*+ 2.1	2.1	6.0	0.1	+10.0	Card on.
6	−0.07	3.86	0.09	0.06	2.8−2.9	17.2−18.0	1.937	*+11.3	+19.7	3.8	0.1	+ 6.6	Tube on.
7	−0.05	3.82	0.10	0.05	2.9−3.0	17.9−18.3	1.943	*− 1.1	− 5.5	8.7-6.1	0.4	+ 5.0	Tube on.
0	+0.05	3.62	0.20	0.06	1.5−1.6	15.9−18.4	1.145	*+ 3.2	+ 4.4	5.8-6.2	1.0	+ 7.8	Tube on.
4	−0.02	3.77	0.04	0.06	4.7−4.9	19.3−19.7	2.960	*− 2.2	—	8.8	0.7	− 6.8	Six-magnet systems. Bakelite bearings.
7	+0.12	3.84	0.14	0.03	2.1−2.3	17.7−19.7	1.386	− 6	.0	3.0	0.3		Bakelite bearings. *AF*
5	+0.10	3.70	0.06	0.05	4.2−4.5	18.0−18.2	2.936	*+ 0.6	+ 0.3	8.8	0.7	+ 8.6	Agate bearings. New journals. Tube on.
0	+0.04	3.63	0.18	0.06	1.2−1.5	15.7−18.4	1.047	+ 7.3	+13.3	14.4	0.7	+ 5.9	Tube on.
6	+0.04	3.83	0.09	0.02	3.8−4.0	17.6−18.5	2.585	*+ 0.1	− 1.7	5.9	0.4	+11.7	Tube on.
4	0.005	3.83	0.06	0.04	2.5−2.6	18.5−18.9	1.651	*+ 1.1	+ 4.3	9.0	0.2	+ 4.2	

3.76 ± 0.09 ± 0.05.
ranged from $16^{s}.30$ to $16^{s}.64$.
umber of sets = 159.

* Observation of *I* made on day preceding that in which rotations began.

which would be made if the rotor were not reversed. Column 27 gives the approximate value of the deflection d produced on reversing the rotor with the gear ratio 2 — 1 (speed approximately 60 r.p.s.); column 28, the approximate value of the absolute sensibility A; column 29, the exact value of the ratio Q of rotor sensibility to absolute sensibility; column 30, the approximate values of the mean intensity of magnetization of the rotor in c.g.s. units; column 31, the approximate range of motion of the spot of light on the magnetometer scale when the rotor is turned through a complete revolution with extreme slowness; column 32, the approximate amount by which the magnetometer zero is changed when the speed changes from a few revolutions per second to the highest speed (except that for Nickel III, these quantities, obtained from rough observations, may all be spurious, as many of them doubtless are). Column 33 gives the error ΔB in the B current, as has already been explained.

The value of λ in each group in which the rotor was reversed was obtained by giving equal weights to the observations for mark east and mark west, usually equal or nearly equal in number. The means were obtained by giving each group a weight proportional to the number of sets it contains.

It is clear from the table that the discrepancy between the values of λ for the positions AF and EQV no longer exists; also, that in the mean no variation of λ with speed is apparent. This latter fact alone is strong evidence that the chief systematic errors are eliminated, as most of them produce deflections which are nearly independent of the speed, or proportional to the square of the speed, while the deflection produced by the effect under investigation is proportional to the speed.

The most reliable results are those obtained with the gear ratio 2—1 and are collected in Table XIV. The last column gives the numerical departures $\Delta'''\lambda$ of the values of λ for the individual rotors from the mean for all the rotors, and the mean value of these departures. $\Delta'\lambda$ and $\Delta''\lambda$ have the same meanings as in Table XIII. All quantities are given weights proportional to the number of sets of observations involved. Although $\Delta'''\lambda$ is nearly twice as great as $\Delta''\lambda$, an examination of the values of λ obtained with different rotors of the same or nearly the same material, and in different groups of observations (Table XIV) on the same rotors, reveals discrepancies making uncertain any conclusion that real differences in λ for the different rotors and materials exist.

If the values of λ for all the rotors in Table XIV are given equal weights, we obtain the mean 3.75 in place of 3.76.

TABLE XIV.

λ FROM OBSERVATIONS AT THE HIGHEST SPEED.

Rotor	No. sets	$-\lambda \times 10^7$	$\Delta'\lambda \times 10^7$	$\Delta''\lambda \times 10^7$	$\Delta'''\lambda \times 10^7$
Yensen iron	18	3.91	±0.07	±0.04	0.15
Armco iron	10	3.67	0.06	0.04	0.09
Norway iron	6	3.69	0.12	0.04	0.07
Steel III	17	3.76	0.07	0.03	0.00
Steel IV	21	3.81	0.06	0.05	0.05
Steel I	9	3.75	0.10	0.07	0.01
Mean for iron and steel (6 rotors)	(81)	3.79	0.07	0.04	0.06
Nickel III	10	3.64	0.08	0.02	0.12
Nickel I	9	3.75	0.08	0.06	0.01
Mean for nickel (2 rotors)	(19)	3.69	0.08	0.04	0.05+
Cobalt II	13	3.84	0.10	0.05	0.08
Heusler's alloy I	10	3.62	0.20	0.06	0.14
Permalloy	16	3.78	0.09	0.04	0.02
Hopkinson's alloy	10	3.63	0.18	0.06	0.13
Preuss's alloy (Fe—Co)	6	3.83	0.09	0.02	0.07
Bloch's alloy (Co—Ni)	4	3.83	0.06	0.04	0.07
Mean for 159 sets	=	3.76	±0.09	±0.05	±0.07
Mean $\lambda/2\pi$	=	$1.06 \times m/e$	± 2.4%	± 1.2%	±1.9%

Almost exactly the same mean was obtained from the experiments of the immediately preceding series (Table XII), but the work done in the final series has increased the degree of certainty of the results very greatly.

It is a source of great satisfaction to us that our early work by the method of electromagnetic induction, including the earliest work, of 1914, agrees so closely with all of our recent determinations. The freedom of this early work from systematic errors is due largely to the great symmetry of the experimental arrangement for detecting the changes of magnetization produced by the rotation.

61. **Recent experiments on rotation by magnetization.** We have already referred to the effect converse to ours and discovered in the following year by Einstein and de Haas, viz., rotation by magnetization. A considerable amount of work, some of which has already been mentioned, has now been done on this effect. The most thorough investigations are those of Emil Beck,[33] who used a method of resonance greatly improved over the original method used by Einstein and de Haas; Chattock and Bates,[34] who used a ballistic method improved over that of J. Q. Stewart; and Sucksmith and Bates,[35] who used a null resonance method.

On the simplest classical theory and the assumption of a single simple magneton, these experiments determine the ratio R of its angular momentum to its magnetic moment. On the same theory, as already stated, this value of R should agree with our value of $\lambda/2\pi$; and both should be equal to $2\,m/e$ for a Bohr magneton.

All the most reliable of these experiments, as a matter of fact, give values for R not far from half this quantity, viz., m/e, like our own value of $\lambda/2\pi$.

The mean value found by Beck for iron and nickel is $1.10 \times m/e$; that found by Chattock and Bates, for the same metals, is $1.00 \times m/e$, which they consider correct within less than 1%. The discrepancies between the individual observations from which their final means are derived, however, are so large that we do not feel confident their error is so small. For iron, nickel, and Heusler's alloy, Sucksmith and Bates have found $1.00 \times m/e$, with an apparent error of about 1%.

The mean value of $\lambda/2\pi$ obtained from our rotation experiments, as given in Table XIV, is $1.06 \times m/e$. We do not see how the error can be greater than 2%.

62. **Acknowledgments.** In the course of this work we have received a great deal of assistance from others, many of whom are mentioned in what precedes. We are particularly indebted to Mr. C. A. Kotterman, now of the Sloane Physical Laboratory, who assisted us in many ways in all but the first and last parts of the investigation; to Mr. G. H. Jung, now of the University of California, whose faithfulness and great skill made the degree of mechanical perfection achieved in our latest work possible; to Mr. C. C. Ennis and Mr.

---

[33] *Ann. d. Phys.*, **60**, 109, 1919; see also G. Arvidsson, *Phys. Zeit.*, **21**, 88, 1920.
[34] *Roy. Soc. Phil. Trans. A*, **223**, 257, 1923.
[35] *Roy. Soc. Proc. A*, **104**, 499, 1923.

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THE value for the atomic weight of germanium, which we have recently found by analysis of germanium tetrachloride, 72.60[1], is 0.1 unit higher than the one in current use, and nearly 0.2 unit higher than the value recently found by Müller.[2] The simplest way to secure further light on the subject seemed to us to prepare and analyze the tetrabromide; for while similar constant errors might result from inadequate purification and in the analysis of such similar compounds as the chloride and the bromide, yet it seemed unlikely that these errors would be of equal magnitude in both cases. We believe that the concordance of the result of the present investigation with that of the previous one furnishes very strong if not irrefutable evidence that the atomic weight of germanium has the value 72.60.

In its general outlines the experimental procedure of this research followed very closely the methods perfected in this laboratory for the preparation and analysis of other volatile mineral halides.[3]

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[1] Baxter and Cooper, These Proceedings, **59**, 235, 1924; Jour. Phys. Chem., **28**, 1049, 1924.

[2] Müller, Jour. Amer. Chem. Soc., **43**, 1085, 1921.

[3] Baxter, Weatherill and Scripture, These Proceedings, **58**, 245, 1923; Baxter and Fertig, Jour. Amer. Chem. Soc., **40**, 1228, 1923; Baxter and Scott, These Proceedings, **59**, 21, 1923; Baxter, and Cooper, Reference 1.

PURIFICATION OF MATERIALS.

Reagents.

Water. Ordinary distilled water was twice redistilled, once from alkaline permanganate, and once from very dilute sulphuric acid. The condensers were block tin tubes, fitted to pyrex flasks with constricted necks which served as stills. The connection was made by a water seal, no cork or rubber being used. The water was collected in pyrex flasks, generally just previous to use.

Nitric Acid. Concentrated C. P. nitric acid was distilled through a quartz condenser, the first two-thirds, as well as the last tenth being rejected. If necessary, the fractional distillation was continued until the product was free from chloride.

Silver. This substance was prepared by standard methods. These consisted in brief of the following processes: double precipitation as chloride, followed by reduction with alkaline sugar solution, fusion of the metal on charcoal, solution in nitric acid and repeated crystallization of silver nitrate, precipitation with ammonium formate, fusion on pure lime, electrolytic transport, fusion on pure lime in hydrogen, etching, drying in a vacuum at 500°. Details of these processes may be found in earlier papers from the Harvard Chemical Laboratories.[4] Recent work by Baxter and Parsons [5] and Baxter [6] as well as earlier investigations have shown that these processes yield a product of adequate purity.

Two different specimens were used. The first had been prepared by Mr. M. J. Dorcas and was employed in Analyses 1 to 11. The second was purified especially for this investigation and served in Analyses 12 to 16.

Sodium Hydroxide. Pure material was prepared by metathesis from solutions of barium hydroxide and sodium carbonate, both of which had been recrystallized until practically free from chloride. The solution was freed from precipitate by centrifugal settling, and after evaporation in platinum dishes was allowed to crystallize. Since the product was found to contain a trace of chloride it was recrystallized several times in platinum dishes until the chloride had been completely removed.

Nitrogen.— This gas was prepared free from hydrogen by a modification of the Wanklyn process. Air was charged with ammonia and passed over hot copper catalyst. The excess of ammonia was removed by wash bottles containing dilute sulphuric acid. Hydrogen resulting from catalytic decomposition of the ammonia was next burned with hot copper oxide, and the gas was then purified and dried by dilute silver nitrate solution, sodium hydroxide, concentrated sulphuric acid and resublimed phosphorus pentoxide. Finally last traces of oxygen were removed by hot copper. The apparatus, which was constructed entirely of glass, is described in detail by Baxter and Grover.[7]

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[4] See especially Richards and Wells, Pub. Car. Inst. No. 28, p. 16, 1905.
[5] Baxter and Parsons, Jour. Amer. Chem. Soc., **44**, 577, 1922.
[6] Baxter, ibid., **44**, 591, 1922.

Bromine. The source of bromine was a very concentrated solution of zinc bromide which had been made from pure zinc and pure bromine some years before by Dr. J. H. Hodges in an investigation on the atomic weight of zinc.[8] This bromine had been first distilled from aqueous potassium bromide in order to remove chlorine. Next one-fourth of the product was converted to potassium bromide by means of recrystallized potassium oxalate, and the remaining three-fourths of the bromine was distilled from solution in this potassium bromide. All the product was then converted to potassium bromide by means of potassium oxalate, and the potassium bromide was fused in a platinum dish with enough recrystallized potassium permanganate to oxidize all organic matter. In order to obtain bromine the purified potassium bromide was dissolved in a solution of pure sulphuric acid. The excess of potassium permanganate contained by the salt caused a small quantity of bromine to be liberated. This bromine was removed by distillation, since it might have contained a trace of iodine. Enough pure permanganate to liberate the greater portion of the bromine was then added, and the bromine was distilled into a receiver cooled with ice. In this last step the bromine received a third distillation from a bromide. Pure zinc was treated under water with this bromine to prepare the zinc bromide solution.

To recover the bromine, portions of the zinc bromide solution were treated with recrystallized potassium permanganate and dilute sulphuric acid in insufficient amounts to liberate all the halogen, and the bromine was separated by distillation. This fourth and last distillation from a bromide was considered necessary only because the solution had been standing for a long period. After separation from the aqueous layer the pure bromine was redistilled and dried for some days over resublimed phosphorus pentoxide. In all the above operations cork and rubber were rigidly excluded from the apparatus.

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[7] Baxter and Grover, Jour. Amer. Chem. Soc., **37**, 1037, 1915.
[8] Baxter and Hodges, Jour. Amer. Chem. Soc., **42**, 1242, 1921.

Recovery and Purification of Germanium.

At the beginning of this investigation the greater proportion of the germanium material was in the form of germanic acid dissolved in a nitric acid solution of sodium nitrate and sometimes silver nitrate resulting from the earlier analyses of germanium tetrachloride. This material was arsenic-free so far as we knew. A smaller proportion consisted of fractions rejected during the distillation of the germanium tetrachloride because of their arsenic content. Most of these had been hydrolyzed and precipitated with hydrogen sulphide.

The germanic acid solutions were evaporated until a large portion of the germanic acid had separated. The flasks in which the evaporation took place were then connected with a condenser delivering into a receiver containing water chilled with ice. By means of a separatory funnel sufficient concentrated hydrochloric acid was admitted to the flask to dissolve the germanic acid on boiling, and the solution was distilled until sodium chloride began to separate in the still. At this point the contents of the still were tested with hydrogen sulphide and found to be free from germanium. The germanium tetrachloride and hydrochloric acid solution contained in the receiver was too strongly acid to permit hydrolysis and remained perfectly clear. From this distillate germanium hydroxide was precipitated by adding an excess of ammonia. The precipitate was washed by decantation until its colloidal tendency became so great as to prevent settling, and then was collected on a filter paper and dried at 110°.

The ammoniacal filtrate and washings were evaporated to small bulk and distilled with hydrochloric acid as described above. If the germanium tetrachloride collecting in the receiver hydrolyzed at any time, the resulting precipitate of dioxide was set aside and the combined precipitates obtained in this way were washed, collected on a filter and dried at 110°.

The residues containing arsenic were freed from the latter element by distillation from a solution in which the arsenic was in the higher state of oxidation, according to the procedure of Dennis and Papish.[9] Most of this material was in the form of sulphides. These were washed, collected and dried, and roasted at about 500° to eliminate the greater part of the arsenic. The product was treated with an excess of sodium hydroxide, and after saturation with chlorine, the solution was distilled with an excess of hydrochloric acid. The germanium dioxide forming in the receiver was treated as above and the material remaining in the solution was precipitated with ammonium hydroxide.

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[9] Dennis and Papish, Jour. Amer. Chem. Soc., **43**, 2131, 1921.

In order to avoid loss of germanium during reduction through the presence of chloride in the various precipitates the dioxide was next ignited in a current of air saturated with moisture at room temperature. The material was contained in alundum boats placed in a quartz tube. After the tube had cooled, the air was replaced with pure dry electrolytic hydrogen and the tube was again heated to incipient redness until reduction was nearly complete. Then the temperature was raised to the melting point of germanium for a short time. At the end of this preliminary reduction the comparatively small globules of metal were fused into large globules by filling the alundum boats and heating them in a current of hydrogen until the germanium had melted. The final product weighed 48 grams. There seemed to be good reason to believe that this germanium was free from impurities, especially arsenic, because samples of the dioxide precipitated by ammonia and by direct hydrolysis of the distilled chloride gave no indication of arsenic lines when their arc spectra on graphite electrodes were photographed with a Féry quartz spectrograph.

Preparation and Distillation of Germanium Tetrabromide.

Germanium tetrabromide was prepared by heating the metal in a current of nitrogen charged with bromine. Pure dry nitrogen, prepared as described on page 220, was passed through pure dry bromine (see page 221) in a bubbling flask, and then through a hard glass tube containing the germanium in alundum boats. The tube was inclined downward and connected at its lower end by means of a ground joint with a condensing bulb, followed by a U-tube chilled with ice. Since the boiling point of the tetrabromide is 185.9°[10] it was unnecessary to chill the condensing bulb. Fused or well-ground glass connections without any lubricant were used throughout the system.

After the nitrogen train and the germanium tube had been swept out with nitrogen for some time the bromine was introduced in the bubbling flask through a side tube, and the germanium was heated to about 250°, since as already stated by Dennis and Hance,[11] rapid combination between the metal does not take place below 220°. The bromine flask and connecting tubes were warmed to increase the bromine concentration until the metal glowed brightly from the heat of reaction. The tetrabromide condensed in the cool portion of the hard glass tube beyond the metal and flowed down into the condensing bulb. This transfer was assisted by occasionally warming the tube. As long as a considerable amount of unchanged metal remained in the boats very little bromine passed into the distillate. The process was continued until the metal had nearly disappeared and unchanged bromine was passing through the system. Then the condensing bulb was sealed off from the reaction tube. About sixty-five cubic centimeters of tetrabromide were obtained. Besides free bromine the product contained hydrogen bromide and either germanic acid or oxybromide owing to some contact with moisture. Arsenic, tin, antimony, and silicon also were to be feared. In the following table are given the boiling points of some probable impurities, together with the effect on the observed atomic weight of germanium of a proportion of 0.1 per cent of each impurity alone.

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[10] Dennis and Hance, Jour. Amer. Chem. Soc., **44**, 301, 1922.
[11] Reference 10, page 300.

TABLE I.

BOILING POINTS OF INORGANIC BROMIDES.

	Boiling point degrees	Effect of 0.1% on atomic weight of germanium
HBr	− 69.0	−0.087
Br_2	58.6	−0.094
$SiBr_4$	153.0	−0.055
$GeBr_4$	185.9	...
CBr_4	190.0	−0.076
$SnBr_4$	201.0	+0.038
$AsBr_3$	220.0	+0.020
$SbBr_3$	280.0	+0.070

The germanium tetrabromide was now purified by fractional distillation in a fashion very similar to that used in purifying other inorganic halides in this laboratory.[1, 3]

Dennis and Hance[12] have found that, if excess of bromine is removed from the tetrabromide by means of mercury, mercuric bromide vaporizes to some extent with the tetrabromide when the latter is distilled, but that the bromine may easily be eliminated by fractional distillation. We therefore avoided the use of mercury.

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[12] Reference 8, page 300.

Since our experience with the tetrachloride led us to expect that other impurities could be removed by fractional distillation, we depended entirely on this process henceforth.

The bulb containing the tetrabromide was sealed to two U-tubes and the second U-tube was connected through a tube containing solid sodium hydroxide to an efficient water pump. While immersed in a bath of concentrated sulphuric acid at about 120° the bulb was exhausted and the tetrabromide was allowed to boil gently for some time. In this way the air in the bulb was flushed out and at the same time a considerable proportion of the free bromine and other more volatile impurities were expelled from the tetrabromide. In the first U-tube, which was chilled with ice, a fraction of tetrabromide of about two cubic centimeters condensed. The glass tube connecting the bulb to the first U-tube was sealed off at a constriction while the tetrabromide was boiling.

In order to remove permanent gases still more completely the bulb A (Fig. 1) was attached through a special valve *a* with which it had been originally provided to three half-liter globes 1, 1, 1, followed by a second special valve *b*. The globes were exhausted to a pressure of 0.001 mm. by means of a Gaede mercury pump through a side tube *d* which was sealed while the pump was operating. Then the bulb of tetrabromide was surrounded with boiling water and connection with the exhausted globes was made by opening the special valve *a* nearest the bulb. Bromine together with a small amount of tetrabromide immediately evaporated into the globes 1, 1, 1, again flushing out the gases in the bulb. The globes were sealed off at *c* not long after the valve was opened. Another bulb *B* attached through a vertical tube to a fractionating column *f* filled with glass pearls was next sealed to the tetrabromide bulb *A*. Beyond the fractionating column were condensing bulbs 2, 28, and a special valve *h*.

The details of a special valve are shown in Figure 1. The sealed-in capillary *o* is closed at the end and is scratched with a file at several points to facilitate breaking. To open the valve the closed tube of glass weighted with mercury *n* is allowed to strike the capillary with some force.[13]

After the bulb *B* and the column *f* had been exhausted through *g*, the valve *b* was opened. Even when the tetrabromide was surrounded with boiling water, distillation did not at first take place, apparently owing to choking with bromine vapor. But when one of the small bulbs, 2, was temporarily chilled with liquid air, so that the bromine was largely removed from the system, distillation of the tetrabromide proceeded readily. After all but a few cubic centimeters of tetrabromide had been collected in *B*, the original bulb *A* was disconnected by sealing the capillary *e*.

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[13] This valve was apparently devised by Bruner and Bekier [Zeit. Elektrochem., **18**, 369, 1912] and has been found very useful in this laboratory. References 1, 3.

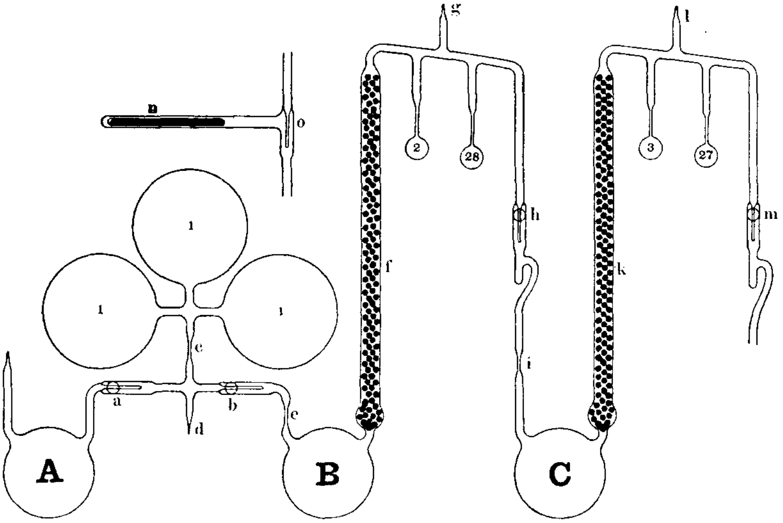

FIG. 1.

Bulb *B* was now attached to a third bulb *C* with fractionating column and these were exhausted through *l*. Bulb *B* containing the tetrabromide was then surrounded with boiling water, while the small bulb 2 was chilled with liquid air. Refluxing in the column *f* took place vigorously while a liquid consisting chiefly of bromine condensed in the small bulb. By continually agitating the system the greater part of the bromine content of the tetrabromide was eliminated and collected in 2, which was sealed off at the capillary while still chilled. The main bulk of tetrabromide was then transferred from *B* to *C* by opening the valve *h* and warming *B*. The Hempel column operated very efficiently with negligible choking when merely air-cooled. When the residue in *B* had been reduced to about two cubic centimeters the capillary *i* was sealed.

Eleven more fractional distillations were now carried out with apparatus of the general type indicated in the right-hand side of Figure 1. In each distillation one or two least volatile fractions were removed from the tetrabromide by condensation in small bulbs before connecting each exhausted system with the succeeding one, and when all but about two to three cubic centimeters of bromide had been removed during a fractionation, the still was sealed off from the receiver and the residue condensed in a small bulb. The complete outline of the distillation is shown in Figure 2. In this diagram each fractionation is indicated by a lettered circle, while the fractions removed are shown by small circles numbered in the order of decreasing volatility; that is, in distillation *G* the most volatile fraction is 8 and the least volatile 23. The atomic weight of germanium found from each sample is also shown in Figure 2, the upper number in each case being that found by comparing the bromide with silver, the lower by weighing the silver bromide formed. Fractions 9, 15, 17 and 22 either were lost by the cracking of the bulbs or were preserved for future examination.

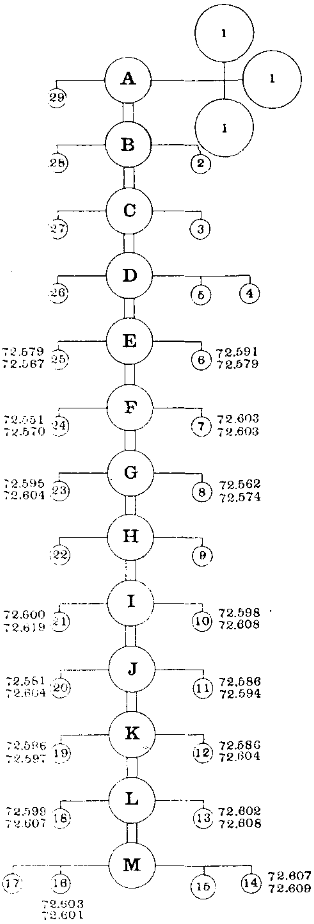
1
1
1
A
29
B
28
2
C
27
3
D
26
5
4
E
72.579
72.567
25
6
72.591
72.579
F
72.551
72.570
24
7
72.603
72.603
G
72.595
72.604
23
8
72.562
72.574
H
22
9
I
72.600
72.619
21
10
72.598
72.608
J
72.561
72.604
20
11
72.586
72.594
K
72.596
72.597
19
12
72.586
72.604
L
72.599
72.607
18
13
72.602
72.608
M
17
16
72.603
72.601
15
14
72.607
72.609

FIG. 2.

PURITY OF GERMANIUM TETRABROMIDE.

As stated above, the removal of the bromine in the less volatile fractions was very rapid. Fractions 2 and 3 consisted very largely of bromine, Fraction 4 contained a small proportion, but Fraction 5 was essentially colorless. So far as bromine and hydrogen bromide were concerned we therefore felt justified in using subsequent fractions for analysis.

Since silicon tetrabromide is more volatile than the germanium compound the more volatile fraction, 3, weighing 5.0 grams, was analyzed for silicon. The contents of the bulb were added to pure water in a platinum crucible and the crucible was filled with freshly distilled constant-boiling hydrochloric acid. After evaporation of the solution nearly to dryness over an electric hot plate the crucible was three times more filled with hydrochloric acid and the solution evaporated. In this way the germanium was expelled while the silicon compound must have been hydrolyzed to silicic acid. The crucible was then ignited and weighed by substitution. In order to expel any silica the crucible was next filled with hydrofluoric acid which had been freshly distilled through a platinum condenser, and after evaporation of this acid and ignition, the crucible was reweighed. A loss in weight of only 0.01 mg. was observed and this result was not altered by a second evaporation with hydrofluoric acid. This seemed to be conclusive evidence that the germanium tetrabromide was never contaminated with a perceptible amount of silicon tetrabromide.

The residues from the first four fractionations, fractions 26 to 29, were hydrolyzed with ammonia and the solutions evaporated to dryness. When the arc spectra of these products were produced on graphite electrodes and photographed with a Féry quartz spectrograph, arsenic lines could not be discovered in any of the spectra, even in that of fraction 29. Fractions 26 and 27 seemed to contain no metals except germanium. Fractions 28 and 29, however, showed the merest traces of tin and antimony. The least volatile fraction of the fifth distillation was therefore believed to be pure enough for analysis.

ANALYSIS OF GERMANIUM TETRABROMIDE.

The analysis of germanium tetrabromide followed closely the procedure used in the analysis of the tetrachloride. After being weighed the bulb containing germanium tetrabromide was broken under an excess of sodium hydroxide and the glass was collected and weighed. The solution was diluted to considerable volume and made acid with nitric acid, and then was precipitated with a solution of a weighed, very nearly equivalent amount of pure silver. The point of exact equivalence between bromide and silver was then found with the assistance of a nephelometer. In all the experiments the silver bromide was collected and weighed.

The bulb containing the material for analysis was first scrubbed with soap solution and then was allowed to stand for some time in sulphuric acid-dichromate solution. After thorough rinsing it was soaked for at least twenty-four hours in the purest water. From this point the bulb was not touched with the fingers. Next the bulb was placed in a basket of platinum wire which had already been weighed under water with an accuracy of one milligram, and the bulb and basket together were weighed under water of known temperature. Rinsing with pure water and draining were followed by drying, first over sulphuric acid, finally over potassium hydroxide which had been fused with permanganate to destroy organic matter. The bulb was then weighed by substitution and the density of the air in the balance case at the time of weighing was found by weighing a sealed standardized glass globe.[14]

A 4 normal solution of pure sodium hydroxide containing a one hundred per cent excess over that necessary to form sodium bromide and sodium germanate was filtered through a platinum sponge crucible into a heavy walled, two-liter Erlenmeyer pyrex flask provided with a carefully ground glass stopper. The weight of glass in the bulb containing the tetrabromide was estimated in computing the amount of sodium hydroxide necessary. The weighed bulb was carefully introduced, and, before the stopper was inserted, the flask was warmed gently so that ultimately the pressure should be slightly inward. After cooling the flask with tap water the inner walls of the flask were wetted with sodium hydroxide solution by inclining the flask and rotating, and then the bulb was broken by shaking the flask. A dense fog immediately appeared in the flask. Again the flask was cooled and although the fog disappeared in the course of fifteen minutes the flask was allowed to stand for about three hours longer. The resulting solution was perfectly clear except for glass fragments.

The fragments of the glass bulb were collected on a weighed, platinum sponge crucible. The solution was filtered directly into the glass-stoppered Erlenmeyer precipitating flask, and the glass was washed by decantation ten or twelve times with about one hundred cubic centimeters of boiling hot water and once with cold water before being transferred to the crucible. Drying at 300° for eighteen hours in an electrically heated porcelain air bath preceded the weighing of the crucible both empty and with the glass. In order to make sure that no germanium hydroxide was retained by the crucible and glass, the two were further treated with hot 4 normal nitric acid by slowly pouring from 400 to 800 cc. through the crucible. This treatment usually produced a slight loss in weight of a few hundredths of a milligram, but a second similar treatment produced no further change of appreciable magnitude.

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[14] Baxter, Jour. Amer. Chem. Soc., **43**, 1317, 1921.

In similar experiments with the tetrachloride[15] it was found that during the treatment of the glass with alkali, solution took place at the rate of 0.007 mg. per gram per hour of treatment with alkaline germanate solution, combined with the nitric acid treatment. A positive correction of this magnitude was therefore applied to the weight of glass obtained in each analysis. This correction did not exceed 0.03 mg. in any experiment.

From the observations with the bulb before breaking and the weight of glass corrected as above and for the buoyant effect of the air, the weight of germanium tetrabromide was found. The weight of silver necessary to precipitate the bromide was computed, and was weighed out, chiefly in the form of a very few large buttons, the final adjustment being made with small electrolytic crystals. After careful solution of the silver in halogen-free nitric acid and elimination of nitrous acid by heating the solution, in a flask provided with a spray trap in the form of a column of bulbs ground into the neck of the flask, twenty-five cubic centimeters additional of concentrated nitric acid were added, together with sufficient water to make the solution nearly tenth normal in silver. The acid solution was then added slowly with continual agitation to the germanium solution which had been diluted to less than tenth normal concentration during the washing of the glass and also had been made acid by adding an excess of twenty-five cubic centimeters of nitric acid through a funnel tube extending to the bottom of the solution. The mixture was then allowed to stand for several days with occasional shaking, and the solution was tested for excess of bromide or silver in a nephelometer. If an excess of either was found, the deficiency of the other was made up by adding hundredth normal silver nitrate or potassium bromide until the endpoint had been reached. Since at the endpoint the test portions appeared almost free from precipitate, there seems good reason to believe the bromine to have been thoroughly freed from chlorine.

Occasional testing of the solution was continued for a period of at least one month in every analysis. The manipulations of precipitation and testing of the solutions were always carried out in ruby light. In using the nephelometer all the precautions noted by Richards and

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[15] Reference 1, page 246.

Wells[16] were observed, such as preparing the comparison tubes under as nearly as possible identical conditions of temperature, concentration and time.

In every analysis the silver bromide was collected on a platinum sponge crucible and weighed. First the solubility of the silver chloride was reduced by adding eight milligrams excess of silver nitrate, and after being shaken vigorously the system was allowed to stand overnight. Filtration of the supernatant liquid through a large weighed platinum sponge crucible was followed by thorough washing of the precipitate by decantation with a solution containing seven milligrams of silver nitrate and five cubic centimeters of nitric acid per liter. In Analyses 29 and 30 no nitric acid was added to the washing liquid and in Analysis 30 forty milligrams of silver nitrate were added to the original solution to lower the solubility of the silver bromide. The precipitate was transferred to the crucible with pure water, and after being dried in an electrically heated porcelain oven for about eighteen hours at 160° it was weighed. Moisture retained by the precipitate was determined by fusing the main bulk and finding the loss in weight.

No correction is applied for silver bromide dissolved in the mother liquor and silver nitrate washings, but the solubility in the pure water used for transfer was assumed to be 0.1 mg. per liter. The precipitating flask was rinsed with ammonia and after the resulting solution had been diluted to 100 cc., its bromide content was found by comparison with standard bromide solutions in a nephelometer after adding nitric acid and silver nitrate.

In the tables the original weight of silver bromide has been corrected for bromide introduced in the comparison of the germanium bromide with silver. The results of all the analyses undertaken are given.

Weighings were made on a No. 10 Troemner balance, sensitive to 0.02 mg. with a load of fifty grams. A 5 milligram rider was used to determine quantities less than this and more than 0.05 mg. Interpolation from zero points was employed only for amounts less than 0.05 mg.

The weights were of gold-plated brass, except the fractional weights which were of platinum, and were compared by the Richards substitution method.[17]

All weighings were by substitution. In the case of the bulb and silver, the weights were substituted for the object weighed. In the case of the glass and the silver bromide the crucibles were substituted for similar counterpoises. A small quantity of impure radium bromide was kept in the balance case to prevent electrostatic effects.

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[16] Richard and Wells, Amer. Chem. Jour., **31**, 235, 1904; **35**, 510, 1906.
[17] Richards, Jour. Amer. Chem. Soc., **22**, 144, 1900.

Vacuum corrections were applied as follows:

TABLE II.

VACUUM CORRECTIONS.

	Density	Vacuum correction per gram
Weights	8.3	. . .
Air at 0° and 760 mm.	0.001293	. . .
Glass	2.5	+0.00033
Silver	10.49	−0.000031
Silver bromide	6.47	+0.000041

RESULTS.

TABLE III.

ATOMIC WEIGHT OF GERMANIUM.

Series I — $GeBr_4 : 4 Ag$

Ag = 107.880 Br = 79.916

Number of analysis	Fraction of $GeBr_4$	Weight of $GeBr_4$ in vacuum gram	Weight of Ag in vacuum gram	Weight of Ag added or subtracted in solution gram	Corrected weight of Ag in vacuum gram	Ratio $GeBr_4 : 4 Ag$	Atomic Weight of Germanium
1	6	6.39771	7.03767	+0.00045	7.03812	0.909008	72.591
2	25	6.79020	7.46994	+0.00020	7.47014	0.908979	72.579
3	7	10.89713	11.98727	+0.00030	11.98757	0.909036	72.603
4	24	7.72394	8.49697	+0.00100	8.49797	0.908916	72.551
5	8	9.90366	10.89499	+0.00085	10.89584	0.908940	72.562
6	23	11.17556	12.29401	+0.00010	12.29411	0.909017	72.595
7	10	9.59408	10.55436	−0.00010	10.55426	0.909024	72.598
8	21	10.69340	11.76371	−0.00015	11.76356	0.909028	72.600
9	11	10.71709	11.78974	+0.00030	11.79004	0.908995	72.586
10	20	11.79501	12.97529	+0.00075	12.97604	0.908984	72.581
11	12	10.50295	11.55424	+0.00020	11.55444	0.908997	72.586
12	19	11.94787	13.14344	+0.00025	13.14369	0.909019	72.596
13	13	10.11814	11.13101	−0.00035	11.13066	0.909033	72.602
14	18	8.38344	9.22244	0.00000	9.22244	0.909026	72.599
15	14	9.93784	10.93224	−0.00005	10.93219	0.909044	72.607
16	16	9.28357	10.21251	+0.00005	10.21256	0.909035	72.603
					Average	0.909005	72.590
				Average, omitting Analyses 4 and 5		0.909016	72.595

TABLE IV.

THE ATOMIC WEIGHT OF GERMANIUM.

Series II — $GeBr_4 : 4\,AgBr$

Number of analysis	Fraction of $GeBr_4$	Weight of $GeBr_4$ in vacuum	Weight of AgBr in vacuum	Loss on fusion	Dissolved AgBr	Corrected weight of AgBr in vacuum	Ratio $4AgBr : GeBr_4$	Atomic Weight of Germanium
		gram	gram	gram	gram	gram		
17	6	6.39771	12.25240	0.00030	0.00012	12.25222	0.522167	72.579
18	25	6.79020	13.00453	0.00033	0.00011	13.00431	0.522150	72.567
19	7	10.89713	20.86771	0.00027	0.00038	20.86782	0.522198	72.603
20	24	7.72394	14.79273	0.00039	0.00010	14.79244	0.522155	72.570
21	8	9.90366	18.96701	0.00044	0.00014	18.96671	0.522160	72.574
22	23	11.17556	21.40114	0.00048	0.00026	21.40092	0.522200	72.604
23	10	9.59408	18.37240	0.00037	0.00021	18.37224	0.522205	72.608
24	21	10.69340	20.47707	0.00046	0.00020	20.47681	0.522220	72.619
25	11	10.71709	20.52364	0.00038	0.00025	20.52351	0.522186	72.594
26	20	11.79501	22.58742	0.00045	0.00020	22.58717	0.522200	72.604
27	12	10.50295	20.11287	0.00042	0.00046	20.11291	0.522199	72.604
28	19	11.94787	22.88061	0.00047	0.00017	22.88031	0.522190	72.597
29	13	10.11814	19.37595	0.00037	0.00022	19.37580	0.522205	72.608
30	18	8.38344	16.05424	0.00041	0.00017	16.05400	0.522203	72.607
31	14	9.93784	19.03070	0.00034	0.00014	19.03050	0.522206	72.609
32	16	9.28357	17.77797	0.00029	0.00029	17.77797	0.522195	72.601
						Average	0.522190	72.597
				Average, omitting Analyses 20 and 21			0.522195	72.601
						Average of Series I and II		72.598

DISCUSSION OF RESULTS.

In the following table the results are arranged in the order of decreasing volatility of the fractions analyzed.

TABLE V.

THE ATOMIC WEIGHT OF GERMANIUM.

Fraction	Atomic weight $GeBr_4 : 4\,Ag$	Atomic weight $GeBr_4 : 4\,AbBr$	Average
6	72.591	72.579	72.585
7	72.603	72.603	72.603
8	72.562	72.574	72.568
10	72.598	72.608	72.603
11	72.586	72.594	72.590
12	72.586	72.604	72.595
13	72.602	72.608	72.605
14	72.607	72.609	72.608
16	72.603	72.601	72.602
18	72.599	72.607	72.603
19	72.596	72.597	72.597
20	72.581	72.604	72.593
21	72.600	72.619	72.610
23	72.595	72.604	72.600
24	72.551	72.570	72.561
25	72.579	72.567	72.573

Since the more probable low boiling impurities, bromine, hydrogen bromide and silicon tetrabromide, if present, would lower the observed atomic weight, while the possible high boiling impurities, the bromides of tin, arsenic and antimony, would raise the observed atomic weight, it is natural to look for such a trend with changing volatility of material. Actually no such trend can be discovered. The variations seem to be fortuitous, except possibly in the case of the two least volatile fractions 24 and 25. But these two fractions give low values for germanium instead of high values. While these two results may have been affected by some unknown low boiling impurity, the fact that other scattered results in the series are as low as these makes it unnecessary to assume such an explanation. Moreover, the visual evidence as to the absence of bromine and the negative test for silicon together with the spectroscopic evidence is in favor of uniform purity of all the fractions.

However, if any difference in purity exists, the middle fractions must have been the purer. Elimination of the three lightest fractions, 6, 7, 8, and the three heaviest, 23, 24, 25, raises slightly the averages of the two series to 72.596 and 72.605, with the combined average 72.601. These six fractions happen also to be the first six analyzed, and it is possible that lesser familiarity with the process may have been in part responsible for the fact that all the experiments which yielded low results are included among them.

After Analyses 11 and 27 had been completed the flask which was used at first for breaking the tetrabromide bulb was changed for a new one, since the first had been somewhat scarred in earlier work. We have no reason to believe that this change was of importance.

Furthermore a different sample of silver was used in Analyses 12–16 and 28–32 from that employed in the others; but since both specimens had been prepared by identical processes, here again there is little likelihood that the results were affected by the change. In fact the ratios of silver used to silver bromide obtained in the two parts of each analysis support this view. These ratios are given in the following table. In Analyses 6, 7, 8, 23 and 25 the silver bromide was tested for germanium by photographing the arc spectra on graphite electrodes in a Féry quartz spectrograph. No evidence of even traces of germanium could be found.

TABLE VI.

Ag : AgBr

Analyses	Fraction of $GeBr_4$	Ratio Ag : AgBr
1 and 17	6	0.574436
2 and 18	25	0.574436
3 and 19	7	0.574452
4 and 20	24	0.574481
5 and 21	8	0.574472
6 and 22	23	0.574466
7 and 23	10	0.574468
8 and 24	21	0.574482
9 and 25	11	0.574465
10 and 26	20	0.574487
11 and 27	12	0.574479
12 and 28	19	0.574454
13 and 29	13	0.574462
14 and 30	18	0.574464
15 and 31	14	0.574456
16 and 32	16	0.574450
	Average	0.574463
Average from Analyses 1–11		0.574466
Average from Analyses 12–16		0.574457

The average of the first eleven experiments differs from that of the last five by only 0.0016 per cent. Since the latter is almost identical with the value found by Baxter [18] and by Hönigschmid and Zintl,[19] 0.574453, these five experiments may on the whole be considered the most satisfactory of all. The average atomic weight of germanium found in these last five experiments in these first series is 72.601, and in the second 72.604.

The value 72.60 thus seems to represent fairly the result of our analysis of germanium tetrabromide. This is identical with the value found by analysis of the tetrachloride. As pointed out in the paper on the tetrachloride this result is in accord with the mass spectrograph results of Aston [20] who estimates the proportions of the isotopes of germanium, 70, 72 and 74 to be 2 : 4 : 5. These figures lead to an average atomic weight of 72.55.

SUMMARY.

1. Pure germanium tetrabromide has been prepared by fractional distillation in exhausted vessels.

2. The atomic weight of germanium has been found to be 72.60 by analysis of germanium tetrabromide. This value is identical with that previously found by analysis of the tetrachloride.

We are especially indebted to the Bache Fund of the National Academy of Sciences for generous assistance in providing much of the necessary apparatus and materials, and to the New Jersey Zinc Company for the gift of germaniferous zinc oxide.

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[18] Baxter, These Proceedings, **42**, 201, 1906.

[19] Hönigschmid and Zintl, Ann., **433**, 201, 1923.

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THE LATENT HEAT OF VAPORIZATION OF LIQUID OXYGEN-NITROGEN MIXTURES.

By Leo I. Dana.

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INTRODUCTION.

This research deals with the determination of the latent heat of vaporization at atmospheric pressure of liquid oxygen, nitrogen, and of mixtures of the two in all proportions. The best of the previous determinations for pure oxygen and nitrogen and those reported herewith are listed in Table I.

TABLE I.

Investigator		Latent Heat
	OXYGEN.	
J. S. Shearer[1]	1903	59.5
T. Estreicher[2]	1904	57.8
I. Dewar[3]	1906	51.1
H. Alt[4]	1906	50.92
H. Barschall[5]	1911	51.3
W. H. Keesom[6]	1914	50.8
L. I. Dana	1922	51.01
	NITROGEN.	
J. S. Shearer[1]	1903	49.8
I. Dewar[3]	1906	50.4
H. Alt[4]	1906	47.65
L. I. Dana	1922	47.74

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[1] Phys. Rev., **17**, 469, 1903.

[2] Bull. de l'Acad. des Sc. de Cracovie, p. 183, 1904.

[3] Proc. Roy. Soc., **76** A, 325, 1906.

[4] Ann. der Phys., **19**, 739, 1906.

[5] Zeit. für Elektrochemie, **17**, 345, 1911.

[6] Comm. Phys. Lab. Leiden, No. 137e, 1914.

The general method adopted by the above-mentioned investigators was to pass an electric current through a coil immersed in a Dewar flask filled with the liquid. By measuring the power input, and the amount of liquid evaporated, either from the volume of the gas or the weight of the liquid, the latent heat could be evaluated. Dewar dropped a mass of metal of known specific heat into the liquid and measured the volume of the evaporated gas. In all the cases except those of Alt and Barschall, the heat leaks were large. As Alt pointed out, the evaporation due to conduction down the current leads, while the evaporation proceeded, was obviously not the same as that without power input.

Several attempts[7, 8, 9] have been made to measure the latent heat of mixtures by methods identical with those used for pure substances. None of the results, however, can be said to possess definite meaning. In the first place, no accurate thermodynamic definition of the latent heat was given or followed; it is possible to define the latent heat of a binary mixture in several ways. Secondly, the method of placing a heating coil in a large volume of the liquid mixture and evaporating without stirring cannot yield definite results. The reason for this is that the changes of composition which take place in one portion of the liquid (say near the heating coil) are not communicated to all portions except by slow convection. In such an instance, not only would the existence of a steady state be improbable, but the results would depend on the position of the heating coil.

The calorimetric method adopted in this research is the method of continuous flow. The principal feature of the calorimeter consists of the thermal isolation of a small volume of the liquid by means of a glass bulb, surrounded by a vacuum jacket, communicating with a large volume of the liquid through a capillary tube. See Figures 1 and 2. Heat is supplied electrically through a heating coil in the bulb. Issuing from the top of the latter is a narrow glass tube through which the cold gas from the boiling liquid flows with so high a velocity that the main source of heat leak — conduction down the heating coil leads — is diminished to a negligible proportion. When a binary mixture is being evaporated, the important property of the calorimeter to be noticed — assuming that the proper steady state exists in the bulb — is that the composition of the vapor arising from it is identical with that of the main body of liquid in the outside bath.

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[7] J. S. Shearer, Phys. Rev., **17**, 469, 1904.

[8] Fenner and Richtmyer, Phys. Rev., **20**, 77, 1905.

[9] G. Witt, Archiv für Matematik, Astronomi och Fysik, Upsala, **7**, No. 32, 1, 1911–12.

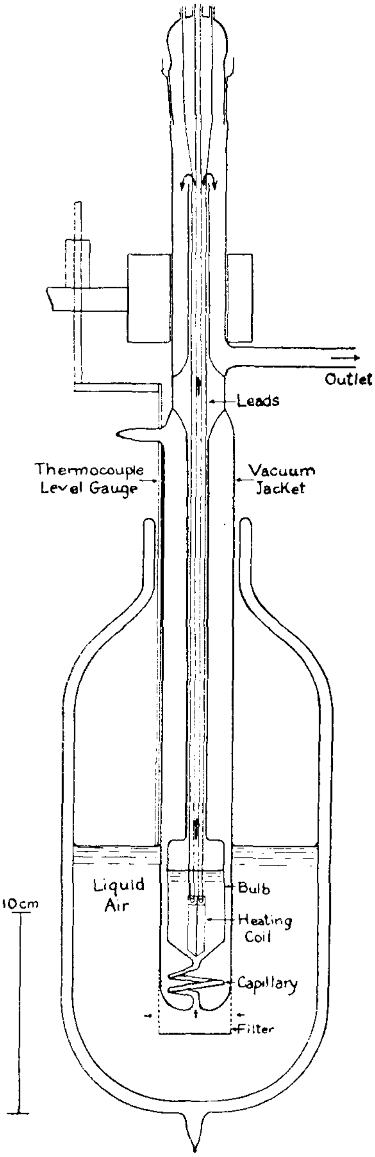
Outlet
Leads
Thermocouple
Level Gauge
Vacuum
Jacket
10cm
Liquid
Air
Bulb
Heating
Coil
Capillary
Filter

Fig. 1.

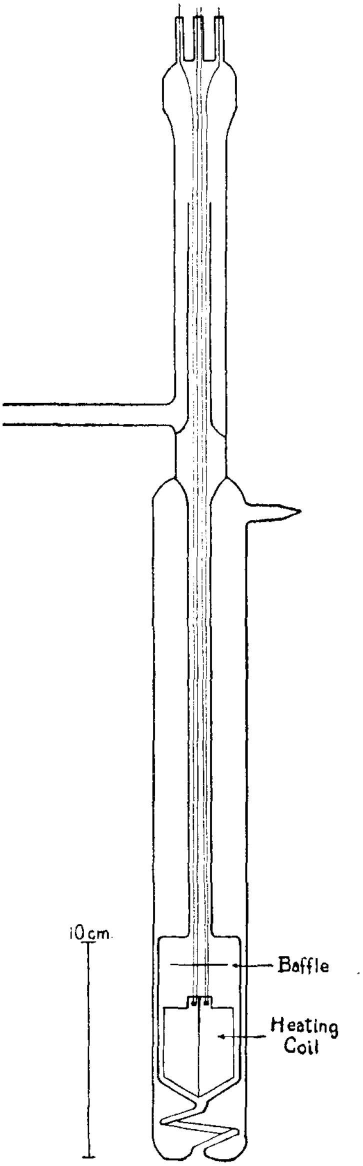
10 cm.
Baffle
Heating
Coil

Fig. 2.

Power is measured by determining the current and voltage with a high precision potentiometer. After the evaporated gas passes through a circuit which saturates it with water vapor and brings it to a definite temperature, its volume is measured by a laboratory gas meter of the wet type which has afforded a fairly high degree of precision. A chronograph records the rate of rotation of the meter drum. As the liquid evaporated, the calorimeter was lowered by a special mechanism, thus maintaining a constant level of liquid in the bulb.

DEFINITIONS OF THE LATENT HEAT.

There is no need to enter into detail concerning the definition of the latent heat of vaporization for a pure substance. For a binary mixture, however, the definition is a far more complicated matter, since in addition to three fundamental variables such as pressure, temperature, and volume, there is also the composition; thus one would expect a number of definitions equal to the number of possible methods of evaporation.

Consider the temperature-composition diagram for mixtures of liquid oxygen and nitrogen at constant pressure, Figure 3. The composition x represents the actual mole-fraction of oxygen which (as far as the perfect gas laws may be considered to hold for the gases under normal conditions) is equal to the per cent of oxygen by volume as determined by ordinary gas analysis. One definition of the latent heat is the heat required to evaporate an infinitesimally small amount from a pool of liquid of composition x_1 at constant temperature and pressure, into the vapor phase x_2 normally in equilibrium with the liquid.[10, 11] Two other definitions result from the method of beginning with a liquid of composition x_1, and evaporating the liquid completely — in the first case, at constant temperature, and in the second, at constant pressure. The latent heat determined in this investigation can be called the latent heat at constant composition, or the vertical instead of the horizontal latent heat. That is (looking at Fig. 3), the change of state is from the liquid state 1 to the vapor state 1′ and the heat per unit mass supplied to the liquid measures the total change in heat-content in passing from the state 1 to 1′. By the heat-content is meant the thermodynamic function $H = E + pv$, where E is the internal energy, p the pressure, and v the specific volume.

---

[10] v. d. Waals-Kohnstamm, Lehrbuch der Thermodynamik, Band II, p. 112.

[11] J. P. Kuenen, Verdampfung und Verflüssigung von Gemischen, p. 38.

How the process of evaporation can be carried out in the calorimeter seems at first sight somewhat obscure, and we will attempt to describe it in more detail. As stated before, the composition of the vapor leaving the bulb is the same as that of the liquid entering the capillary from the outside bath, provided that a steady state has been reached in the bulb and that no liquid or vapor is ejected back into the bath. This fact was verified by experiment: first, by analysis of the liquid and vapor, and second, by weight of the separate components before mixing. This condition must hold if the law of conservation of matter holds, but it seems paradoxical in view of the fact that the vapor boiling off from a binary mixture is, in general, of different composition from the liquid. Nevertheless, if we take the composition of the liquid in the bath as x_1, it is evident (Fig. 3) that the vapor which normally would come off from this liquid would be of composition x_2, but, since the vapor issuing from the bulb is of composition $x_{1'} = x_1$, the liquid in the bulb must be x_3. This is actually the case. As the liquid x_3 in the bulb loses vapor x_1, which is richer in nitrogen than x_3

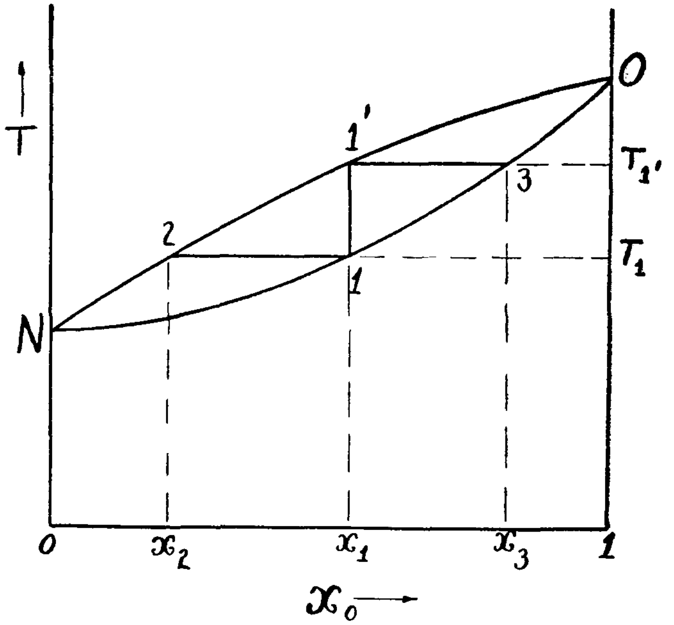

FIG. 3.

liquid x_1, also richer in nitrogen than x_3, enters from the capillary to make up for the loss. The temperature of the liquid in the bulb is always higher than that of the outside liquid by an amount given by the vertical ordinate between the two curves in Figure 3.

For this method to work it would seem to be necessary that the bulb be reasonably small and that good stirring of the liquid take place in order that a steady state should be attained in a short time. In our case, sufficient stirring was produced by the boiling. A test of the correctness of the conditions in the bulb is furnished by observing whether the composition of the vapor arising from it is independent of the rate of evaporation. One of the main difficulties was to produce this result. Obviously, this state of affairs is not sufficient for constancy of the apparent or measured latent heat with the rate of evaporation, for the heat leaks or other disturbances may be different at different rates.

DESIGN OF THE CALORIMETERS.

The principles and factors governing the design of the calorimeters will now be considered. A number of models were constructed of which three were used for final data. In Table II the dimensions in cm. of the important parts are given.

TABLE II.

Calor. no.	Bulb		Capillary	
	Length	Diameter	Length	Inside diameter
III	5.5	3.3	13	0.25
VI	5.0	2.8	20	0.13
VII	7.0	3.5	15	0.13

Pyrex glass was used for the calorimeters. In order to reduce the amount of waste liquid at the end of each run, the bulb was placed as low as possible in the jacket. The inside of the jacketed space was silvered and evacuated to cut down convection and radiation. The use of a narrow tube for conveying the evaporated gas resulted in a high velocity of flow, thus diminishing the heat conduction down the copper leads. The tube was of length adequate to make negligible the conduction down the glass. As shown in Figures 1 and 2, the cold gas passed through a circuitous path at the top of the calorimeter so that its cooling power could be utilized.

As the largest heat leak encountered is the heat conduction down the copper leads the diameter had to be chosen as small as consistent with low development of Joulean heat in the leads. The currents were of the order of one ampere. Number 28 copper wire seemed to be the best compromise. The wires, insulated over their whole length by very fine, long, glass capillaries, were soldered to heavy copper terminals, enclosed by tubes at the head. For potential leads, Number 30 constantan, also insulated with glass capillaries, was chosen. The resistance of these leads had to be taken into account in the potential measurements, and did not vary appreciably with the temperature.

It is to be observed that the measurement of the conduction leaks must be made while the calorimeter is running, for the cooling power of the cold gases is intended to take care of these leaks. A thermocouple, with one junction in the liquid, was installed along the central tube of the calorimeter. One element of the couple was copper, of the same diameter as that of the leads to the heating coil; fine constantan wires were soldered to the copper 10 cm. apart, and the e.m.f.'s were read with the power on. At a power input of 8 watts the junction 10 cm. above the liquid read 6° C higher than the liquid, and at 20 watts it read 2.5° C higher. Now if we assume, at worst, the temperature gradient to be linear from 10 cm. above to the surface of the liquid, the conduction down the glass tube is not more than 1 part in 10,000 of the power input, and that down the copper leads is at most 3 parts in 10,000. The possibility has also to be considered that a part of the Joulean heat developed in the leads can enter the liquid; rough calculations show that it is not more than 2 parts in 10,000. Corrections have been applied for these leaks, although, as far as the accuracy to which the final data is given, they may well have been omitted.

With pure oxygen and nitrogen the temperature in the bulb is the same as that outside in the bath; thus, in these cases, there is no question of radiation to the lower half of the calorimeter. Approximate calculations show that radiation from above is negligible. The results for the latent heats with calorimeter VI were the same when silvered or unsilvered, even with mixtures. But calorimeter VII showed a small effect with mixtures due to radiation to the surroundings, which calculation evinced to be possible as regards order of magnitude; on silvering the effect disappeared. As a control on the losses through the vacuum by radiation, convection, and glass conduction, the leads and heating coil were removed immediately at the close of a run, and the attempt was made to collect any gas evolved, but no appreciable quantity came off.

The same heating coil served for calorimeters III and VI. The leads to the coil were attached to the ground glass stopper head at the top — surrounded by a mercury cup — and the whole combination formed a unit which could readily be removed. In calorimeter VII, however, the heating coil was sealed into the bulb so that the whole calorimeter formed one piece, as shown in Figure 2. The heating coil for calorimeters III and VI consisted of about 55 cm. of number 35 B. and S. gage manganin wire, having a resistance of about 15 ohms. It was strung with glass beads and wound closely on a mica frame — the coil having a final diameter of about 1.2 cm. and a length of 3.0 cm. The glass beads served the double purpose of insulation and the production of quiet boiling. Previous investigators found that the power input with a bare wire was sharply limited. With our coils, however, the beads prevented bumping, and the boiling remained quiet and regular even with powers as high as 20 watts. Although the data obtained with the first coil with various rates of evaporation indicated that very little, if any, superheating of the vapor or spattering of the liquid existed, nevertheless, the fear arose that such difficulties might occur with so concentrated a heating coil. Therefore, the heating coil for calorimeter VII had quite different constants. It was made considerably longer — 125 cm., 16 ohms resistance — and was distributed on a mica frame throughout the volume of a larger bulb. The wire was completely covered with small pyrex glass beads with sharp edges. If there was superheating of the liquid or of the vapor, it should be considerably less with such a coil than with the previous coil. At the same time, the mica baffle plate, which remained fixed above the heating coil, should reduce to some extent the spattering if it existed in appreciable amount. Fortunately the results with both types of calorimeter were identical within the experimental error. As a check against superheating of the liquid, the temperature in the bulb, while evaporating liquid oxygen, was observed to be the same, within 0.01° C, as that of the liquid outside; and, as a check against superheating of the vapor, it was found that with oxygen the same value of the latent heat resulted when the level of the liquid stood at the top of the heating coil as when it stood 1.5 cm. above. In several cases, in which, by accident, the heating coil remained only partly immersed, very much higher values for the latent heat than normally found were recorded.

In determining the proper dimensions of the capillary, we have to take into account a number of considerations. Since with mixtures the temperature in the bulb is higher than in the liquid outside, the dimensions of the capillary must be such that the heat conducted away is negligible. If the internal diameter is too large, the liquid has a tendency to kick back when the evaporation is proceeding. This was apparently the case with calorimeter III, as the composition of the vapor was not constant with the rate of evaporation, and consequently it could not be used for mixtures. To prevent the tube from becoming clogged by small particles of ice it was necessary to use a filter, just slipping over the bottom of the calorimeter. It consisted of a cylinder of fine gauze, closed at the bottom by a thin brass plate. It was not desirable to have the gauze at the bottom because the mesh sometimes allowed bubbles of gas to pass through, or the sharp points of the mesh acted as centers of formation of bubbles which found their way up to the capillary, thus introducing extra gas into the line. Finally the capillary had to be free from sharp kinks so that the flow of liquid would be perfectly regular. The velocities of the liquid through the capillary were rather high; e.g., for calorimeter VI, and for the fastest rate of evaporation (20 watts) the flow amounted to 7 cm. per second.

Liquid Levels.

It is evident that as the liquid in the bulb evaporates its level would fall quite rapidly if it were not kept constant by the influx of new liquid and by lowering the calorimeter. In maintaining the level we have first to consider the proper height of the liquid above the coil, in order that the coil should not burn out, and that the vapor should not become superheated. For safety the liquid was held 1 to 1.5 cm. above the heating coil. It requires a certain pressure head to force the gas through the pipes and meter, to overcome the resistance of the capillary to the flow of fluid, and to balance the difference of densities with mixtures when the liquid in the bulb is richer in oxygen than the liquid outside and consequently denser. The last three factors determine the natural difference in level between the outside and inside while operating. Fortunately the pressure head necessary to force the gas through the meter and tubes was not high — it varied from 0.8 to 1.6 cm. of water — for the liquid from the surface to the bottom of the filter is waste liquid as far as a run at a particular time is concerned. Assuming Poisseuille's law to hold, the pressure head required to force the liquid through the capillary can be calculated. For calorimeter VI at the maximum rate of evaporation it is about 4 mm. The correction for differences in densities depends on the compositions and densities, but the maximum effect is only 1.5 mm.

A copper-constantan thermocouple, connected to a micrometer screw, served as a level gauge. The conduction of heat along the copper was so great that when the junction rested just above the liquid surface a thermocouple millivoltmeter read slightly less than when the junction just touched the liquid. Thus, by the kick of the needle, one could tell when the junction came into contact with the surface, to 0.1 mm. For the pure components the question of the level is not of much importance, but it is vitally so for mixtures.

In the first set-up, the calorimeter was attached to a vertical screw, and whenever the millivoltmeter indicated that the liquid level had fallen, the calorimeter was lowered 0.5 mm. or so by turning the screw. With this procedure, or even with more careful hand regulation, one could not, with mixtures, maintain constant for any length of time the composition of vapor issuing from the bulb. It soon became clear that the changes of composition followed the discontinuous changes in level. Now, the fundamental condition for this experiment is that a steady state exist in the bulb and that no liquid or vapor be ejected back. For a steady state evaporation, the volume of the liquid in the bulb should remain constant, which condition means that the level with respect to the bulb should be kept invariable. What effect a discontinuous change in level can have upon the composition becomes evident by a simple computation. For example, when the outside liquid contains 50% oxygen the liquid in the bulb has the composition 76% oxygen. If we assume that the liquid stands 3 cm. high in the bulb, and that the calorimeter is suddenly lowered 0.1 mm., then the readjustment of the levels to the previous values will result in the injection into the bulb of liquid of composition different from that already present, producing a decrease of its oxygen content by 0.8%, and at the same time of that of the issuing vapor by an approximately equal amount. Hence we see the importance of lowering the calorimeter at a rate equal, as near as possible, to the rate of descent of the outer level.

A specially constructed mechanism, driven by a direct current motor, maintained a constant rate. It consisted of a series of worms and gears which turned a fixed vertical screw, thus lowering a nut to which the calorimeter was attached. To provide for a wide range of speeds, a variable resistance was inserted in series with the motor armature. In operation the rates of descent of the calorimeter varied from 0.2 to 0.5 mm. per minute. From the physical constants of the liquid, and the dimensions of the calorimeter and Dewar flask, the rate of lowering could be calculated approximately for a definite mixture and power input. The rates of rotation of the vertical screw were plotted as a function of the two variables, power and composition, thus affording quickly an idea of the proper rate; and the attainment of the exact speed ensued in a short while by adjusting the rheostat and watching the level gauge. By this method, the total change in level during a whole run did not amount to more than a few tenths of a millimeter, thus giving satisfactory constancy of composition and good results for the latent heats.

Gas Measurement.

Coming from the calorimeter, the gas is, of course, perfectly dry and colder than room temperature, so that in passing through the meter it would produce gradual changes in the level of water and the temperature of the meter. It was, therefore, necessary to pass the gas through a conditioning circuit, which could accommodate a flow of two to five liters of gas a minute, consisting of a saturator, a condenser, and a cooling coil. The saturator consisted of a pyrex glass tube, and was kept at 40° or 50° by a heating coil wound on the outside. Then the gas passed through an all-metal condenser, cooled by tap water; and the flow of water could be regulated to maintain the temperature of the gas in issuing from the condenser about 1° higher than the meter temperature. Finally, the gas entered a coil of copper tubing immersed in water, and the temperature of the water was adjusted to keep the temperature of the gas just entering the meter within 0.1° of that of the meter. Temperatures were measured by differential thermocouples of very fine wire, protected from radiation, with one junction in the meter water and the other in constrictions in the pipes.

The laboratory gas meter of the wet type was not adopted without misgiving, for the feeling prevailed that it could not yield a satisfactory degree of precision. The attempt to use a true "rate of flow" meter instead of a semi-integrating type did not meet with success. Waidner and Mueller[12] have made a thorough study of the laboratory gas meters and assert that, if properly used, they are good to 0.2%. With very careful manipulation of the meter, however, our data for the latent heats show a better agreement, indicating a higher precision in use. In addition to the attention paid to the condition of the gas, the meter was carefully leveled before each run, and the water level set with a magnifying glass. The calibrated meter thermometer could be read to 0.05° C. It is possible for the volume of gas delivered per revolution to depend on the rate of revolution; consequently, the meter must be calibrated as a function of the rate of flow. It is of the utmost importance to perform this calibration so as to prevent this factor from producing an apparent variation of the latent heat with flow.

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[12] Bureau of Standards. Technologic Paper, No. 36, 1914.

For the calibration a large tank of over 100 liters in volume, immersed in a vat of water, was filled with dry air at a pressure of two atmospheres. Then the gas was passed at a constant rate, as indicated by an auxiliary flow meter, through the conditioning apparatus and meter exactly as under the conditions of use. From a tall mercury open manometer connected to the tank and read by a cathetometer, and from the barometer readings, the initial and final pressures were determined. Temperatures of the gas in the tank were given by the temperatures of the vat water, allowing, of course, time to reach equilibrium. During a run the temperature of the water did not change more than several hundredths of a degree. Enough gas was passed through the meter to make exactly 35 revolutions of the drum. A number of determinations at several rates of flow were made during two separate calibrations. Unfortunately in the first case, a tank of rather thin sheet metal was used, which, it is now quite certain, became deformed slightly under pressure, with the result that its volume in use differed from that found subsequently by filling with a known volume of water. In the second calibration a much more rigid tank was used. Both calibrations showed about the same variation with flow, but only the second, determined immediately after the main series of latent heat data, is given here.

TABLE III.

Calibration of Meter.

Time per revolution in seconds	Liters per revolution	Mean
90	2.8142	
	2.8156	
	2.8120	2.8139
65	2.8176	
	2.8163	2.8170
55	2.8196	
	2.8218	
	2.8198	
	2.8177	2.8197
45	2.8212	
	2.8230	
	2.8211	
	2.8167	2.8205

The results of this calibration are plotted on Figure 4. All the latent heats are calculated using the value of the meter constant given on the straight line on this plot.

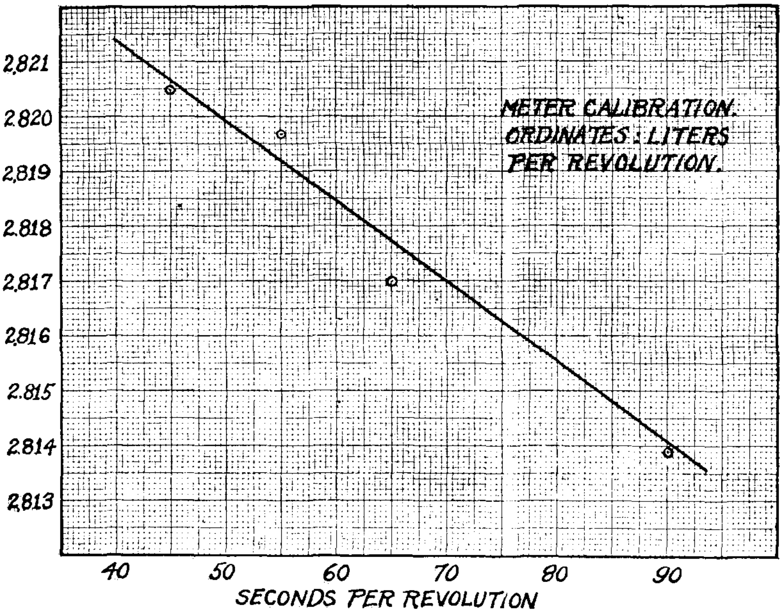

Fig. 4.

Accessory Apparatus.

For the measurement of the current and potential difference at the heating coil a four-dial White double potentiometer, having a range of 0–50 millivolts, was available. The fifth place was read by deflection of the galvanometer which had a sensitivity of 2.5 mm. per microvolt at 1.25 meters. In all of the measurements the conventional precautions of shielding and of the elimination of stray e.m.f.'s were taken. For the current measurement, the drop across a 0.01-ohm Reichsanstalt standard provided a convenient method. The heating coil was shunted by a 10,000-ohm and 10-ohm Bureau of Standards form of standard resistance, which possessed recent certificates, and the potential taps were on the 10-ohm standard. See Figure 5. Correction, of course, had to be made for the current shunted through the potential leads and standard coils. The Weston standard cell for the potentiometer was compared with two other certified Weston cells, and the readings of the potentiometer itself were checked by an auxiliary potentiometer made up of certified B. S. resistances and standard cells. The results of the intercomparison left no doubt that the power was correctly measured in international watts to at least 0.05 %.

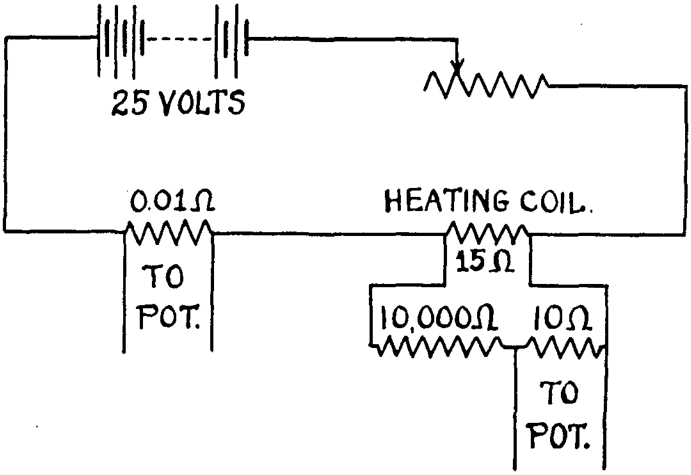

FIG. 5.

The rate of revolution of the meter was determined on a chronograph. By fastening a projecting piece of platinum foil to the zero point of the meter, and also covering the needle with foil — this terminal being grounded to the frame — the time of passing the zero could be communicated to the chronograph. Second beats were furnished by a standard clock, and the position of the meter signal could be easily read to 0.1 second. In practice the total time for a definite number of revolutions, usually in a period of 40 minutes, was ascertained, giving the mean time per revolution.

For the analysis of the mixtures, an electrical method based on the thermal conductivities of gases and developed by the Bureau of Standards,[13] was set up and used as an operating indicator. Actual analyses were made by absorbing the oxygen in a copper ammonium solution in a Hempel pipette.[14] The range was divided into two parts by using two burettes and the analyses were good to 0.1% of oxygen. The method of sampling deserves discussion. It was decided to be best to gather an average sample over a whole run, in order to smooth out possible slight irregularities in the composition resulting from the unsteady motion of the calorimeter. At the outlet of the meter gas was tapped off at a constant rate by an 8-liter siphon bottle. A capillary tube flowmeter insured that the same fraction of the issuing gas passed out at any instant. Instead of using water in the siphon bottle, it was filled with a heavy, white, pure mineral oil in which gases are only very slightly soluble. However, since the analysis of the sample occurred immediately after collection, all danger from selective solubility was avoided.

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[13] E. R. Weaver, et al., J. Ind. Eng. Chem., **12**, 359, 1920.

Liquefaction of Gases.

None of the evaporated gas was saved, as the cost and trouble of collecting it would have been far greater than that of using new gas. Electrolytic oxygen, under pressure in 200 cu. ft. capacity cylinders, constituted the source of oxygen. This gas contained several tenths of a per cent of hydrogen and several tenths of nitrogen; and — it could be assumed with considerable certainty — negligible traces of argon and other rare gases. The analysis after liquefaction showed usually not more than 0.2% of gases insoluble in the oxygen absorbent. Especially since the liquid was made to boil vigorously before use by the insertion of a metal rod, it should be expected that practically no hydrogen remained. It could, therefore, be assumed that nitrogen alone constituted the impurity and the small correction necessary was made for it. We owe to the kindness of Mr. Claude C. Van Nuys of the Air Reduction Company, New York, a number of cylinders of rather pure nitrogen from a liquid air column. This gas showed less than 0.1% of oxygen and a trace of argon.

A reasonably rapid method of liquefaction was desirable, for two liters of pure liquid had to be prepared for each day's run. After passing through a reducer and a potassium hydroxide drier, the gas entered under pressure a copper coil immersed in liquid air. The gas liquefied in the coil and passed out through a double-walled, evacuated, and silvered tube into a Dewar collecting bottle. When pure oxygen was liquefied by means of liquid containing nitrogen, the liquefied oxygen usually came out colder than corresponded with its boiling point at atmospheric pressure, and it reached its equilibrium temperature only after a considerable time. Such liquid could give an erroneous latent heat. When, on the other hand, the oxygen was liquefied with liquid oxygen from a rectifying column, it of course always came out warmer, but nevertheless quickly reached its proper temperature because of the cooling produced by evaporation. At any rate, before any liquid could be used it was equilibrated by the introduction of a heavy metal rod for a few minutes, thus inducing violent boiling.

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[14] W. L. Badger, J. Ind. Eng. Chem., **12**, 164, 1920.

EXPERIMENTAL PROCEDURE.

Before each run the calorimeter had to be taken down, the bath filled with filtered liquid, the calorimeter set up again, and the meter adjusted. After the start, usually not more than twenty minutes elapsed before the gas temperatures were in proper adjustment, the meter water saturated with the gas, and general thermal equilibrium attained. As a rule, a run at a particular flow lasted about forty minutes. During the interval periodic readings of current, voltage, temperatures, pressures, etc., were taken; at the same time, the meter remained connected to the chronograph record. In the course of a run, the power fell slowly (not over 0.5% at the most) on account of the falling e.m.f. of the battery. The other variables might also change slightly, but since the small variations of the factors were practically linear, and since these factors enter into the result to the first power, no error could be introduced by taking a mean value over the whole period. To show that no irregularities occurred by taking the mean, the latent heat was calculated on the basis of data recorded over five-minute periods for an hour. The values thus obtained showed very little difference among themselves, and the mean of them all agreed with the total mean.

In order that the same general conditions of operation should exist for each rate of evaporation, several determinations at different rates were always made on the same day. In this way, a more accurate idea of the manner of change of the apparent latent heat with rate could be acquired. With pure substances, data for three or four rates in succession were recorded; but when mixtures were used, the longer time required to reach the correct speed of the screw, and the time consumed for analysis, left no opportunity to work at more than two rates per day.

In the case of mixtures, the composition of the liquid in the bath changed by about 0.6% per hour as a result of heat leaking into it; this leak of course did not enter the calorimeter. Thus, during a run of forty minutes the change in composition amounted to 0.4%. The average sample of the gas took account of this change, and the variation of latent heat with composition is sufficiently small so that no error of consequence from this source was introduced.

RESULTS.

It appeared desirable to adopt one pure substance as a standard, first, to test over a number of months the constancy and reproducibility of the apparatus as a whole for the latent heat, and, second, to be able to test out doubtful points without introducing the complications arising from composition. Accordingly, the latent heat of oxygen was determined off and on for several months.

The formula used to calculate the latent heats from the observed data is as follows:

$$\text{Latent heat} = \left(\frac{Po}{J\ To}\right)\frac{W\ Tm\ t}{D\ N\ Pm} = 0.66504\ \frac{W\ Tm\ t}{D\ N\ Pm}$$

where

Po = normal atmospheric pressure, 760.0 mm. Hg.

J = electrical equivalent of heat, 1 calorie (15° C) = 4.184 watt-seconds.

To = the absolute temperature of the ice-point, 273.13° C.

W = the mean corrected power in international watts.

Tm = the mean absolute temperature of the gas in the meter.

t = the mean time in seconds per revolution of the meter.

D = the density of the gas passing through the meter, under normal conditions, corrected for gravity.

N = the number of liters of gas delivered per revolution of the meter.

Pm = the mean partial pressure of the gas in the meter.

$Pm = (B + h/13.6 - w)$, where

B = the corrected barometric pressure, in mm. Hg.

h = the excess of pressure in the meter over atmospheric expressed in mm. of water.

w = the pressure of saturated water vapor in mm. Hg at the temperature of the meter.

In deriving the above formula, we have assumed that the laws of perfect gases can be applied to the separate gases and to their mixtures over the ordinary range of room temperature and of atmospheric pressure. The latent heats are given in calories (15° C) per gram.[15] The density for oxygen[16] under normal conditions has been taken as 1.4289, and for nitrogen as 1.2507. Corrections have been applied for gravity. The latent heat depends, of course, on the atmospheric pressure; but for ordinary fluctuations the effect is small. The correction is practically the same for oxygen, nitrogen and mixtures. It can be calculated from the vapor pressure curve[17] and from the variation of the latent heat with the temperature.[18] The effect is 0.0025 calories per gram per mm. Hg.

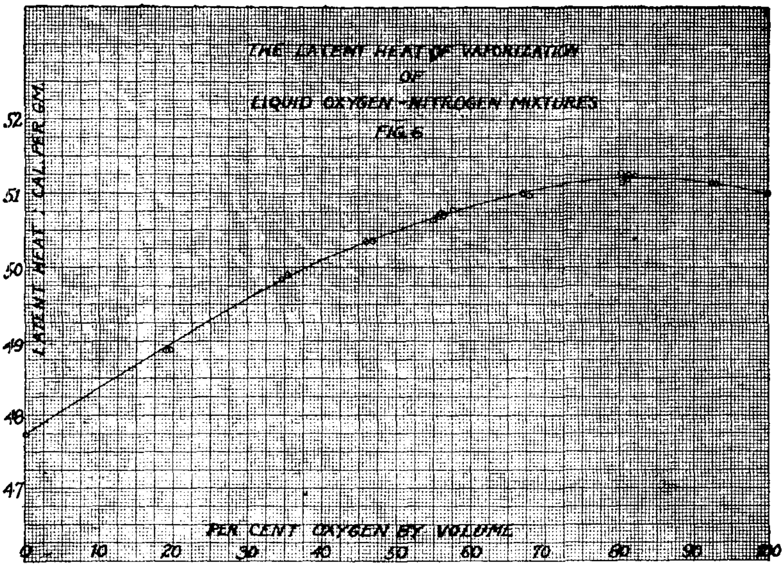

Fig. 6.

In tabulating the final data we will give the itemized mean values for all the separate factors entering into the latent heat. N, the number of liters per revolution, can be obtained from Figure 4 by interpolation of t. In Table IV are tabulated the final corrected data for oxygen. Although the last place in the latent heat is very uncertain, it has been thought best to include it. The values for nitrogen are given in Table V. In Table VI we list the final observed values for the latent heats of mixtures of various compositions and at different rates of evaporation. These results are plotted on Figure 6.

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[15] 1 calorie (15° C) = 4.184 watt-seconds. See Wärmetabellen der Phys. Tech. Reichsanstalt, by Holborn, Scheel, and Henning.

[16] Males et Gonzalez, Comptes Rendus, **175**, 6, 1921.

[17] Wärmetabellen, loc. cit., p. 32.

[18] H. Alt, loc. cit.

TABLE IV.

THE LATENT HEAT OF VAPORIZATION OF OXYGEN AT 1 ATMOSPHERE.

Date	Cal. no.	W	P_m	T_m	t	Latent heat 15° Cal.
Nov. 16	VI	8.507	745.9	293.06	92.171	50.981
1921		10.202	746.0	293.33	76.818	50.954
		13.587	746.1	293.64	57.701	50.970
		19.145	746.2	293.86	40.956	50.967
Dec. 1	VI	8.522	740.7	295.75	90.674	51.048
		10.219	740.7	296.02	75.596	51.042
		13.635	741.1	296.13	56.661	50.989
		19.948	741.5	296.28	38.772	50.997
Dec. 3	III	8.711	738.2	293.74	88.934	50.985
		10.164	738.4	293.80	76.274	50.984
		13.482	738.6	293.84	57.581	50.998
		19.813	738.6	293.86	39.242	51.032
Dec. 7	III	8.561	740.5	293.83	91.068	51.000
		10.161	740.7	293.00	76.800	51.030
		10.109	740.9	293.17	77.132	51.004
		13.348	740.9	293.31	58.427	50.990
		19.832	741.3	293.44	39.423	51.063
Feb. 13	VI	8.188	744.0	293.07	95.577	51.017
1922		11.050	745.3	293.13	70.960	50.978
		14.629	746.5	293.19	53.774	51.029
Feb. 16	VII	8.394	741.3	292.97	93.096	51.065
		11.101	741.6	293.08	70.372	51.000
		16.175	742.1	293.25	48.383	51.021
					Mean Value	51.006

As indicated before, we could by taking temperature measurements in the bulb and in the bath obtain the temperature-composition diagram for oxygen-nitrogen mixtures at atmospheric pressure. Unfortunately, this was not done except for two mixtures, for it was

TABLE V.

THE LATENT HEAT OF VAPORIZATION OF NITROGEN AT 1 ATMOSPHERE.

Date	Cal. no.	*W*	*Pm*	*Tm*	*t*	Latent heat 15° Cal.
Oct. 3	III	10.073	734.7	293.12	62.922	47.704
1921		15.069	734.3	293.28	42.118	47.767
Jan. 20	VI	7.860	748.8	293.83	81.806	47.685
1922		10.532	749.0	293.98	61.134	47.709
		13.837	749.3	294.03	46.578	47.709
Feb. 23	VI	7.900	740.9	291.53	81.434	47.813
		10.964	738.5	291.87	58.464	47.787
		15.181	737.7	292.08	42.204	47.181
April 10	VI	7.901	734.6	293.48	80.045	47.711
		10.290	734.2	293.66	61.427	47.693
		14.379	733.7	293.74	43.983	47.720
					Mean Value	47.737

expected that the data would be secured from other sources to a higher degree of precision than could be readily obtained in our investigation. The temperature readings in Table VII were made with copper-constantan couples of fine wire placed in calorimeter VI. A single-junction couple, inserted in the central tube, had its junction in the liquid in the bulb, while a double-junction couple, strapped to the outside of the calorimeter, gave the temperature of the bath. While liquid oxygen and liquid nitrogen were being evaporated in the calorimeter, the couples were calibrated in place; in addition, the e.m.f.'s at the boiling point of water and at the transition temperature of sodium sulphate were found. For the boiling points of oxygen and nitrogen, we chose the latest values determined at Leiden.[19] ΔT signifies the vertical temperature difference between the two curves in the $T-x$ diagram, Figure 3. For comparison we include the values determined by Baly.[20] It is believed that our values are good — relatively at least — to 0.03° C. That there are disagreements between these and Baly's results is evident here and also in some preliminary work by the late Mr. A. K. Dunbar on the oxygen-nitrogen equilibrium diagram in which departures in the same direction were found.

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[19] Comm. Leiden, No. 152, d. P. G. Cath.

[20] Baly, Phil. Mag., **49**, 517, 1900.

TABLE VI.

The Latent Heat of Vaporization of Liquid Oxygen-Nitrogen Mixtures at 1 Atmosphere.

Date	Cal. no.	*W*	*Pm*	*Tm*	*t*	Comp. % oxygen	Latent heat 15° Cal.
Feb. 21	VI	8.400	755.6	293.46	92.606	80.25	51.280
1922		11.050	754.9	293.80	70.376	81.2	51.254
		15.500	754.8	294.00	50.223	82.1	51.237
Feb. 24	VI	8.063	752.0	292.63	94.168	67.0	51.007
		14.908	752.5	292.97	51.032	67.9	50.968
Feb. 25	VI	8.187	752.0	292.60	90.638	54.9	50.637
		10.889	751.6	292.68	68.275	55.6	50.665
		14.825	751.9	292.86	50.282	56.1	50.731
Feb. 27	VI	7.928	743.5	293.40	90.708	46.0	50.346
		14.927	743.8	293.74	48.307	46.85	50.352
Feb. 28	VI	7.997	746.1	293.34	84.447	18.85	48.867
		14.104	747.0	293.43	48.076	19.4	48.894
Mar. 6	VI	8.127	747.4	293.10	96.054	92.5	51.154
		14.874	747.2	293.47	52.539	92.8	51.147
Mar. 22	VI	7.950	742.1	292.56	88.289	34.6	49.837
		14.374	742.5	292.97	49.002	35.4	49.900
Mar. 27	VII	7.831	751.1	293.88	94.568	56.6	50.710
		15.110	750.6	294.16	49.171	57.6	50.773
Mar. 28	VII	8.341	742.3	293.89	91.344	80.4	51.156
		15.040	741.4	294.32	50.729	81.1	51.212

TABLE VII.

Temperature Measurements.

	Temperature of liquid, °K					
	Outside		In bulb		ΔT	
Composition % oxygen	This research	Baly	This research	Baly	This research	Baly
Pure oxygen	90.13	90.96	90.13	90.96	0	0
46.7	81.10	81.36	85.20	85.32	4.10	3.96
34.6	79.96	80.12	83.58	83.68	3.62	3.56
Pure nitrogen	77.31	77.54	77.31	77.54	0	0

Discussion of Results.

In any calorimetric experiment the problem of heat leaks and other thermal disturbances provides the chief source of difficulties and errors. In this research we have first of all tried by design to avoid these troubles; in the second place, to make, as far as practicable, independent measurements of possible heat leaks; and, thirdly, to regard the final results — considered particularly as a function of the rate of evaporation — as criteria of the absence of irregularities. The first two factors have been previously considered. As to the latter, the most thorough tests were made with liquid oxygen, and it was assumed and borne out by the results that the same conclusions held for nitrogen and mixtures. If the extraneous influences such as conduction, convection, radiation, superheating of liquid and vapor, and spattering and carrying of spray enter into the experiment, then it can be shown that the manner of variation of the latent heat with the rate of evaporation should throw some light on the degree to which these influences enter. In general, the curve will not be a straight line. An examination of the final results discloses small variations with flow in different directions, but with a tendency towards a minimum; in no case, nevertheless, is the difference between two results in a single run more than 0.15 %. We conclude that the measurements show the effect on the average to be less than the experimental error or less than 0.1 %. To be able to judge accurately of the presence of disturbing factors, measurements of higher absolute precision than the magnitude of these are required. For example, suppose that a constant leak of 0.2 % existed at the lowest flow; then by increasing the rate by 100 %, the observed latent heat would change only by 0.1 %.

The average deviation of the mean of twenty-three determinations of the latent heat of oxygen is 0.025 calories per gram or 0.05 %. The average deviation from the mean of eleven determinations for nitrogen is 0.042 calories per gram or 0.09 %. From a large-scale drawing of Figure 6, the deviations of the points from the curve have been estimated; the average amounts to about 0.03 calories per gram or 0.06 %. If we consider the precision and accuracy to which the separate factors involved in the latent heat have been measured, we can get an idea of the accuracy of the final results. In Table VIII, the errors are estimated — with experience and calibrations as a basis — in per cent of the latent heat. From this table we conclude that the results should be accurate in absolute measure to about 0.2 %, and relative to themselves to about 0.1 %. It is to be observed that a large amount of

TABLE VIII.

Estimation of Errors.

Factor	Accidental error or deviation from mean	Maximum possible systematic error	Maximum probable systematic error
W	0.03 %	0.05 %	0.03%
Tm	0.03	0.05	0.03
t	0.01	0.02	0.01
D	0.01	0.02	0.015
N	0.05	0.20	0.10
Pm	0.015	0.05	0.02
Heat leaks or other effects	—	0.15	0.10

preliminary data has been taken, although not recorded here, which fully substantiates the final values.

What has been measured in this investigation is the difference in heat-contents ($H = E + pv$) between the liquid state of a mixture and the vapor state of the same composition vertically above it on the temperature-composition diagram. If we knew the heat of mixture of the liquids, we should be able to determine the $H - x$ diagram for liquids. The heat-contents obtained in this investigation then allow us to get the $H - x$ diagram for the vapors. A knowledge of both of these curves and of their difference seems to be fundamental in ascertaining the numerous possible latent heats of a mixture.

Now, we can devise an hypothetical process which, starting from the initial state 1 (Fig. 3), will finally lead to the state 1′; and since the change in heat-content is independent of the path, we can obtain a possible expression for the change in heat-content from 1 to 1′. This circuitous path provides a clearer insight into the evaporation process and shows that we may be able to procure information other than the latent heat from the experiment.

The process is to be carried out in four main steps by means of an hypothetical thermodynamic engine, shown in Figure 7. Starting with the main chamber filled with a mixture containing x moles of liquid oxygen and $(1 - x)$ moles of liquid nitrogen at the temperature T_1 (Fig. 3), and, assuming that the semi-permeable membrane M_1 is permeable to liquid oxygen alone while the membrane M_2 is permeable to nitrogen alone, the membranes are then pushed in, separating the oxygen and nitrogen. This first step is to be performed isothermally at T_1, at the same time allowing the liquid oxygen to remain at its saturation pressure P_O, corresponding to T_1, and the nitrogen at

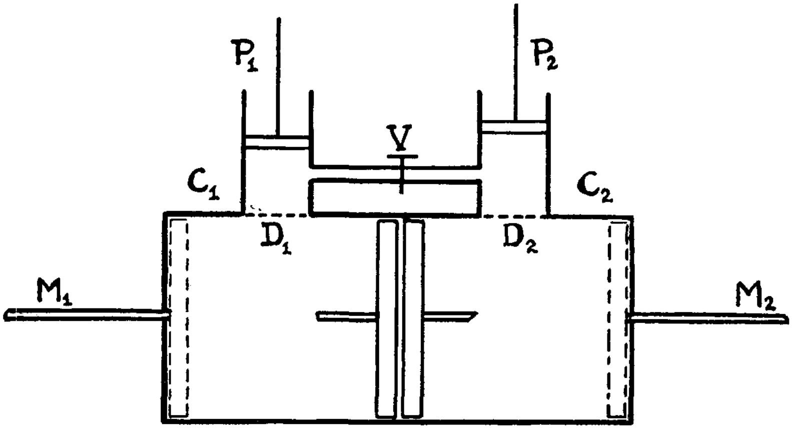

Fig. 7.

its saturation pressure P_N, corresponding to T_1. Secondly, x moles of oxygen are evaporated at T_1 and P_O by removing the diaphragm at D_1, pulling out the piston P_1, and adding the necessary heat. Similarly, $(1 - x)$ moles of liquid nitrogen are evaporated at T_1 and P_N. Thirdly, x moles of oxygen vapor are superheated in chamber C_1, from T_1 to $T_{1'}$ at P_O' (Fig. 3); and $(1 - x)$ moles of nitrogen vapor are superheated from T_1 to $T_{1'}$ at P_N. Finally the oxygen is compressed from pressure P_O to atmospheric at $T_{1'}$; and the nitrogen is expanded from P_N to atmospheric at $T_{1'}$. Then the valve V is opened, and the gases are allowed to mix at $T_{1'}$, while the pressure is maintained at atmospheric.

The expressions for the change in heat-content for the individual steps can be written down, but all the terms cannot be evaluated because of the lack of thermodynamic data.

I. $H = E + pv.\quad dH = dE + p\,dv + v\,dp.\quad \int dH = \Delta_1 H$

Then

$$\Delta_1 H = x \int_{P=1}^{P=P_O} \left\{ v - T\left(\frac{\partial v}{\partial T}\right)_p \right\} dp$$
$$+ (1 - x) \int_{P=1}^{P=P_N} \left\{ v - T\left(\frac{\partial v}{\partial T}\right)_p \right\} dp - Q_M$$

The first two terms represent the "work" terms measuring the heat-content required to bring the liquid components from atmospheric pressure to slightly above atmospheric for nitrogen, and slightly below for oxygen; consequently these terms tend to cancel. We will denote them by W. To evaluate them we need to know the equations of state for the liquids. Q_M represents the heat of mixture of x moles of liquid oxygen and $(1 - x)$ moles of nitrogen at T_1. Thus $\Delta_1 H = W - Q_M$.

II. $p =$ constant; hence $v\,dp = o$, and $dH = dE + p\,dv = dQ$

Denoting by L_O and L_N the latent heats of the individual components at T_1, we get

$$\Delta_2 H = \int dQ = x\,L_O + (1 - x)\,L_N$$

III. $dH = dE + p\,dv = dQ = C_p\,dT$

$$\Delta_3\,H = \int dQ = x \int_{T_1}^{T_{1'}} C_{p_O}\,dT + (1 - x) \int_{T_1}^{T_{1'}} C_{p_n}\,dT$$

where C_{p_o} and C_{p_n} are the specific heats at constant pressure of oxygen and nitrogen vapors respectively. Since the specific heats at constant pressure do not vary rapidly with the temperature or pressure, they can be taken as constants over the range T_1 to $T_{1'}$ — which is not more than 5° C — and they can be assumed equal to their values at atmospheric pressure. The largest step in heat-content is in step II, and since III does not amount to more than 2% of II, the above approximations are justified.

$$\therefore\ \Delta_4 H = x\,C_o(T_{1'} - T_1) + (1 - x)\,C_n\,(T_{1'} - T_1)$$

IV. We get expressions similar to those in I.

$$\Delta_4 H = x \int_{P=P_O}^{P=1} \left\{ v - T\left(\frac{\partial v}{\partial T}\right)_p \right\} dp$$
$$+ (1 - x) \int_{P=P_N}^{P=1} \left\{ v - T\left(\frac{\partial v}{\partial T}\right)_p \right\} dp + q_m$$

where q_m is the heat of mixture of the vapors at $T_{1'}$. As before, the the first two terms tend to cancel. Denote them by w. Then $\Delta_4 H = w + q_m$. For ideal gases $\Delta_4 H$ would vanish.

The net change in heat-content for all the steps is

$$\Delta H = \Delta_1 H + \Delta_2 H + \Delta_3 H + \Delta_4 H$$
$$\Delta H = W + w - Q_m + q_m + x\,L_o + (1 - x)\,L_N$$
$$+ x\,C_o\,(T_{1'} - T_1) + (1 - x)\,C_n\,(T_{1'} - T_1)$$

Thus the sum of all these terms should be equal to the latent heat found in this investigation. At the present time we cannot evaluate the first four terms. To evaluate the remainder, it is obvious that we must know the temperature-composition diagram. A preliminary calculation of these terms has been made using Baly's data, and the best data that could be found for specific heats and the variation of latent heat with temperature, with the result that the calculated curve of latent heats falls not more than 0.1 % from the experimental curve. We can probably conclude that the sum of the first four terms in ΔH is of the order of 0.1 %. Thus from the last equation we have a method of calculating from the individual latent heats and specific heats of the vapors the latent heats of the mixtures with an accuracy probably at least as good as that of the experimental determinations on the mixtures.

The reason for the maximum in the latent heat curve, Figure 6, becomes evident on referring back to the previous discussion. The latent heat terms alone when plotted against composition form very roughly a straight line. On adding the terms for the superheating of the vapor, the curve acquires a maximum. Thus no peculiar physical significance is to be attached to this maximum.

SUMMARY.

A new calorimeter has been designed for the determination of the latent heats of vaporization at low temperatures of pure liquids and their binary mixtures. The calorimetric method adopted is the continuous flow method at constant pressure. A latent heat has been defined and measured which involves the change in heat-content from the liquid state to the vapor state vertically above it on the vapor line in the temperature-composition diagram.

The latent heat of vaporization of liquid oxygen at one atmosphere has been found to be 51.01 calories[21] (15° C) per gram, while that of liquid nitrogen has been determined as 47.74 calories per gram. In the following table (p. 267) are given the values of the latent heats of the mixtures which were read from a smooth curve drawn through the experimental points.

The absolute accuracy of the results is estimated as not less than 0.2 %, while the shape of the curve is believed to be good to 0.1 %.

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[21] 1 calorie (15° C) assumed equal to 4.184 international watt-seconds.

TABLE IX.

THE LATENT HEAT OF VAPORIZATION OF LIQUID OXYGEN-NITROGEN MIXTURES AT 1 ATMOSPHERE.

Per cent oxygen	Latent heat	Per cent oxygen	Latent heat
0	47.74	55	50.67
5	48.06	60	50.83
10	48.37	65	50.97
15	48.68	70	51.08
20	48.98	75	51.17
25	49.28	80	51.23
30	49.57	85	51.23
35	49.83	90	51.18
40	50.07	95	51.12
45	50.28	100	51.01
50	50.48		

A few observations on the temperature-composition diagram are also given. With the aid of data on the individual latent heats and on the specific heats of the superheated vapors it is shown to be possible to calculate the latent heats of the mixtures, with probably a high accuracy.

I wish to take this opportunity to express my appreciation of the excellent conditions under which I have been able to perform this work in the Jefferson Physical Laboratory at Harvard University. To Professor H. N. Davis I am indebted for a number of suggestions and for his constant help and interest in facilitating the progress of the investigation. Acknowledgment is also made of an appropriation by the Research Corporation of New York from which a part of the expenses for this research were paid.

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PIEZOELECTRIC CRYSTAL OSCILLATORS APPLIED TO THE PRECISION MEASUREMENT OF THE VELOCITY OF SOUND IN AIR AND CO_2 AT HIGH FREQUENCIES.

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By George W. Pierce.

Received July 18, 1924. Presented October 14, 1925.

1. **Reference to Previous Paper.**— In a previous paper[1] I have described a method of producing sustained high-frequency electric and mechanical vibrations by a novel combination of a plate of piezoelectric crystal with a thermionic vacuum tube, and have shown how to employ the apparatus in the calibration of wavemeters. These vibrations are of extraordinary constancy as to frequency so that it has seemed desirable to apply the apparatus to other measurements.

The present account describes the precision measurement of the velocity of sound at high frequency.

2. **Preparation of the Piezoelectric Crystal Plate.**— Quartz was used as the piezoelectric substance. A plate was cut from the natural crystal of quartz with the orientation suggested by P. and J. Curie,[2] as is here shown in Figure 1. After the natural crystal had

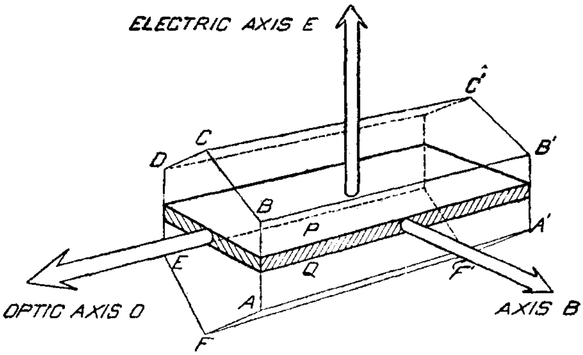

Figure 1. Orientation of axes in natural quartz crystal. Method of sectioning.

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[1] G. W. Pierce, Piezoelectric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters, Proc. Am. Acad. of Arts and Sciences, **59,** No. 4 (1923). (Reprints may be purchased from the Librarian of the Academy, 28 Newbury Street, Boston, Massachusetts.)

[2] Pierre and Jacques Curie, Comptes Rendus, **91,** 383 (1880); also, Oeuvres de Pierre Curie, Paris, 1908.

been trued up by crosswise cuts so as to form a prism with the hexagonal ends ABCDEF and A′B′C′. . F′, which are respectively perpendicular to the natural edges AA′, BB′ of the crystal, a rectangular slab was obtained from the prism by two parallel cuts P and Q, which are perpendicular to a natural face such as ABB′A′.

3. Mounting of Plate Between Electrodes.—The rectangular slab so obtained has three axes represented in the diagrams by arrows: the *optic axis O* (parallel to the lengthwise natural edges of the crystal), the *electric axis E* (parallel to two opposite natural faces of the crystal), and the *third axis B* (perpendicular to the optic axis and the electric axis). This slab is placed between two metal electrodes M' and M'', as shown in Figure 2. The crystal may rest on M'', while M' may best be supported above the crystal so as not quite to touch it. This is done in order to leave the crystal free to execute mechanical vibrations, without too much restraint from the electrodes.

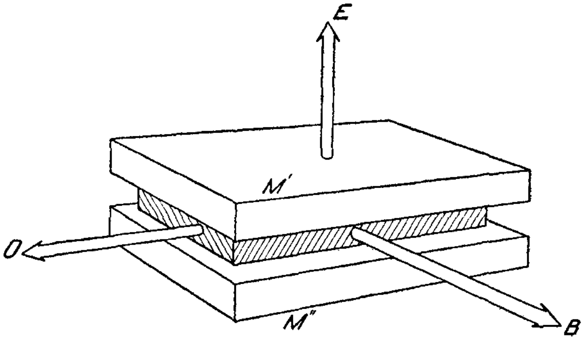

FIGURE 2. Electrodes M' and M'' with plate of crystal between.

The manner of supporting the upper plate M' is shown in Figure 3. M' is attached to an upper bakelite plate, which in turn is supported on columns attached to a lower bakelite base. Bolts, nuts and spiral springs, as shown, permit the adjustment of the clearance between M' and the crystal slab. In Figure 3, the axis B points toward or away from the observer.

Another method of mounting the crystal slab (which is here represented as circular) is shown in Figure 4, in which the plates M' and M'' are in vertical planes, and the plate M'' is perforated with a hole about 1 cm. in diameter, so as to permit the radiation of sound through this hole. In this figure the plate M'' is in the form of a spring clamp and may rest upon the crystal with sufficient pressure to hold the crystal in place, or if desired, the pressure may be removed by the screw S and the crystal supported independently by a small shelf below.

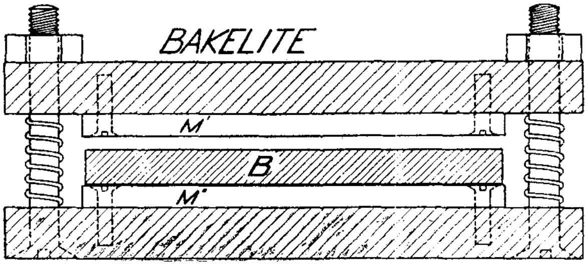

FIGURE 3. Mounting of plate of crystal in clamp with adjustable electrode distances. This arrangement for emitting and receiving sound of frequency determined by dimension in direction of axis B.

In Figure 4, the electric axis E points through the perforate electrode M'', while the axes B and O are in the plane of the crystal slab with orientation that is immaterial for the present purposes.

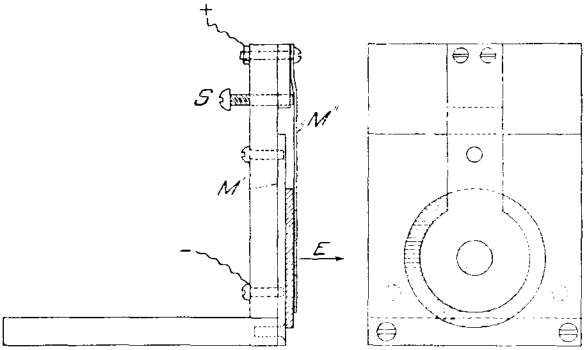

FIGURE 4. Side view and front view of crystal plate mounted in a vertical plane having a perforate electrode for emitting and receiving sound of frequency determined by thickness of plate in direction of electric axis E.

4. The Piezoelectric Action.—As to the action of the piezoelectric crystal, attention is called to the discovery of Curie that a difference of electrical potential of proper sign established between electrodes such as the plate M' and M'' (that is, an electric force of proper sign established in the direction of the electric axis E of Fig. 2) causes the piezoelectric quartz crystal to expand along the axis E and contract along the axis B. No change occurs along the optic axis O. A reversal of the electric force causes a reversal of these expansions and contractions. Curie also found that an expansion along E or contraction along B produced by an external mechanical pressure developed an electromotive force between the plates (i.e., along E) opposite in sign to that which would produce the given expansion. That is to say, an electric field along E produces distortion of the crystal in directions E and B, and a distortion so produced in these directions reacts to diminish the field along E. In case of alternating forces the amplitude and phase of these effects depend upon the mechanical constants of the crystal and upon the frequency of the applied e.m.f.

Langevin[3] with great ingenuity showed how to use such a piezoelectric crystal body as a source of sound particularly in water, and Cady[4] made a thorough and beautiful investigation of the crystal oscillators and crystal resonators, and adapted them to use as constants of electrical frequency.

5. The Electric Circuits for Producing Sustained Vibrations. — In my work above cited I showed a simple form of connections of the crystal vibrator to a thermionic vacuum tube so as to produce sustained electrical and mechanical vibrations of the system with a period determined by the dimensions of the slab of crystal and independent of the electrical constants of the circuit. Cady had already described a means of doing this by a different type of circuit, which is described in his paper above referred to in the Radio Institute Proceedings. My circuit which for some purposes has certain advantages is illustrated in Figure 5. The piezoelectric crystal vibrator marked "crystal" has one of its electrodes connected to the plate P and the other electrode connected to the grid G of a thermionic vacuum tube, having a filament F heated by the battery marked "*A Bat.*" The plate is supplied with current by the battery marked "*B Bat.*" A microammeter, A, and a telephone "*Tel*" shunted by a bypass condenser C are included in the plate circuit. The element marked "*load*" in the figure was described in my original paper as a resistance of about 30,000 ohms or a large inductance, say 20 milhenries. This system produces sustained electric oscillations in the circuit and mechanical oscillations of the crystal vibrator with a period of one mode of natural mechanical vibration of the crystal body. This period in my original investigation was the mechanical period of compression and recovery of the crystal in its shortest dimension (which was along the electric axis). Such a system radiates sound in the direction of the electric axis *E*.

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[3] Langevin, Brit. Pat. Specifications, N.S., **457**, No. 145,691 (1920).

[4] W. G. Cady, The Piezoelectric Resonator, Proc. Inst. Radio Engineers, **10**, 83 (1922).

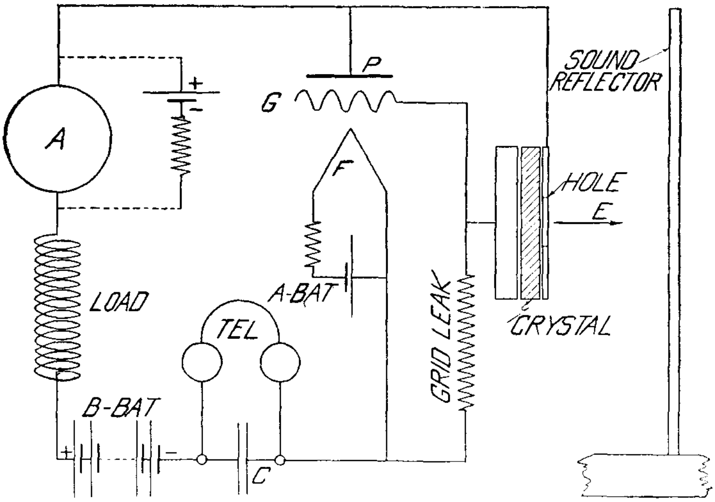

FIGURE 5. The author's type of electric circuit best suited for crystal frequency determined by dimension *E*.

It is also possible to cause the system to oscillate with the period determined by a larger dimension of the vibrator (that is in the direction of the axis *B*).

To attain this vibration of longer period I find that the best circuit is that shown in Figure 6 which is similar to Figure 5 except that the crystal vibrator is connected between the grid and some point below the inductance, as for example, the positive end of the B battery. The telephones, or their equivalent, acting as a choke, and the bypass condenser shunting them, are, in this case, usually necessary to give the proper reaction to make the system oscillate. This arrangement causes the crystal to vibrate with a period determined by the mechanical frequency of the crystal slab, expanding and contracting along the direction of the B axis, so that sound is radiated in this direction.

Other types of piezoelectric oscillating circuits will be described elsewhere.

6. Exploration of Sound Waves.— In Figures 5 and 6 is also shown a sound reflector which may be moved toward or away from the crystal so as to explore the standing sound waves. *No additional apparatus for detecting the sound is necessary, for the reflected sound wave falling on the emitting face of the crystal vibrator, even when the reflector in some cases is at a distance of 300 half waves of sound from the vibrator, reacts on the crystal with sufficient force to cause the current in the milliammeter A to fluctuate visibly in accordance with the phase of arrival of the reflected wave.*

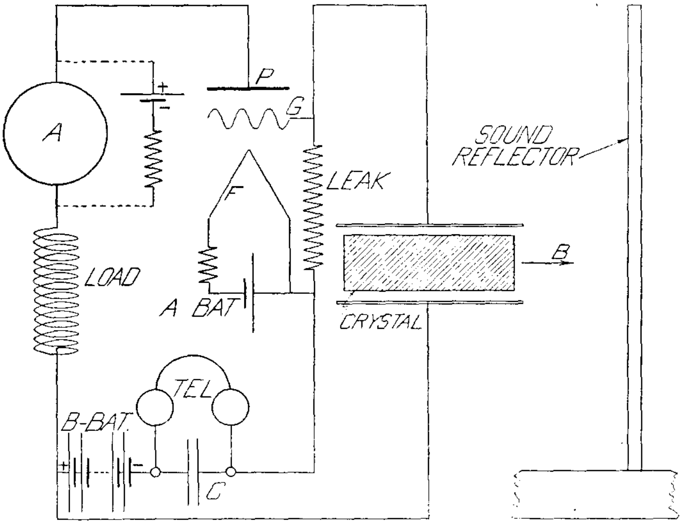

Figure 6. Type of electric circuit best suited for crystal frequency determined by dimension B.

When a small dry-battery vacuum tube (UV 199) is used the normal current in A when the crystal is oscillating is about 0.5 milamperes and the fluctuations range from 1.0 milampere (or circuit stops oscillating) when the mirror is close to the crystal down to zero when the mirror is at infinite distance from the crystal. To render the fluctuations more evident for precision measurements, a Weston microammeter, with one division equal to 4 microamperes, is used at A and is shunted by a potentiometer and battery combination to make the normal current through *A* small, say 10 divisions (40 microamperes). The fluctuations in the current in *A*, as the reflector is moved, may then amount to the whole additional scale, 90 divisions, when the reflector is 20 half wavelengths of sound away, and to about 25 divisions when the reflector is 100 half wavelengths away. These magnitudes depend on the frequency and area of radiating face of the crystal. The microammeter gives a sharp maximum at each half wavelength of displacement of the reflector.

It is seen that the apparatus oscillates of itself with a highly constant fixed frequency determined by a mode of mechanical vibration of the crystal plate. It thus produces sound waves and at the same time, by the strength of the plate current, determines the relative phase of the direct and reflected sound waves in air, so that the distance between successive loops of the standing sound wave for a range of 100 or more half wavelengths may be measured with great precision, by merely changing the distance between the reflector and the radiating face of the crystal, and making proper readings. These distances were changed by a calibrated precision screw, illustrated in Figure 7, by which the mirror could be moved about 50 centimeters and its position read with an accuracy of one one-thousandth of a millimeter, but such accuracy was not ordinarily required or attainable in the location of maxima. A modification of the mounting is shown in Figure 8, in which the mirror and crystal are contained in a gas-tight box.

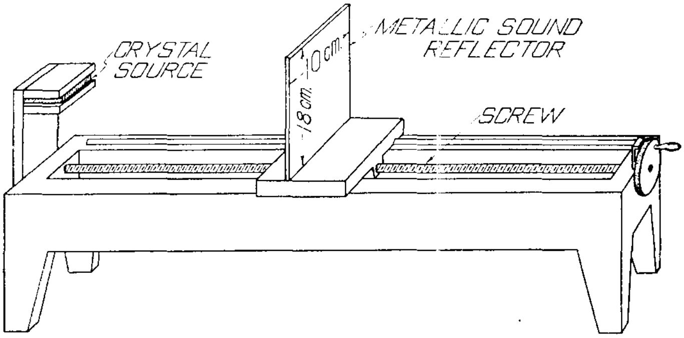

FIGURE 7. Mounting of crystal and mirror with screw drive.

7. Example of Microammeter Readings Plotted Against Crystal-to-Mirror Distance.—To show the nature of the observations of the standing waves reference is made to Figure 9, which is a plot of divisions of the microammeter in the plate circuit of the oscillator as ordinates, against the scale reading in millimeters of crystal-to-mirror distance. Three half waves are plotted so that an idea may be had of the accuracy of location of the maxima. In this particular experiment, in which the sound frequency was 98183 cycles per second, the maxima could be located to better than 1/20 millimeter, and since the train of standing waves in this case was explored for 140 millimeters, this degree of precision of locating the maxima gives the

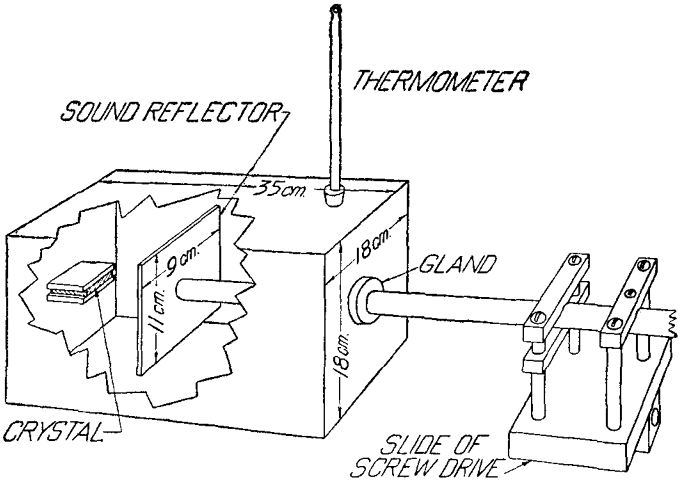
THERMOMETER
SOUND REFLECTOR
35cm.
18cm.
GLAND
9 cm.
11 cm.
18cm.
CRYSTAL
SLIDE OF SCREW DRIVE

FIGURE 8. Crystal and mirror in brass box for containing gas.

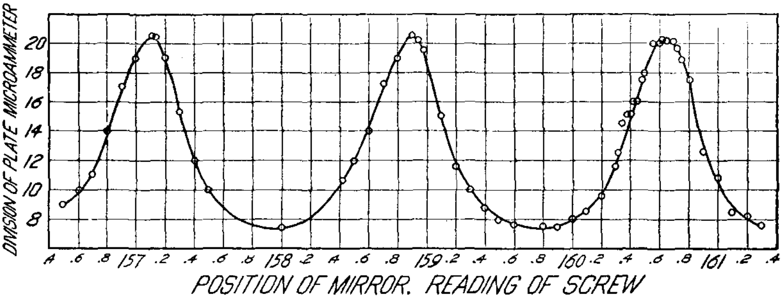

FIGURE 9. Plot of readings of microammeter in plate circuit of piezo-electric oscillator against readings of position of reflecting mirror.

wavelength to about 1/30 of one per cent. In the case of some of the other frequencies a still greater degree of precision was attainable.

For the present purpose it was not necessary to make readings in sufficient number to plot the complete standing wave system. It was only necessary to determine the positions of the mirror for maxima of the microammeter, and to locate accurately such of these positions as were to be used in the calculations. Usually maxima at intervals of five, ten, or twenty half wavelengths were employed.

8. Method of Averaging Observations.— Suppose that we have set the reflecting mirror at a series of positions of maxima schematically represented as *positions* 0, 1, 2, ... n, in Figure 10, and let the distances, which we shall call *intervals*, between successive *adjacent positions* be a_1, a_2, ... a_n. These intervals are attempts to measure the same quantity x (say), and question arises as to the best method of averaging the intervals to get this quantity x. If we merely take

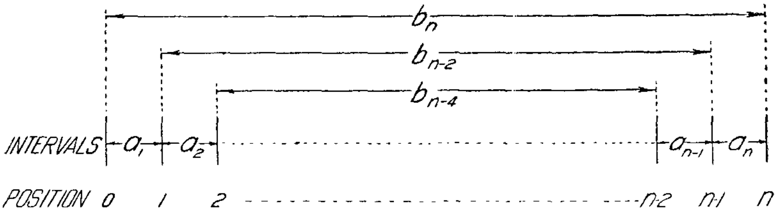

FIGURE 10. Diagram used in discussion of method of averaging.

$x = \dfrac{a_1 + a_2 + \, . \, . + a_n}{n}$ we see that $x = \dfrac{\text{distance from 0 to } n}{n}$; that is to say, this method of averaging, if employed, makes the result depend on the settings 0 and n alone and ignores all the intervening settings, which might as well not have been made. This method of averaging is evidently not correct.

Assuming that the positions 0, 1, ... n are all located with equal accuracy we may obtain the proper method of averaging by regarding the distance between any pair of positions, for example, the position 2 and position 8, as data for determining the fundamental interval, giving to this distance a weight proportional to its length ($8-2=6$) and averaging for all such pairs of positions, with due precaution to count every pair only once. Starting with position 0 we have distances 0 to 1 = a_1, 2(0 to 2) = $2(a_1 + a_2)$, etc. Next starting with position 1, we have (1 to 2) = a_2, 2(1 to 3) = $2(a_2 + a_3)$, etc. These are collected in Table I.

TABLE I.

Weighted Distances for Determining Fundamental Interval.

Starting with Position			
0	1	2	...
a_1	a_2	a_3	...
$2(a_1+a_2)$	$2(a_2+a_3)$	$2(a_3+a_4)$	...
$3(a_1+a_2+a_3)$	$3(a_2+a_3+a_4)$	$3(a_3+a_4+a_5)$	...
....	....	....	...
$n[a_1+a_2+\ldots\ldots+a_n]$	$(n-1)[a_2+a_3+\ldots\ldots+a_n]$	$(n-2)[a_3+a_4\ldots\ldots+a_n]$	...
Starting with Position			
....	$n-3$	$n-2$	$n-1$
....	a_{n-2}	a_{n-1}	a_n
....	$2(a_{n-2}+a_{n-1})$	$2(a_{n-1}+a_n)$	
....	$3(a_{n-2}+a_{n-1}+a_n)$		

The sum of all these quantities Σ, say, is

$$\begin{aligned}\Sigma = {} & [1 + 2 + 3 + \ldots\ldots + n]\ [a_1 + a_2 + \ldots\ldots + a_n] \\ & + [2 + 3 + \ldots + n - 1]\ [a_2 + a_3 + \ldots\ldots + a_{n-1}] \\ & + [3 + 4 + \ldots + n - 2]\ [a_3 + a_4 + \ldots\ldots + a_{n-2}] + \text{etc.}\end{aligned}$$

Now, as in Figure 10, putting

$$\begin{aligned} b_n &= a_1 + a_2 + \ldots\ldots + a_n, \\ b_{n-2} &= a_2 + a_3 + \ldots\ldots + a_{n-1}, \text{ etc.,}\end{aligned}$$

and noting that

$$(1 + 2 + 3 + \ldots\ldots + n) = \frac{(1 + n)\, n}{2},$$

$$(2 + 3 + \ldots\ldots + n - 1) = \frac{(1+n)\,(n-2)}{2},$$

we obtain, after dividing out the common factor $\frac{1+n}{2}$,

$$\Sigma = n\, b_n + (n-2)\, b_{n-2} + (n-4)\, b_{n-4} + \ldots\ldots \tag{1}$$

This sum may be otherwise written in the form

$$\Sigma = n\, a_1 + 2\,(n-1)\, a_2 + 3\,(n-2)\, a_3 + \ldots\ldots + n\, a_n \tag{2}$$

If now the fundamental interval is x, which $a_1, a_2, \ldots\ldots. a_n$ are attempts to measure, the value of x may be obtained from (1) or (2) by dividing Σ by the sum of the coefficients of the a's, giving

$$x = \frac{n\, b_n + (n-2)\, b_{n-2} + (n-4)\, b_{n-4} + \ldots}{n + 2(n-1) + 3(n-2) + \ldots + n}, \tag{3}$$

or

$$x = \frac{n\, a_1 + 2\,(n-1)\, a_2 + 3\,(n-2)\, a_3 + \ldots\ldots\ldots\ldots + n\, a_n}{n + 2\,(n-1) + 3\,(n-2) + \ldots\ldots + n}. \tag{4}$$

Equation (3), *or the alternative equation* (4), *gives the weighted mean value of the quantity x, of which $a_1, a_2, \ldots.$ are the observed values.* Equation (4) is the more convenient and is used below.

9. Temperature Reduction.—For convenience in reducing wavelengths and velocities measured at temperature $t°$ C. to the corresponding values at 0° C., Table II has been compiled.

TABLE II.

Temperature Reduction Table.

$v_0 = v_t \times \theta$, where $\theta = 1/\sqrt{1 + 0.00367t}$.

$t°$ C.	θ Temperature factor	0.1 × diff. Interpolation
15	0.97365	
16	190	−17
17	025	
18	.96857	
19	692	−16.5
20	527	
21	.96365	
22	198	−16.5
23	036	
24	.95875	
25	713	−16.1
26	553	

10. Sample Set of Observations on Wavelength of Sound in Air.— Table III contains a sample set of measurements of the wavelength of sound in air at 205620 cycles per second. With the mirror 3.5 cm. from the crystal a position of maximum deflection of the microammeter was noted. The mirror was then moved twenty half

TABLE III.

Run No. 144. Wavelength of Sound in Air.
Frequency 205620 Cycles per Second.
Steps 20 λ/2. Humidity 86%.

Temp. degrees C.	Step 20λ/2 cm.	Weight	Weight × step
22.8	1.6812	17	28.580
	1.6805	2 × 16	53.776
	1.6832	3 × 15	75.744
	1.6830	4 × 14	94.248
	1.6767	5 × 13	108.986
22.92	1.6736	6 × 12	120.499
	1.6830	7 × 11	129.591
	1.6830	8 × 10	134.640
	1.6761	9 × 9	135.764
	1.6962	10 × 8	135.696
23.00	1.6672	11 × 7	128.374
	1.6861	12 × 6	121.399
	1.6814	13 × 5	109.298
	1.6730	14 × 4	93.688
22.98	1.6806	15 × 3	75.627
	1.6754	16 × 2	53.613
22.90	1.6829	17	28.609
22.92		969	Σ = 1628.23

$$\lambda = \frac{1628.23}{969 \times 10} = 0.16803 \text{ cm.}$$

Temperature factor = 0.96046

$$\lambda_0 = 0.16138 \text{ cm.}$$

wavelengths farther away. This was found to be a displacement of 1.6812 cm., which is recorded as the first value in the table. The mirror was then displaced a second step of twenty half wavelengths, giving 1.6805, and so on for seventeen steps, encompassing a wave-train of 340 half wavelengths. This table thus epitomizes the exploration of a stationary wave system of 340 maxima. Averaging results as in § 8 we obtain the wavelength as 0.16803 cm. at 22.92° C., which reduced to 0° C. gives $\lambda = 0.16138$ cm.

The data of Table III are for a single run. Five other similar runs were made at this frequency, and the complete set of values of wavelength at 0° C. are recorded as λ_0 in Table IV.

TABLE IV.

Collection of Results of Wavelength and Velocity of Sound in Air at 0° C.

Frequency 205620 cycles per second.

Humidity per cent	Run no.	λ_0 cm.	Residual
87	141	0.16146	0.00015
80	142	.16118	13
79	143A	.16115	14
86	143B	.16137	6
86	144	.16138	7
84	153	.16132	1
Mean 84		0.16131	0.00009

$$\lambda_0 = 0.16131 \pm 0.00003$$

$$v = 331.69 \pm 0.06 \text{ at } 0^\circ \text{ C.}$$

In the third column is the value of λ_0 for each of the six runs, having an average of $\lambda_0 = 0.16131$ cm. In the fourth column are the residuals, which are the amounts by which the individual values differ from their mean. The probable error of the mean E_a computed by the approximate formula

$$E_n = 0.6745 \frac{\text{mean residual}}{\sqrt{n-1}},$$

where n = number of separate values, is 0.00003 cm. Multiplication of λ_0 (reduced to meters) by the frequency gives for the velocity of sound at 0° C. at this frequency, 205620 cycles per second, the value

$$v = 331.69 \pm 0.06 \text{ meters per second.}$$

This result was obtained in free air without any enclosure about the gas. The mean humidity during the several runs was 84 per cent.

In a similar way other high frequencies were employed. For comparison a determination of the velocity of sound in air at the audio frequencies 995.88 and 2987.6 cycles per second was also made. For this purpose a novel method was employed, a description of which follows.

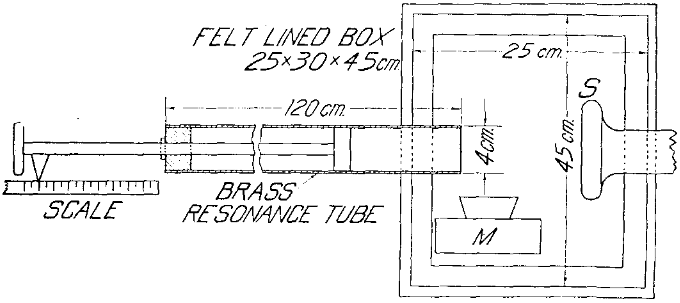

FIGURE 11. Apparatus for measuring velocity of low-frequency sound. S = magnetophone source, M = microphone detector.

11. Low-Frequency Sound Required a Special Method.— The crystal oscillator method with crystals at hand could not be employed at frequencies below about 40000 cycles per second, so that for low frequencies resort was had to a resonance-tube method. This method as ordinarily employed is inaccurate. A modification was introduced that resulted in a considerable improvement. This consisted in the use of a superimposed interference effect that gave a position of practical silence, or null point, in the middle of what would ordinarily be the sound maximum.

12. Null Method.— A sketch of the apparatus for this purpose is given in Figure 11. A brass resonance tube 120 cm. long and 4 cm. internal diameter, and provided with a piston and scale, was employed. The source of sound was a magneto telephone at S, which was driven by a tuning fork of known frequency. The source S, the mouth of the resonance tube, and a microphone receiver M were enclosed in a felt-lined wooden box, and the distances between them were adjusted so that the sound direct from *S* to *M* just neutralized the sound from the tube to *M* when the sound emitted by the tube was a maximum. This gave a fiducial point of silence in the middle of what would otherwise be a maximum and permitted very accurate settings with sound of frequency 995.88 cycles per second. The microphone *M* communicated through a transformer with a head telephone receiver in which the observer listened. By means of an electric filter in this circuit, the fundamental frequency of the sound (995.88 cycles per second) could be eliminated and settings could then be made on the harmonic of 3 × 995.88 = 2987.6 cycles per second, with which the settings were still more accurate.

13. Sample Sets of Low-Frequency Sound Measurements.—Table V contains a sample set of observations at 995.88 cycles per second analyzed by the method of § 8.

TABLE V.

Run No. 67 in Air at 995.88 Cycles per Second.

Temp. degree C.	Setting at minima in cm.	Mean setting	$\lambda/2$ cm.	Wt.	$\lambda/2 \times$ wt.
21.7	20.32				
	.32				
	.33	20.323	17.274	4	69.096
	37.58				
	.60				
	.61	37.597	17.339	6	104.034
	54.86				
	.96				
	.96				
	.96	54.936	17.307	6	103.842
	72.26				
	.21				
	.26	72.243	17.190	4	68.760
	84.44				
	.47				
	.39	89.433			
				20	345.732

$$\lambda/2 = 345.732 \div 20 = 17.287 \text{ cm.}$$

$$\lambda = 34.573 \text{ cm.}$$

$$\text{Temperature factor} = 0.96250$$

$$\lambda_0 = 33.276 \text{ cm.}$$

Other values of this wavelength λ_0 are collected in Table VI, in which they are given weights proportional to the number of actual settings at each position of the piston.

TABLE VI.

Collection of Results for Frequency 995.88 Cycles per Second.

Humidity per cent	Run no.	λ_0 cm.	Weight	Residuals
36	66A	33.350	1	0.019
36	66B	33.266	1	.065
36	67	33.276	3	.044
35	69	33.320	3	.011
52	72	33.412	3	.081
52	73	33.328	6	.003
55	74	33.340	6	.009
Weighted mean		33.332		0.027

Whence $v = 331.94 \pm 0.07$ m/sec. at 0° C. Not corrected for effect of tube.

TABLE VII.

Collection of Results for Frequency 2987.6 Cycles per Second.

Humidity per cent	Run no.	λ_0 cm.	Weight	Residuals
36	66C	11.105	2	0.007
36	67B	11.110	1	.002
35	68A	11.115	6	.003
35	68B	11.111	3	.001
Weighted mean		11.112		0.003

$v = 331.98 \pm 0.03$ m/sec. at 0° C.

The weighted mean value of velocity of sound in air at 0° C. (Table VI) obtained at this frequency is 331.94 meters per second with a probable error of 0.07 meters per second, uncorrected for effects of the tube.

This apparatus proved to be especially accurate when applied to the measurement of the velocity of the harmonic frequency 2987.6, as is shown by the results in Table VII.

14. Dimensions of Crystal Vibrators.—Figure 12 gives a dimensional sketch of the different crystal vibrators employed. The wavy line emerging from the right-hand face of each crystal vibrator indicates the emitting surface and the direction of radiation of sound. In each specimen the optical axis is vertical in the sketch and the

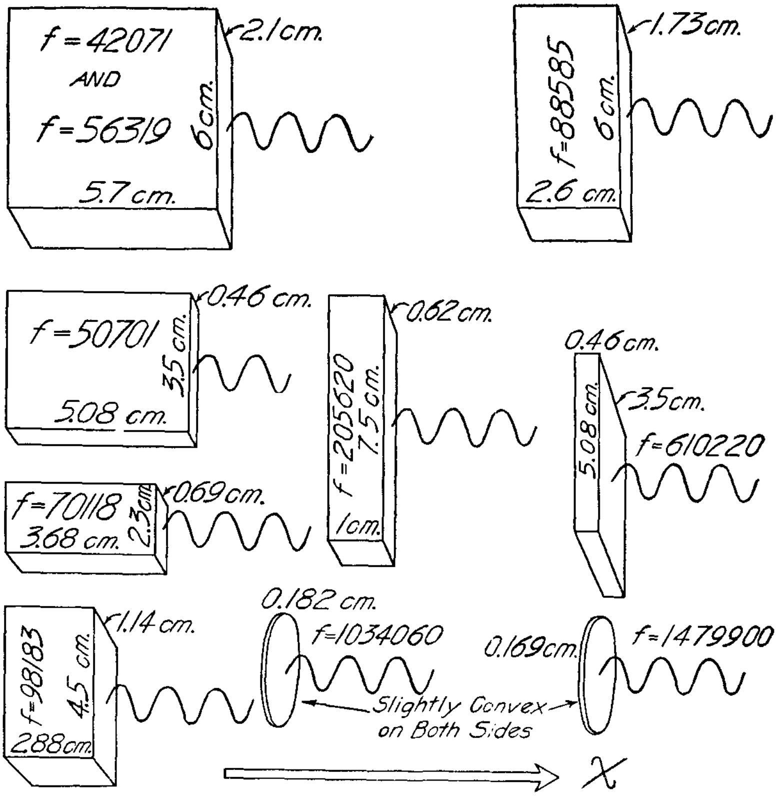
f=42071
AND
f=56319
6cm.
5.7 cm.
2.1cm.
f=88585
6cm.
2.6 cm.
1.73cm.
f=50701
3.5 cm.
5.08 cm.
0.46 cm.
f=205620
7.5 cm.
1cm.
0.62cm.
0.46cm.
5.08 cm.
3.5cm.
f=610220
f=70118
2.3cm
3.68 cm.
0.69cm.
f=98183
4.5 cm.
2.88cm.
1.14 cm.
0.182 cm.
f=1034060
0.169cm.
f=1479900
Slightly Convex
on Both Sides
X

FIGURE 12. Dimensions of crystals for different frequencies.

electric axis is perpendicular to the paper for all except those of frequencies 610220, 1034060, and 1479900, in which the electric axis is in the direction of sound radiation (to the right in the sketch). The frequency, designated by f, is marked on each vibrator. The upper left-hand specimen was used for two different frequencies, which could be differentiated one from the other, and each employed at will, by throwing in or out a suitable condenser in shunt to the inductance marked "*load*" of Figure 6. This same vibrator was originally somewhat longer in the dimension running right and left in the sketch and then gave the frequency 41009 cycles per second. After being used at this frequency it was ground down to give the frequencies indicated in the sketch.

Table VIII contains the frequencies of the several crystal vibrators and also the dimension x (in direction right to left in sketch). This x

TABLE VIII.

Relation of Frequency to Dimension x.

Axis of radiation	x Determining dimension in cm.	f cycles per sec.	fx
B	5.7	42071 56319	23.98×10^4 32.10
B	5.08	50701	25.76
B	3.68	70118	25.80
B	2.61	88585	23.12
B	2.88	98183	28.29
B	1.00	205620	20.56
B	0.46	610220	28.07
E	0.182	1034060	18.82
E	0.169	1479900	25.01
		Average	25.14×10^4

is the dimension in the direction of propagation of the sound. The product of f in cycles per second and x in centimeters is contained in the last column. The average of this product is about 25×10^4, and this may be employed as a very rough guide in cutting the crystal vibrators for a required frequency.

After the crystals are cut the vibrators are ground with coarse carborundum powder and finished with fine carborundum, fine emery, and in the case of thin specimens with rouge, to such fineness of surface as may be required for efficient oscillation. Repeated measurements of frequency during grinding enable a skillful operator to attain a desired frequency to better than 1/100 of 1 per cent, if standards of that precision are available.

15. Standards of Frequencies.—This subject is treated in the previous paper[5] to which reference is made for details. A standard tuning fork, here designated fork "*S*," made by the Western Electric Company, was used to operate electrically a high-speed siphon recorder made by the General Radio Company, and was thus chronographed four times for a period of 300 seconds each and once for a period of 250 seconds, and was found to have a frequency of 49.916 at 18.2° C., with a probable error of 1/400 of 1 per cent. Immediately preceding and following this operation, the 20th harmonic of the fork was observed to beat 2.44 times per second with a second fork, driven by a vacuum tube circuit, as shown in the previous paper. This second fork, here designated fork "*P*," was thus found to have a frequency

$$f_{\text{fork } P} = 20 \times 49.916 - 2.44 = 995.88 \pm 0.02$$

at 18° C.

Next a standard piezoelectric crystal oscillator permanently mounted was calibrated in terms of fork *P*, by the use of an electric oscillator and resonant-circuit wavemeter as intermediary. The wavemeter was calibrated by setting it to resonance with the electric oscillator when the latter was at a beat zero with the 6th to the 18th harmonic of fork *P*, and immediately thereafter the readings of the wavemeter were again taken at resonance with the electric oscillator when each of the harmonics of the electric oscillator between the 25th and the 58th was at beat zero with the standard piezoelectric crystal oscillator. By careful interpolation this gave

$$f_{713} = 420710 \pm \frac{1}{200} \text{ per cent.}$$

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[5] Pierce, Proc. Am. Acad. of Arts and Sciences, **59,** No. 4 (1923).

This crystal oscillator will be known as No. 713, which is approximately its electric wavelength.

In terms of this crystal and its harmonics and multiples, the entire wavemeter was now carefully calibrated for the range of frequencies between 6000 and 6,000,000.

16. Measurement of Frequencies of Crystal Vibrators.—Immediately following or preceding the exploration of the sound wave system the frequency of each crystal vibrator was measured by one or more of the following methods:

I. *Harmonic Bracketing with Standard Crystal.* This consists of obtaining a wavemeter setting on an electric oscillator at beat zero with some harmonic of the unknown crystal X and then obtaining, on both sides of this wavemeter setting and as near to it as possible, a wavemeter setting on the electric oscillator at beat zero with some two harmonics of the standard crystal No. 713. Interpolation gives the wavemeter correction and the value of the frequency of the unknown.

II. *Audio-Frequency Beats.* Intercomparison of crystal vibrators was made in some cases by measuring on an audio-frequency meter the beat frequency of the fundamental of one crystal with some harmonic of another crystal.

III. *Superheterodyne Method.* When the beat frequency of II was above audibility, a third oscillator frequency could be made to beat with the rectified beat frequency of the two crystals. Measurement of the third frequency (which was a difference) could then be made with sufficient accuracy by the wavemeter.

IV. *Subheterodyne.* When the beat frequency of II was too low for audibility it could be made audible by an oscillator beating with both crystals (say 1000 per second with one and 1002 per second with the other), then the beating of the two audio frequencies with each other could be counted (as 2 beats per second).

V. *Direct Check of Crystals against Tuning Fork P.* This is a method similar to that used in § 13.

These various methods will be designated in the tables of results. As an example of Method I an electric oscillator beating with the fundamental of a crystal X and two harmonics of the oscillator beating with crystal No. 713 gave readings as in Table IX. The fundamental wavelength of No. 713 is 713.08 meters, based on 3×10^8 meters per second as the velocity of the waves, so that by regarding λ of column three as approximately correct it is found that n of the fourth column is 5 and 14/3, as entered. These last two numbers are then multiplied by 713.08 and entered as λ_{correct} for No. 713. A comparison of columns three and five gives the correction increments 0.3 and 0.9 entered as δ for No. 713. Interpolation between these gives $\delta = +0.8$ for X, which added to the observed λ gives $\lambda_{\text{correct}} = 3386.6$, for which the frequency is 88585. Note that the wavemeter calibration was in error less than 1/30 of 1 per cent in the range of Table VIII.

TABLE IX.

Determination of Frequency of Crystal X.

Crystal	Condenser divisions of W.M. No. 14 Coil E	λ Meters by calibration table	n	λ correct	δ	f
X	10.409 10.410 10.410	3385.8		3386.6	0.8	88585
713	11.530 11.532 11.530	3565.1	5	3365.4	.3	
713	10.048 10.047 10.047	3326.8	14/3	3327.7	.9	

17. Simultaneous Exploration and Frequency.—In some cases two crystal vibrators were mounted one above the other on the sound-exploring apparatus, their two sound patterns explored simultaneously, and the beat frequency between one of the vibrators and some harmonic of the other continuously observed by an audio-frequency meter. This was a safeguard against possible changes in frequency or in the effects of temperature or composition of the air.

18. Negligible Change of Frequency of the Crystal Oscillator Due to Reaction of Reflected Sound.—The reflected sound changed the plate current of the piezoelectric oscillator. The crystal of frequency 42071 was made to have this value so that its 10th harmonic would have the frequency of the standard crystal No. 713, which is 420710. It was found that by slight adjustment of the clearance of the electrodes of 42071 its 10th harmonic gave 1.4 beats per second with No. 713, when the sound reflector was adjusted at a position such that the plate ammeter of the oscillator was at a minimum. When now the reflector was moved along the sound pattern in space until the ammeter deflection was a maximum the beat frequency was 8 per second. It thus appears that the effect of the mirror was a change of frequency of 6.6 cycles per second in 420710, or 0.66 cycles per second in 42071, which is 1/700 of 1 per cent.

19. Negligible Effect of Humidity on the Velocity of Sound.—According to standard tables[6] the density of air at 20° C. is about 0.4 per cent less at 50 per cent humidity than for dry air at the same temperature. If the moisture affected the velocity of sound only by changing its density we should expect that the velocity in air at 50 per cent humidity would be about 0.2 per cent more than the velocity in dry air. On the contrary, I have not been able to detect any effect of humidity on the velocity of sound, although the following careful measurements were made. In the metal box shown in Figure 8 seven runs were made with air dried by phosphorus pentoxide so that a hygrometer read zero humidity and seven similar runs were made with the air at 50 per cent humidity. This was done at a frequency of 98183 cycles per second and gave for the wavelength reduced to 0° C. the values in Table X. The effect of humidity up to 50 per cent is seen to be less than the probable error which is less than 1/100 of 1 per cent.

It should be noted, however, that in this experiment in order to obtain dry air in the metallic sound chamber, it was found necessary to remove the felt lining, so that we have not a sound pattern determined solely by the distance from the crystal to the mirror. The velocity of sound at 0° C., calculated from the above wavelength values and the frequency, comes out to be 331.56, whereas in open air or in a box lined with nonreflecting material the velocity at this frequency is found to be 331.77. The absence of lining of the box thus makes in the absolute value of the velocity an error of 1/15 of 1 per cent, but it is not believed that this could have any effect in making the apparent velocity the same in dry air and in damp air.

Many other experiments made in the course of this research have failed to disclose any effect of humidity and indicate that the effect of humidity up to 80 per cent on the high frequency velocity of sound is below 1/50 of 1 per cent. This is a novel result contrary to existing ideas.

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[6] Kaye and Laby.

TABLE X.

Sound Wavelength at 0° C. at 98183 Cycles per Second for Dry Air and Moist Air.

Dry air	50% Humidity
0.33759 cm.	0.33780 cm.
.33774	.33760
.33777	.33776
.33770	.33789
.33766	.33762
.33775	.33753
.33770	.33765
Average	
0.33770 cm.	0.33769 cm.
Probable error	
0.00001 cm.	0.00003 cm.

20. Results on Velocity of Sound in Air at 0° C.— Table XI gives the results on air at 0° C. All of these values except those at $f = 995.88$ and $f = 2987.6$ were obtained in free air without any container whatever.

TABLE XI.

Velocity of Sound in Air at 0° C.

f Frequency cycles per second	v Velocity meters per second at 0° C.	Probable error in meters due to errors in wavelength	Probable error in meters due to errors in frequency	Method of determining frequency
995.88*	331.94*	0.07	0.02	Fork *P*
2987.6*	331.98*	.03	.02	Fork *P*
41009	332.45	.03	.07	Beats with 205620
42071	332.37	.06	.01	Beats with No. 713
50701	332.47	.06	.07	Beats with 205620
56319	332.32	.08	.07	Bracketed by No. 713
70118	331.98	.02	.02	Beats with No. 713
88585	331.97	.03	.05	Bracketed by No. 713
98183	331.77	.07	.02	Checked *vs.* Fork *P*
205620	331.67	.06	.07	Bracketed by No. 713
610220	331.81	.07	.06	Bracketed by No. 713
1034060	331.76	.03	.05	Superheterodyne
1479900	331.64	.05	.04	Bracketed by No. 713

* Measured in a resonance tube and uncorrected for the diameter of the tube.

These results are plotted in Figure 13.

21. Effect of Frequency on the Velocity of Sound in Air.— For the present omitting from consideration the two values at $f = 996$ and $f = 2988$ which were measured in a tube, and for which there may be an uncertain tube correction, we see that between $f = 50000$ and $f = 100000$ the velocity falls from $v = 332.47$ m/sec. to $v = 331.78$ m/sec., which is a decrease of 1/5 of 1 per cent. The velocity falls further to $v = 331.67$ at $f = 200000$, then seems to rise again to $v = 331.81$ m/sec. at $f = 600000$, and thence to fall to $v = 331.64$ at

$f = 1500000$. It is to be noted, however, that in the range from $f = 100000$ to $f = 1500000$ the straight broken line departs from the observed values by a maximum of about 1/40 of 1 per cent or 0.09 m/sec., which is about the probable error of observations. The actual existence of the maximum at $f = 600000$ may, therefore, be questioned.

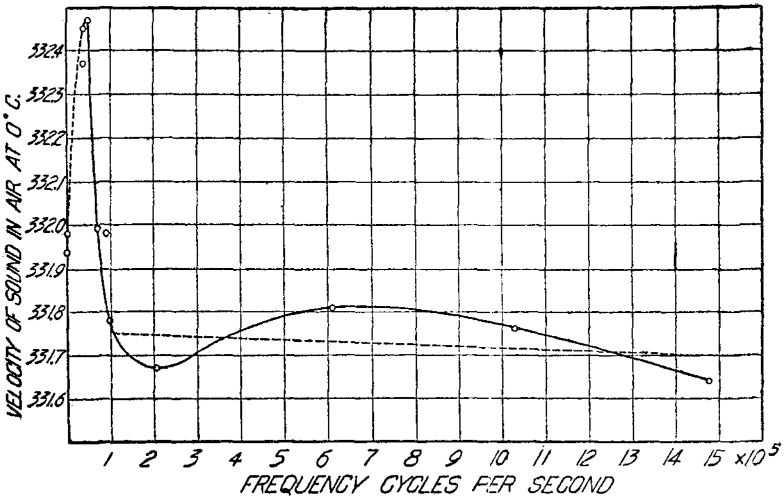

FIGURE 13. Velocity of sound in air at 0° C. Plotted against frequency.

22. Precautions in Measurements.—To make sure that the frequencies and experimental conditions did not vary between observations, one set of measurements of velocity at $f = 50701$ and $f = 205620$ was made with the two crystals mounted one above the other so that their sound patterns could be explored simultaneously. Each of the crystals was provided with its own vacuum tube, load inductance, and microammeter. The two load inductances were placed in inductive relation to each other so that in a telephone in circuit with the crystal $f = 50701$ beats could be heard between the harmonic $f = 4 \times 50701$ of one crystal and the fundamental $f = 205620$ of the other. These beats, measured on a special bridge-type of audio-frequency meter, remained between 2835 and 2837 cycles per second so that, if the indicated frequencies are correct, we should have

$$4 \times 50701 + 2836 = 205620$$

$$\text{i.e.} \qquad 205640 = 205620$$

which is in agreement to within 1/100 of 1 per cent. While this agreement of frequency was being maintained, the sound wavelength measurements gave $\lambda_0 = 0.65555$ cm. at $f = 50701$, and $\lambda_0 = 0.16129$ cm. at $f = 205617$, whence the calculated velocities of sound at 0° C. are 332.37 m/sec. at $f = 50701$ and 331.64 m/sec. at $f = 205617$. These values show that the higher frequency gives the lower velocity by an amount considerably greater than the errors of observations. The results at this frequency in Table XI contain this run averaged with certain others not made simultaneously.

Another precaution consisted in the interspersing of readings at one frequency with those of another so that gradual changes or improvements of methods should not introduce differences in the results.

23. Effort to Extend the Observations to Higher Frequencies. — With a crystal vibrator giving 3,000,000 oscillations per second, effort was made to obtain the velocity of sound in air, but no reactive effect could be obtained by moving the reflector. Either the crystal was insensitive to reaction, or, what is more probable in view of the results for CO_2 (see § 25), the air was opaque to these frequencies of three million cycles per second.

24. Concerning Previous Measurements of Sound Velocity at High Frequencies.— The only previous measurements of the velocity of sound at high frequencies were by E. Dieckmann[7] in 1908 by the use of a Poulsen Arc as source, a transmission grating for dispersion, and a suspended vane for detector. Frequencies were measured by a wavemeter. The medium was illuminating gas, in which Dieckmann obtained a velocity of 456. m/sec. independent of frequency between $f = 78000$ and $f = 780000$ cycles per second, but the method employed was not capable of high precision.

25. Measurement of the Velocity of Sound in CO_2 at High Frequencies.— The metallic container illustrated in Figure 8 was lined with cotton wool and after long flushing with CO_2 gas from a commercial cylinder of liquid CO_2, the velocity of sound was measured for three frequencies with the results given in Table XII. The water vapor present, which is the chief impurity in commercial CO_2, represented a humidity of 50 per cent at 20° C.

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[7] E. Dieckmann, Ann. der Phys., **27**, 1066 (1908).

TABLE XII.

Velocity of Sound in CO_2 (Commercial) at 0° C.

Frequency cycles per second	Velocity meters per second at 0° C.	Probable error in meters per second
42071	258.82	0.08
98183	258.94	.16
205620*	260.15	.23
1034060	opaque	

* Highly absorbed.

These results are plotted in Figure 14, and show an increase of velocity with increasing frequency from v = 258.8 at f = 42000 to 260.2 at f = 206000. This work with CO_2 is incomplete and further measurements are in progress. In addition to the relatively large change of velocity with frequency this experiment with CO_2 revealed also a very large absorption at the higher frequencies, as is discussed in the next section.

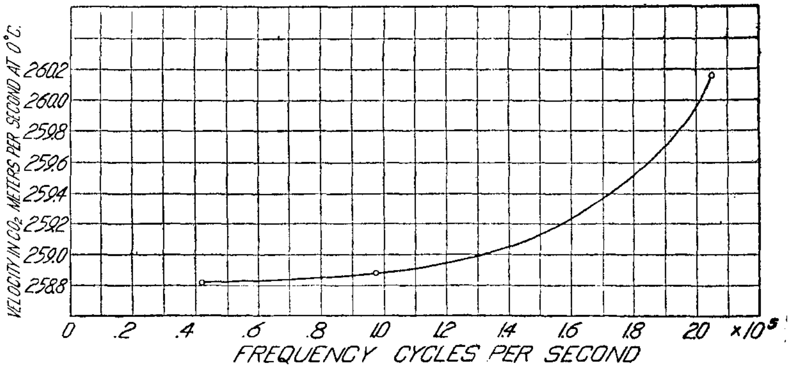

FIGURE 14. Velocity of sound in CO_2 at 0° C. Plotted against frequency.

26. Absorption of High-Frequency Sound by CO_2.— At f = 1034060, CO_2 was found to be practically opaque. No reaction by the reflected wave could be observed even when the mirror was almost in contact with the crystal. At $f = 205620$ the absorption was very great, as the following data show. Figure 15 is a plot of divisions deflection of the microammeter of the plate circuit of crystal $f = 205620$ against crystal-to-mirror distance in CO_2, at 22.7° C. The distance is in millimeters uncorrected for a slight error of the screw. It is seen that the deflection in going from the maximum at 4 mm. distance to the minimum at 4.4 mm. distance drops from 9.2 divisions to 3.5 divisions, representing an amplitude of reaction by the reflected sound equivalent to 5.7 microammeter divisions.

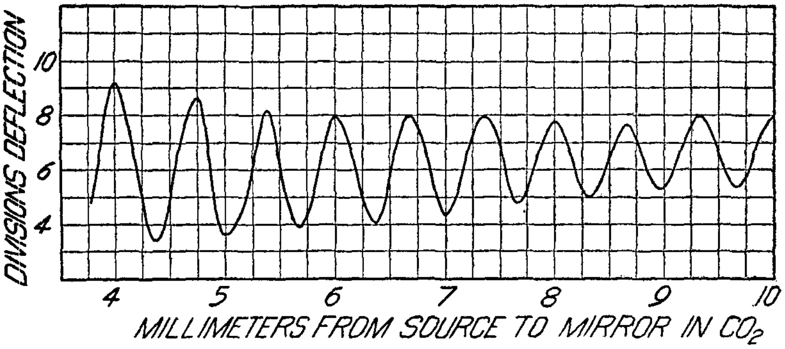

FIGURE 15. Stationary wave system in CO_2.

Immediately after these readings were taken, the CO_2 was removed from the sound box (of Fig. 8) and air substituted. It was then found that with air in the box the reaction due to the reflected wave gave an amplitude of deflection of 10 milliammeter divisions when the crystal-to-mirror distance was 204 mm., which was the greatest distance that the dimensions of the apparatus would permit. We thus have the result that the double transmission through 4 mm. of CO_2 diminishes the sound intensity at $f = 205620$ more than the double transmission through 204 mm. of air. Let us note that the width up and down of the radiating crystal was 6.2 mm. (see Fig. 12) so that in the case of the CO_2, where the mirror was only 4 mm. from the crystal, the departure from a plane wave condition was very small and all the diminution of amplitude was due to nontransmission by the gas; while in the case of the air the mirror was so far from the crystal (204 mm.) that the effect of the departure from a plane wave would be marked, and the diminution of amplitude would be due to absorption and to divergence as well. Thus the absorption of sound at this frequency $f = 205620$ in CO_2 is more than 80 times and may be several hundred times as great as that of air.

At $f = 98183$ a similar experiment shows that 17 mm. of CO_2 reduces the reflected intensity as much as 68 mm. of air, making the absorption of CO_2 at least 4 times that of air. Further experimental work needs to be done on this subject.

27. Concerning Effect of Box Enclosing Gas in Modifying Standing Wave System.— It was pointed out in § 19 that the sound box of Figure 8 when not lined with absorbing material, although the box is enormous compared with the wavelength of the sound, reduced the apparent velocity of sound by 0.6 per cent at $f = 98183$. On the other hand with the box lined with cotton wool 3 cm. thick, the velocity at $f = 205620$ measured in air in the box was found to be 331.70 m/sec., which within the limit of error checks the value 331.77 of Table XI in free air at this frequency. In making absolute measurements the lining of the box is important and should be studied. Cotton was used rather than hair felt, as it was thought to be less hygroscopic, but a careful comparison of different materials has not been made. Incidentally glass wool in considerable thickness was found to be transparent to high-frequency sound.

28. Discussion of Velocity of Sound in Air at Low Frequency. — The direct design of the present research to measure the stationary wave system of sound in free air could not be employed, with the crystal oscillators at hand, at frequencies below $f = 40000$. Resort was had to a resonance tube method, but, although consistent and accurate readings were obtained, the uncertainty of the tube correction to be applied makes it advisable to refer briefly to results obtained by others.

Work of T. C. Hebb. By an ingenious attempt at direct measurement of the wave system in the free air of a large room, T. C. Hebb [8] in 1905 obtained $v = 331.29$ at $f = 2377$.

Explosion and String Galvanometer Registration. By this method E. Eselangon [9] in France in 1917–18, and v. Angerer and Ladenburg [10] in Germany in 1916–18 at sufficiently great distances from the source obtained respectively $v = 330.9$ and $v = 330.78$ as the velocity of sound in dry air at 0° C. I find in a paper by McAdie[11] a reference to a result by D. C. Miller of the value of $v = 330.8$, but I have not found any publication by Professor Miller of this value. If this were also obtained by the same method of explosion and galvanometer registration, we have good evidence for the correctness of 330.8 m/sec. as the speed of explosive waves of low intensity.

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[8] Phys. Rev., **20,** 89 (1905).
[9] Comptes rendus, **168,** 165 (1919).
[10] Ann. der Physik, **66,** 294 (1921).

Resonance Tube Method. The well-known methods using the resonance tubes give a measured wavelength dependent on the diameter of the tube. Helmholtz taking account of viscosity, and Kirchhoff introducing also the effect of the communication of heat to the walls of the tube, both derived the equation

$$v' = v\left(1 - \frac{c}{d\sqrt{\pi f}}\right), \tag{5}$$

in which

f = frequency,
v = velocity in free air,
v' = " " a pipe of diameter d,
c = a constant.

Very discordant results exist as to the applicability of this formula and as to the value of the alleged constant c. If we assume that my two measurements in the tube at $f = 995.88$ and $f = 2987.6$ are both exact, and that equation (5) is correct, and that the velocity is the same at both frequencies, we may eliminate and obtain

$$v = 332.04 \quad \text{m/sec. at } f = 996 \text{ and } f = 2988.$$

This result is larger than that obtained by other experimenters by a somewhat similar use of equation (5), for example: Stevens[12] who in 1902 obtained $v = 331.32$ and recently Grüneisen and Merkel[13] at the Reichsanstalt who obtained $v = 331.57$, by, however, neglecting all of the observations at frequencies below $f = 3480$, which would have given, if kept, a much larger average velocity. In fact their values between $f = 1390$ and $f = 2780$ would have given $v = 331.98$ if these frequencies had been kept and treated as their remaining data were.

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[11] Annals of the Observatory of Harvard College, **86,** 107 (1923).
[12] Ann. der Phys., **7,** 285 (1902).
[13] Ibid., **66,** 293 (1921).

29. Conclusions Regarding Low-Frequency Velocity.— We may hence conclude that the velocity of *explosive waves* of small intensity as registered by galvanometer deflections is 330.8 m/sec. and that the low-frequency velocity measured by standing waves is probably between 331.29 and 332.1 meters per second. The uncertainty surrounding this last value leaves the beginnings of the curve of Figure 13 in doubt, but it seems probable that the dotted line between $f = 0$ and $f = 40000$ represents the real course of the curve.

30. Summary.— I. A simple method is described for (*a*) producing high frequency sound of constant pitch, (*b*) detecting the interference between direct and reflected waves by the reactive response at the source, (*c*) exploring the sound pattern, and (*d*) interchecking the frequencies employed.

II. A null method of measuring the audio-frequency velocity of sound in resonance tubes is described.

III. The velocity of sound in free air and CO_2 has been determined for a range of frequencies between 40000 and 1500000 cycles per second for air and between 400000 and 205000 cycles per second for CO_2. A hitherto undetected change of velocity with frequency has been discovered, as shown in Figure 13 and Figure 14.

IV. Large absorption in CO_2 for frequencies above 205000 cycles per second is shown.

V. The effect of humidity of the atmosphere on velocity of sound is found to be negligible at high frequencies.

31. Theoretical Significance.— It is hoped that additional data may be collected and that at some later date the entire theoretical significance of the results may appear. In the meanwhile the initial increase of the velocity of sound with increase of frequency may be looked upon as due to heat conduction by the gas. In other words, the expansion and compression is not adiabatic for frequencies below 50000 cycles per second for air and below 205000 cycles per second for CO_2. With increase of frequency up to these limits the motion becomes more nearly adiabatic with a resulting increase of velocity.

In the case of CO_2 absorption seems to stop the observations while the curve of velocity against frequency is still rising, but to make sure of this values at frequencies between $f = 100000$ and $f = 200000$ should be obtained. The high absorption in CO_2, if carefully studied, should be of great significance.

In the case of air the frequency of maximum velocity is about

50000 cycles per second and at frequencies higher than this the reduction of velocity may be due to an approach of the period of the sound to the time between collisions of the molecules or atoms of the gas. The existence of two maxima for air (if that at $f = 600000$ is real) may in some way be associated with the two principal ingredients of the air.

Cruft Laboratory, Harvard University,
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Introduction.

The importance of a knowledge of the physical properties of single crystals of the metals requires no argument. Very little is known of the subject, however. Until within a few years, practically all that we had was a determination of the three elastic constants of a natural crystal of copper, and of its electrical resistance and thermal conductivity at low temperatures, and questionable data on the thermal expansion and electrical properties of bismuth. Recently the metallurgists have studied the properties of single crystals particularly with regard to plastic flow when stressed beyond the elastic limit, in order to better understand the behavior of crystalline aggregates when stressed beyond the elastic limit, and we now have a rather complete knowledge of the phenomena of flow in single crystals of aluminum, zinc, and tin.[1] Within a year, and after a good part of this present investigation was completed, Grüneisen and Goens[2] have published data for the elastic constants, thermal expansion, and electrical resistance of single crystals of zinc and cadmium. With the exception of the single crystal of copper, this seems to be the first knowledge of the detailed elastic constants of any metal, as distinguished from the average constants given by an isotropic aggregate. Several years previously I had found very great differences of the linear compressibility of a number of metals in different directions, but had not sufficient data for the detailed constants.[3]

The need of a detailed study of the properties of single crystals is greatest in those metals which do not crystallize in the cubic system, because many of the properties of a cubic crystal are the same in all directions, and therefore will be the same for a single crystal as for an isotropic agregate, of which we already have sufficient knowledge. Among such properties are the linear compressibility, thermal expansion, electrical resistance, and thermal conductivity. Three elastic constants are required for the crystal, however, against two for the aggregate.

In this paper I have studied particularly the more readily obtainable non-cubic metals, namely antimony, bismuth, and tellurium, (trigonal), cadmium and zinc, (hexagonal), and tin (tetragonal). Also, because I was fortunate enough to obtain a fine single crystal of tungsten, I have determined certain of its properties. The properties of these metals which I have examined are as follows. (1) Thermal Expansion. In all of the cases above, except tungsten, two constants are sufficient to determine the thermal expansion. These are the constants which give the expansion along an axis of three-, four-, or six-fold symmetry, and the expansion in every direction at right angles to such an axis. They may most easily be determined by measuring the linear expansion of rods cut along one or the other of these directions. The expansion determined in the following is for a small range of temperature on both sides of room temperature.

(2) Electrical Resistance. The symmetry relations of electrical resistance are like those of thermal expansion, and the requisite constants may also be determined by measurements on samples lying along the axis or at right angles to it. I have determined the specific resistance for the necessary directions, the temperature coefficient of resistance between 0° and 100° C. as a function of direction, and the effect of hydrostatic pressures up to 12000 kg/cm^2 between 0° and 100° C. on resistance in different directions. (3) Elastic Constants. These are much more complicated than thermal expansion or resistance. The trigonal metals have six constants, the tetragonal six, and the hexagonal five. I have determined these constants by measurements of the elastic deformation of properly oriented specimens, and have in addition measured the linear compressibility in two directions to 12000 kg., which gives information about the behavior under pressure of certain combinations of the elastic constants, and in particular gives the cubic compressibility. It is sufficient to measure the linear compressibility in only two directions, as is the case for resistance and thermal expansion.

The chief difficulty which has prevented the measurement of crystal properties of metals in the past is doubtless that of preparing single crystals of sufficient size. In this paper I first describe in detail the method by which the specimens were prepared (I have already sketched this method in general outline[2]), then the general methods of making the various sorts of measurements, and then the detailed data for the individual metals.

METHOD OF GROWING LARGE SINGLE CRYSTALS.

The general method is that of slow solidification from the melt. A tubular electric furnace, in a vertical position, is maintained at a temperature above the melting point of the metal in question. The metal in the molten condition in a suitable mold of glass or quartz tubing is slowly lowered through the bottom of the furnace into the air of the room or into a cooling bath of oil. Solidification thus starts at the bottom of the tube and proceeds slowly along its axis, keeping pace with the lowering. If the lowering is at a speed less than the velocity of crystallization and also slow enough so that the latent heat of solidification may be dissipated by conduction, then the metal will usually crystallize as one grain, provided that only one nucleus started in the tube at the bottom. The final casting has of course the cylindrical shape of the mold, not the geometrical form characteristic of the crystalline system of the metal.

It is, possibly, surprising that this method works for all of the metals of the list, because several of them are known to have polymorphic transitions between the melting point and room temperature (tin and antimony). It is evident that on passing through a transition point the same process repeats itself that took place at the melting point.

In applying this method in practise there are various details to attend to. The most difficult and important is to secure the formation of only one grain at the bottom end. This is not always absolutely necessary, and in much of my early work I took no special precautions in this direction and obtained suitable single crystals after cutting off the bottom ends of the castings. The reason is that one orientation of the grains is usually much more favorable for growth than others, so that even if a number of grains start at the bottom, that grain will eventually win through which is most favorably situated. The most favorable orientation is in almost all cases with the plane of easiest cleavage or slip parallel to the axis of the casting. This does not fix the orientation of the grain uniquely, since this plane of cleavage may have any orientation within 180° about the axis of the casting, so that a number of grains may start all equally well situated for growth; these grains will then persist throughout the entire length of the casting. To ensure, therefore, a reasonably high percentage of success, one must depend on something more positive than mere chance to ensure the formation of only one original grain. I tried a number of schemes with varying success. The best is to draw the lower part of the mold out into a separate chamber, separated from the main part by a capillary 0.1 mm. or so in diameter, as shown in Figure 1. The capillary acts as a filter, allowing only one of the several grains which may have formed initially in the lower bulb to get through into the main part of the mold. Even this will not always ensure success, because if the grain which gets through the filter is particularly unfavorably situated for growth, a more favorably oriented grain is likely to make its appearance spontaneously at some later stage. To avoid this occurrence as much as possible, one may make the lower chamber fairly long, so that the formation of the most favorably

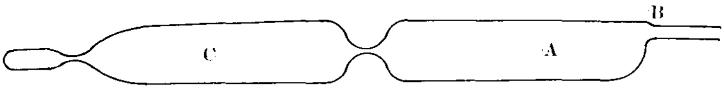

FIGURE 1. The mold used in producing large single crystals.

situated grain may have as much chance as possible to take place below the filter instead of above it.

After the proper starting of the crystal, precautions must still be taken to prevent the appearance of new nuclei, but these precautions are comparatively easy. The most important is freedom from dirt. Specks of dirt clinging to the side of the glass are likely to start new grains. The most likely dirt is small particles of oxide; it is practically impossible to melt a piece of metal which has once solidified and come in contact with the air without the appearance of specks of oxide, even if the melting is performed in vacuum. In order to obtain the molten metal clean, I usually melted the metal in an ante-chamber before the mold proper, separated from the mold by a narrow capillary, which acts as a filter to remove solid dirt. Figure 1 will suggest the general method of procedure. The metal in sufficient quantity is first placed in the ante-chamber *A* through the open mouth of the mold, which is then drawn down at the top end *B* and a glass tube attached by which connections may be made with rubber tubing to vacuum. The mold is now placed in the furnace, the chamber *A* projecting beyond the upper end of the furnace; the mold is evacuated through *B* by a rotary oil pump with which it remains in connection, and the furnace is brought to temperature. The furnace with the mold in it is then brought to a horizontal position by rotating about a suitable axis on which it is mounted, so that now the ante-chamber with the unmelted metal projects in a horizontal position from the furnace. The metal is now melted by an auxiliary gas flame, and raised to a temperature somewhat above that of the furnace. By tipping the furnace, the metal is washed back and forth in *A*. This is an important operation, because in this way large quantities of occluded gas may be eliminated, which otherwise separate during solidification and give an imperfect casting. Some metals are much worse than others in this respect; bismuth particularly is bad, and it may be necessary to manipulate the molten metal for an hour or more before all the gas bubbles have disappeared. After getting rid of the gas, the furnace is rotated back to the vertical position, and the metal allowed to filter through into the mold, admitting air to the antechamber to hasten the filtering. It is well to choose such a quantity of metal that it finally stands a little above the passage from *A* to *C*, thus keeping any oxide on the upper surface and out of the final mold. The glass tube at *B* is sealed off, to prevent further access of air and continued oxidation. Solidification is now started by lowering the mold.

In addition to the complete removal of dirt, the chance formation of new grains is hindered by a proper design of the mold; a too obtuse taper above the lower capillary may start new grains, probably because the temperature does not change uniformly at such a taper.

The proper speed of lowering depends very much on the metal and the size of the mold. In general, the speeds appropriate to this method are very much less than those used in drawing wires out of the surface of molten metal, as would be expected because of the action of the glass walls in restraining the attainment of temperature equilibrium. In general, for castings as large as 2.2 cm. in diameter, which I have made a number of times and which were the largest allowed by the dimensions of the furnace, a speed as low as 4 mm. an hour is desirable, while for small castings I have used speeds up to 60 cm. per hour. It is in general true that it does no harm to err on the side of too great slowness, but this does not seem to be true for bismuth and tellurium, for which there is doubtless a rather definite optimum speed for any given diameter and external conditions.

It is important that air drafts be kept from the emerging mold, as otherwise new centers of solidification may be started. To avoid drafts the mold may be lowered into a pipe closed at the bottom end, with its upper end tightly pressed against the lower end of the furnace. If cooling in an oil bath is preferable to cooling in the air, as it is for cadmium, the pipe may be filled with oil, and a drip supplied at the upper end to remove the oil displaced by the emerging mold.

The removal of the final casting from the mold is important. If the glass and metal are perfectly clean, the metal will stick and cannot be got out. To avoid this the tube must first be greased. A convenient way to do this is to flush the tube with a heavy mineral oil (Nujol) and then to wash this out with several fillings of petroleum ether, finally heating the tube in an inverted position with a gas flame, working from the top down, driving the film of oil before the flame. It will be found that a sufficient amount of oil clings to the glass to prevent the metal from sticking, but it is otherwise imperceptible in amount. The metal may be removed from the mold by cracking off the glass between the jaws of a vice, having first thoroughly scratched the outside of the glass with a diamond, or better, if greater precaution is necessary, by scratching, and then very rapidly heating the outside of the glass locally in a fine oxy-gas flame, and plunging the tube into water, repeating until the glass is completely cracked off. In any event, the removal from the mold must be made with very great care for the soft metals, of which cadmium is the softest of the above list.

Pyrex glass was used for all the metals above, except antimony, which melts at a temperature above that at which pyrex will stand a vacuum. For antimony I first used quartz, and I have to express my great obligation to Mr. E. R. Berry of the Research Laboratory of the General Electric Co. at Lynn for providing me with the quartz, but I later found that a Scotch combustion tubing is sufficiently rigid at the melting temperature, although considerable care is necessary in its manipulation.

The temperature of the furnace is not a matter of much importance, as would be expected with the slow rates of lowering used here. In order to avoid difficulties from a too fine adjustment of temperature, I usually operated 50° or 100° above the melting temperature. The furnace was run from a rotary transformer attached to the municipal power line. Under the particular conditions of this laboratory this proved to be much more constant than any storage battery available. No regulation, other than the initial regulation by rheostats, was attempted. Slow fluctuations of temperature are not of much importance, and rapid changes are eliminated by the thermal inertia of the furnace.

The lowering was by electric motor, working through worm wheels and various gears to give the range of speed desirable. Detailed description of this part of the apparatus is not necessary.

Contrary to my expectations, it turned out that for many purposes castings of small diameter are better than large ones. In fact it is to be emphasized that in most of this single crystal work the specimen should be brought to the final form with as little mechanical work done on it as possible, the reason of course being the extreme plasticity of some of these metals, and the extreme brittleness of others. In making castings smaller than the maximum allowed by the furnace, various devices were adopted for casting a number at a time, thus saving time. Rods 6 mm. in diameter were cast four at a time in tubes sealed to the bottom of the ante-chamber, each 6 mm. tube having its own bulb and filtering capillary at the lower end. Rods 2.5 mm. in diameter were cast sometimes in groups of 25 or more, each individual tube being provided with its own bulb and capillary filter at the lower end, the group of 25 tubes being mounted within a single large containing tube and filled barometer-wise in an inverted position. Devices of this sort will readily suggest themselves. The difficulty with small diameter castings is in getting rid of occluded gas, which may, if not removed, sometimes segregate itself in bubbles large enough to cut entirely across the diameter of the tube.

Incidentally, this method of production of crystals achieves one very desirable end in that it automatically purifies the metal, those impurities which depress the freezing point segregating themselves in the upper end of the casting. Professor Saunders was kind enough to make some spectroscopic examinations, and he always found the lower end of the casting purer than the upper end. Considerable quantities of impurity may be eliminated in this way, as I found by intentionally adding an impurity of lead to tin. The high values of the temperature coefficients of resistance of the metals used here and their low specific resistances are evidence of their unusually high purity.

The principal particular in which this method requires further development is in some precise method of control of the orientation of the casting. No positive control was attempted in this work, but a large number of castings were made, and those were selected which most nearly satisfied the requirements. It has been already mentioned that the favorite method of growth is with the principal cleavage plane parallel to the axis of the casting. This orientation was very exactly attained in a number of instances; in the larger castings of antimony, for example, no specimens were found which were oriented in any other way. More oblique incidences are favored by small diameter of the casting and by rapid cooling, so that some degree of control is possible in making a large number of castings, but I have no method of control over any particular individual. It would doubtless be possible to attain such a control by inoculating with a piece previously crystallized selected to have the proper orientation, but with the particular forms of apparatus which I used no simple method of doing this presented itself, and it seemed simpler to make a large number of castings and choose the best.

Having made the casting, it is necessary to determine whether it is really a single grain, and if it is, what is the orientation with regard to it of the crystallographic axes. This is almost as difficult as the production of the single crystal itself, and many attempts were made to find a simple and rapid method. Of course X-ray analysis would give the information desired, but this is prohibitively long and complicated. The method finally adopted was the simplest possible, and consists merely in an examination of the appearance of the casting in a bright beam of parallel light. The surface of the casting as it comes from the mold is covered with microscopic or sub-microscopic pits whose sides are the plane faces of the natural crystal, the pits thus constituting negative crystals. These plane faces reflect the light regularly in definite directions. If the casting is a single grain the entire surface will flash into illumination simultaneously at particular orientations as it is rotated in a beam of light, whereas if there is more than one grain, the different grains flash out separately. The appearance is much like that of the separate grains of an etched metal under a high power microscope. To make this analysis, it is for some metals necessary to examine the casting immediately after removing the glass, as otherwise the surface becomes tarnished. Etching a tarnished surface will sometimes restore the possibility of the detection of the individual grains, but the original cast surface is always far preferable.

The location of the crystallographic axes may most easily be made by means of the reflection pattern. By determining all the positions of regular reflection, the surfaces of the negative crystals are determined, and so from crystallographic data the axes may be found. The method used in locating the positions of regular reflection is a very simple one, for the general idea of which I am indebted to Dr. Langmuir. The source of light is most simply a nitrogen filled lamp in the ceiling of a high studded and partially darkened room. The crystal rod whose axes are to be located is threaded through a wooden sphere, to which it is attached by the light friction of a roll of paper, or by a touch of wax. The rod is now held in the hands at arm's length toward the floor and rotated until a regular reflection flashes out. The orientation with respect to the beam of light is now marked on the sphere by the following device. A piece of plane mirror, the back of which is smeared with a little printer's ink, is held in one hand, the other maintaining the rod in the position of regular reflection, and the mirror rotated until the source of light is also visible in it. The mirror is now touched lightly to the sphere, the ink on the back side marking the position.

From the complete reflection pattern, determined in this way, the location of the axes may at once be found in terms of known crystallographic data. The details of this pattern will be given under the description of the individual metals. The only caution necessary in connection with the method is to avoid marking positions due to double reflections. Such double reflections can at once be identified, if one is on the lookout for them, because they are not confined to a single direction, but occupy a two dimensional locus. I owe this suggestion to Dr. I. C. Gardner.

Before this final method of location of the axes was adopted, a number of other methods were tried, some of which have interest for their own sakes. For example, much information may be obtained from the manner of mechanical deformation. The impression made by a steel sphere, as in the Brinell test, is not round, but shows the characteristic symmetry of the crystal. Tin shows a four sided impression, and bismuth a three sided. By studying enough of these impressions, on differently oriented surfaces, the axes may be located. The depth of the depression also varies on different faces. Another deformational method, better in some respects, is to make the depression with a chisel edge loaded with a fixed weight. The width of the depression varies characteristically with the orientation of the edge, as well as with the orientation of the surface which receives the impression. The flow figures thrown up by the chisel edge at the ends of the impression furthermore yield a good deal of information. I have also tried determining the variations of scratch hardness, both on different surfaces, and in different directions on the same surface. This method also may doubtless be made to yield the necessary information, but it is more complicated, and gives reproducible results less easily. Since it has been a matter of controversy, however, I may record that I have found perfectly measurable and comparatively large differences of scratch hardness on the same crystal face, and also distinct differences between different crystal faces.

A more rapid method of examination, sufficient for many purposes, is to cut from the crystal a short cylinder, true the flat faces, and compress to plastic flow between polished steel plates. The manner of flow will show whether there is one grain, and will also locate the plane of easiest slip, which is usually perpendicular to the principal axis, and is all the identification of direction necessary for resistance or thermal expansion measurements, or for many measurements of the elastic constants.

If the metal shows easy cleavage, as do zinc, bismuth, antimony, and tellurium, the simplest method is to first check the existence of only one grain by the optical method, and then to locate the plane of easiest cleavage by chipping the corners of the castings. The planes of easiest cleavage give the axis of 6 or 3 fold symmetry. If it is necessary to locate more definitely the other axes, the secondary cleavages may be employed.

The optical method is, however, obviously immensely superior to any of the deformational methods alone, for not only is it rapid and does it give the complete crystallographic orientation, but it does not involve the destruction of any part of the specimen.

METHODS OF MEASUREMENT AND COMPUTATION.

Thermal Expansion. The specimens to be measured were usually in the form of cylinders approximately 2.5 cm. long and 6 mm. in diameter. These were mounted in a steel apparatus, and the temperature of the apparatus and the specimen were changed together, so that effectively the difference of expansion between the steel and the specimen was measured. The differential expansion was determined by means of an optical lever. The mirror was mounted on a knife edge arrangement with an effective length of about 0.4 mm., and the scale was placed at a distance of 4 m., thus giving a magnification of 20,000. The telescope was placed close to the mirror, instead of near the scale as usual, thus doubling the size of the image and making very much easier its location.

The temperature was controlled by means of water baths, rapidly stirred. The specimen in the steel holder was attached to a heavy steel bracket, and the water bath was so arranged that it could be raised from below in a few seconds into position about the specimen. The dimensions of the apparatus were small, and the stirring was rapid, so that temperature equilibrium was attained in less than three minutes after adjusting the bath. Two baths were used, one five degrees above and one five degrees below room temperature. The measurements consisted of a number of readings with the two baths alternately at as short intervals as allowed by the equilibrium requirement. No thermostatic regulation was attempted, but the temperature of the bath was recorded corresponding to each measurement. The mean of a number of such readings gave the data for the calculation of thermal expansion. There was a perceptible drift in the readings due to changes of room temperature. The effect of this may be made to disappear from the average by taking an odd number of readings and operating at regular time intervals. In any event, the effect of drift is comparatively small.

It was necessary to design the apparatus with some care in order to avoid error from unequal expansion in that part of the supporting bracket which projects through the surface of the water. This was attained by so mounting the mirror that the only motion transmitted to it through the surface of the water was the rotation of the shaft attached to the knife edges, which were below the surface of the water, the shaft leaving the surface of the water at right angles. A previous attempt to mount the knife edge above the water was a complete failure.

In using the apparatus it is necessary to know both the magnification of the optical lever, and the absolute thermal expansion of the steel of which the holder is composed, which enters by difference into the final result.

The dimensions of the knife edge mounting for the mirror were determined by producing a known amount of motion with a micrometer screw attached to a vernier head, and measuring the motion of the scale. It was very nearly the same arrangement as I have used previously in calibrating the lever of the linear compressibility apparatus, and the results were gratifyingly regular. The relation between motion of the lever and scale deflection is linear within the range of angles used with a maximum departure of any single reading of not more than 1/4000, which is good considering the high magnification.

Having determined the optical magnification, the thermal expansion of the steel envelope may be found by measuring the apparent thermal expansion of a single known substance. Fused quartz was chosen for this purpose because of its very low absolute expansion, and therefore the presumably high percentage accuracy of a result into which this quantity would enter by difference. The linear expansion assumed for the quartz was 5.7×10^{-7}. The expansion found in this way for the steel lies in the conventional range of values for ordinary mild steel given in tables of constants.

The calibrating data are now all at hand, so that from the apparent expansion of any of the crystal specimens the absolute expansion may be calculated. No attempt was made to obtain readings at more than two temperatures, so that any variation of thermal expansion from constancy cannot be detected by these measurements.

In the following the usual notation is adopted for the thermal expansion: α_{11} is the linear expansion parallel to the six-, three-, or four-fold axis, and $\alpha_{\perp}$ is the linear expansion in any direction at right angles.

Electrical Resistance. The specific resistance was determined in several ways, some involving cutting specimens from a larger crystal in definite orientations, but the simplest, and the one most often used, involved merely the measurement of the resistance of a long cylindrical casting, the length being so great compared with the cross section that any deviation of the lines of flow from uniformity and rectilinearity could be disregarded. The disadvantage of this last simple method is that the orientation of any single specimen cannot be controlled, but a considerable number of specimens must be used of as wide a range of orientation as may be obtained, and then by plotting all the results against the orientation the most probable resistance for the axial and the perpendicular direction may be obtained. It is a further disadvantage that the natural castings are not always geometrically perfect; error from this effect is not important, however. The great advantage of the method is that the specimens have been subject to no machining, so that one may be confident that there has been no alteration of the original crystal structure.

The resistance of the unmachined cast rods was determined by a potentiometer method, and in fact the same electrical measuring apparatus was used which I have previously described and used for determining the effect of pressure on the resistance of specimens of small absolute resistance. For many of the measurements the two current connections and the two potential connections were mounted together in a frame with an arrangement so that the whole specimen could be slid along beneath them, thus exploring the length of the specimen for uniformity. The dimensions could be measured with sufficient accuracy with a micrometer.

The measurements of the effect of pressure on the resistance of rods of different orientations was made by methods previously used and already sufficiently described.[4] The current and potential terminals were in most cases attached to the specimen by soldering, the ratio of length to diameter being sufficient so that no error was introduced by end effects.

In addition to these measurements on natural castings, two methods were attempted for measuring the resistance of machined specimens, which perhaps have sufficient interest to warrant some description. The first of these methods employed a cylindrical specimen, about 1 cm. long and 3 mm. in diameter, turned accurately cylindrical with flat ends. The specimen was soldered with low melting solder at the two ends between two cylindrical copper rods of the same diameter, the whole being carefully made so that there were no irregularities in the external surface. Current was passed in and out at the two ends of the copper, and the potential drop over the middle 2 mm. of the specimen was taken off with needle points pressed against it. If the specimen is machined so as to lie along the crystallographic axis, or at right angles to it, the lines of current flow are straight, and the measurements give immediately the specific resistance. When mounted in a rapidly stirred bath of kerosene to avoid thermal currents, this method is capable of considerable precision, but the difficulty arising from the necessity of careful machining and the accompanying danger of damage to the crystal structure did not justify its extended use, and it was in fact used only with tin and zinc.

The second method attempted to measure the resistance of a massive specimen, on which it was necessary to machine only a single flat surface of the order of 1 cm. in diameter. The method consists in pressing against the surface four needle points mounted in the same straight line at fixed distances apart, using the two extreme needles as current terminals and the two inner points for potential terminals. The points must be close together (extreme points 6 mm. apart in the form used) and the specimen large enough so that the flow is essentially that in a semi-infinite solid. The mathematics involved in the use of the method has some interest, and gives a result somewhat paradoxical at first sight. The apparent resistance measured in this way is greatest for that orientation of the four points which lies along the direction of least specific resistance. If R_x, R_y, R_z are the apparent resistances when the line of points is directed along the x, y, and z axes (using surfaces of different orientations if necessary), then the specific resistance along the x-axis is given by

$$\rho_x = \text{Const} \frac{R_y R_z}{R_x},$$

with analogous formulas for the other directions. If the axis of rotational symmetry lies in the surface, measurements on a single surface suffice.

If the flow is two dimensional (thin sheet) instead of three dimensional, it may be shown that the apparent resistance is independent of the orientation of the four points, no matter how great the variation of specific resistance in different directions. The analysis for the two dimensional case has been used in another paper[5] in connection with measurements of the effect of tension on resistance. The three dimensional analysis is mathematically similar, and it is not necessary to give the details here.

An apparatus was constructed on this principle, and measurements with it verified the mathematics, and gave values for the specific resistance agreeing in general with those found by other methods. It appeared, however, that crystalline disorientation due to machining is a particularly important source of error here, as may be expected from the small dimensions of the apparatus and the fact that the current is thereby confined to the immediate vicinity of the surface. The method was applied to bismuth, cadmium, zinc, and tin. In all cases the machining must be done with extreme care, and since one cannot be sure of the result unless verified by other methods, I abandoned it as a serious method of crystal measurement. It would seem, however, that the method may have uses with other metals whose crystal structure is less likely to damage, on occasions when it would be convenient to obtain the specific resistance of a massive specimen of an isotropic metal merely by pressing four points against it and measuring a potential drop.

The resistances obtained by any of the above methods usually require to be reduced to values of more immediate significance. Thus in measuring the resistance of long cast rods we obtain the specific resistance in directions which are inclined to the principal axis. We need to know the connection between the resistance in any direction and the two principal resistances along the axis of rotational symmetry and any perpendicular axis. In measuring the effect of pressure on resistance, we measure with terminals fixed to the specimen, which undergoes deformations under pressure. The coefficients obtained under these conditions we call the coefficient of "measured" resistance. We desire to find from the coefficient of "measured" resistance the pressure coefficient of "specific" resistance, that is, the effect of pressure on the resistance of a piece of metal of invariable shape. The same remarks apply to the temperature coefficient of resistance. We ordinarily determine the "measured" temperature coefficient; we require to find the "specific" temperature coefficient. Finally, in comparing the results found here for single crystals with those of other observers for aggregates of crystals we must know how to find the resistance of a haphazard aggregate in terms of the principal resistances of the individual crystals.

We denote by ρ_{11} the specific resistance parallel to the axis of three-, four-, or six-fold symmetry (which will always be taken as the z axis), and by $\rho_{\perp}$ the specific resistance perpendicular to that axis. We further shall always denote by θ the angle between any specified direction and the z axis.

Voigt[6] has shown that the resistance in any direction in terms of the principal resistances is given by

$$\rho_\theta = \rho_{11} \cos^2 \theta + \rho_{\perp} \sin^2 \theta. \qquad \text{A.}$$

He has also obtained the relation between the resistance of a haphazard aggregate and that of the individual grains

$$\frac{1}{\rho} = \frac{1}{3}\left[\frac{2}{\rho_{\perp}} + \frac{1}{\rho_{11}}\right] \qquad \text{B.}$$

Next consider the connection between the temperature coefficient of specific resistance in any direction and the temperature coefficient of specific resistance along the axes. Denote by $\rho_\theta(t)$ the specific resistance at temperature t in direction θ, and by β_θ the temperature coefficient of specific resistance in the direction θ. Then by definition of temperature coefficient we have

$$\rho_\theta(t) = \rho_\theta(0)\,[1 + \beta_\theta t].$$

By relation A we have

$$\left.\begin{array}{lrl} & \rho_\theta(t) &= \rho_{11}(t)\cos^2\theta + \rho_\perp(t)\sin^2\theta \\ \text{and} & \rho_\theta(0) &= \rho_{11}(0)\cos^2\theta + \rho_\perp(0)\sin^2\theta. \\ \text{Also} & \rho_{11}(t) &= \rho_{11}(0)\,[1 + \beta_{11}t], \\ & \rho_\perp(t) &= \rho_\perp(0)[1 + \beta_\perp t]. \end{array}\right\}$$

Substituting now these four relations into the preceding equation gives

$$\beta_\theta = \frac{\rho_{11}(0)\beta_{11}\cos^2\theta + \rho_\perp(0)\beta_\perp\sin^2\theta}{\rho_{11}(0)\cos^2\theta + \rho_\perp(0)\sin^2\theta} \tag{C}$$

The relation between the temperature coefficients of resistance in different directions is therefore more complicated than that between the specific resistances themselves. An examination of the derivative shows that between $\theta = 0°$ and $\theta = 90°$ β_θ either continually increases or continually decreases. Its rate of change vanishes at $\theta = 0°$ and is a maximum at some intermediate point not in general at 45°.

It is obvious that exactly the same analysis and the same equation C' applies, replacing β by γ, the *pressure* coefficient of resistance.

We now have to consider the connection between the coefficients of measured and specific resistance. We discuss first the pressure coefficient. There are two sorts of deformation under pressure to consider, that due to change of length and cross section of the rods, and that due to change of angle between the rod and the z axis. Consider a rod originally of unit length inclined at θ to the z axis. Under a uniform hydrostatic pressure p the projections of the rod along x and z, which were originally $\sin\theta$ and $\cos\theta$ become $\sin\theta\,[1 - p(s_{11} + s_{12} + s_{13})]$ and $\cos\theta[1 - p(2s_{13} + s_{33})]$ respectively. (See the following section on elastic constants if the notation is unfamiliar.)

We find from this that for unit pressure the decrease of length is

$$-\frac{\Delta l}{l} = \cos^2\theta(2s_{13} + s_{33}) + \sin^2\theta(s_{11} + s_{12} + s_{13}).$$

But the decrease of volume is

$$-\frac{\Delta V}{V} = 2(s_{11} + s_{12} + 2s_{13}) + s_{33}.$$

Now the cross section decreases by $-\left(\frac{\Delta V}{V} - \frac{\Delta l}{l}\right)$. Hence due to change of dimensions only, the resistance would increase by $-\left[\left(\frac{\Delta V}{V} - \frac{\Delta l}{l}\right) - \frac{\Delta l}{l}\right]$, or

$$2s_{11}\cos^2\theta + 2s_{12}\cos^2\theta + 2s_{13}\sin^2\theta + s_{33}(1 - 2\cos^2\theta) \qquad \text{D.}$$

For those metals whose resistance decreases under pressure, the pressure coefficient of measured resistance is to be increased numerically by D in order to obtain the pressure coefficient of specific resistance, and conversely with those metals whose resistance increases under pressure.

Now consider the correction arising from change of angle. By taking the ratio of the projections of the rod above we find that under unit pressure the angle θ changes by

$$\Delta\theta = \sin\theta\cos\theta[s_{33} + s_{13} - s_{11} - s_{12}].$$

Now we have

$$\rho_\theta = \theta_{11}\cos^2\theta + \rho_\perp\sin^2\theta.$$

Whence

$$\Delta\rho_\theta = 2\sin\theta\cos\theta[\rho_\perp - \rho_{11}]\Delta\theta,$$

and substituting,

$$\Delta\rho_\theta = 2(\rho_\perp - \rho_{11})\sin^2\theta\cos^2\theta[s_{33} + s_{13} - s_{11} - s_{12}]. \qquad \text{E.}$$

The resistance changes by this amount because of change of angle; we correct for it by subtracting $\Delta\rho_\theta$ from the measured change of resistance. In general, the correction E is much smaller than D, and is usually just on the edge of the negligible.

Analysis similar to the above applies to the temperature coefficient, replacing $s_{11} + s_{12} + s_{13}$ by $-\alpha_\perp$, and $2s_{13} + s_{33}$ by $-\alpha_{11}$. We thus find that

$$2\alpha_\perp\cos^2\theta + \alpha_{11}(1 - 2\cos^2\theta) \qquad \text{F.}$$

is to be added to the measured temperature coefficient of resistance to correct for the change of dimensions, and

$$\frac{2(\rho_\perp - \rho_{11})}{\rho_0} \sin^2\theta \cos^2\theta \,[\alpha_{11} - \alpha_\perp] \qquad \text{G.}$$

to correct for the change of angle, the result obtained after applying the correction being the temperature coefficient of specific resistance in the direction θ.

Experimentally the procedure is usually as follows. The specific resistance is measured in a number of directions, and a curve passed through the points by equation A, from which the best values of ρ_{11} and $\rho_\perp$ are obtained. The temperature (or pressure) coefficient of resistance is now measured for two different directions as close as possible to the two axes, and from these results by equations F and G (or D and E) the specific coefficient found in the two directions. These results are now substituted into equations of the type C, giving two equations for calculating the two unknowns β_{11} and $\beta_\perp$ (or γ_{11} and $\gamma_\perp$).

We may find from formula B that the temperature coefficient of specific resistance of a haphazard aggregate is connected with that of the grains by the relation

$$\frac{\beta}{\rho} = \frac{1}{3}\left[\frac{2\beta_\perp}{\rho_\perp} + \frac{\beta_{11}}{\rho_{11}}\right], \qquad \text{H.}$$

with of course a similar formula for the pressure coefficient.

This analysis suggests an interesting possibility with regard to determining the change of the elastic constants under high pressures. By measuring the linear compressibilities we obtain $s_{11} + s_{12} + s_{13}$ and $2s_{13} + s_{33}$ as functions of pressure. Suppose now we measure the effect of pressure on the resistance of rods of four orientations, two of them along the axes. The linear compressibilities enable us to correct the measured coefficients along the axes to specific coefficients. Formula C now enables us to calculate the specific coefficients for the two other directions. But these may also be calculated in terms of the measured coefficients in those directions and the combination of constants $2(s_{11}+s_{12})-s_{33}$ and $2s_{13} + s_{33}$. Comparison of the two results gives the values under pressure of the last two combinations of constants. One is already known from the linear compressibility, and therefore can be used only as a check, but we have in all, three relations from which the behavior under pressure of

s_{33}, s_{13}, and $s_{11} + s_{12}$ may be found. Probably in practise the accuracy demanded in the measurements would be very high.

Elastic Constants. The linear compressibilities were measured by methods already described and applied to a large number of cubic metals.[3] The identical apparatus was used. In general the specimens were machined from a large casting to have a diameter of the order of 6 mm. and a length of 2.5 cm. The proportions are such that there is comparatively little danger of damaging the crystal structure to any depth if the machining is carefully done. The rough rods were usually cut from the large casting with a jeweller's saw by hand, and the machining was done in a jeweller's lathe, run at a high speed and taking a very fine chip. In some cases it was necessary, in order to attain the requisite length, to build up the specimen of two pieces, placed end to end. The same sort of procedure has already been used for other metals, and introduces no error in measurements of this sort, although it does certainly introduce error into other sorts of measurement, as Young's modulus by a compression method, for example. These measurements of linear compressibility are susceptible of a relatively high degree of accuracy, much greater than the measurements of any of the other elastic constants. One reason, of course, is that an elastic limit does not exist for hydrostatic pressure, so that a comparatively large deformation may be produced by high pressures. It is a matter of experiment that the relation between deformation and pressure is linear over so wide a range of pressure that a real increase of accuracy is obtained in this way. There is furthermore a very simple check on the linear compressibility measurements in that the sum of the three linear compressibilities in three mutually perpendicular directions should agree with the cubic compressibility as determined by other methods.

The symmetry of the linear compressibility is the same as that of electrical resistance or thermal expansion, so that only two specimens are necessary, directed along and at right angles to the principal crystallographic axes. Other specimens oriented in different directions would merely give a check on measurements already made.

In discussing further the methods of determining all the elastic constants, it will pay to have before us the detailed relations between stress and strain for the various crystal systems measured here.

For the hexagonal system (zinc and cadmium) there are five constants, and the relation between stress and strain, using the familiar notation of Voigt is:

$$\begin{array}{lllllll}
e_{xx} = & s_{11}X_x & + s_{12}Y_y & + s_{13}Z_z & + 0Y_z & + 0Z_x & + 0X_y, \\
e_{yy} = & s_{12}X_x & + s_{11}Y_y & + s_{13}Z_z & + 0Y_z & + 0Z_x & + 0X_y, \\
e_{zz} = & s_{13} & + s_{13} & + s_{33} & 0 & 0 & 0, \\
e_{yz} = & 0 & 0 & 0 & + s_{44} & 0 & 0, \\
e_{zx} = & 0 & 0 & 0 & 0 & s_{44} & 0, \\
e_{xy} = & 0 & 0 & 0 & 0 & 0 & 2(s_{11} - s_{12}).
\end{array}$$

For the trigonal system (bismuth, antimony, and tellurium) there are six constants, and the relation between stress and strain is:

$$\begin{array}{lllllll}
e_{xx} = & s_{11}X_x & + s_{12}Y_y & + s_{13}Z_z & + s_{14}Y_z & + 0Z_x & + 0X_y, \\
e_{yy} = & s_{12} & + s_{11} & + s_{13} & - s_{14} & 0 & 0, \\
e_{zz} = & s_{13} & + s_{13} & + s_{33} & 0 & 0 & 0, \\
e_{yz} = & s_{14} & - s_{14} & 0 & s_{44} & 0 & 0, \\
e_{zx} = & 0 & 0 & 0 & 0 & s_{44} & 2s_{14}, \\
e_{xy} = & 0 & 0 & 0 & 0 & 2s_{14} & 2(s_{11} - s_{12}).
\end{array}$$

For the tetragonal system (tin) there are six constants, and the relation between stress and strain is:

$$\begin{array}{lllllll}
e_{xx} = & s_{11}X_x & + s_{12}Y_y & + s_{13}Z_z & + 0Y_z & + 0Z_x & + 0X_y, \\
e_{yy} = & s_{12} & s_{11} & s_{13} & 0 & 0 & 0, \\
e_{zz} = & s_{13} & s_{13} & s_{33} & 0 & 0 & 0, \\
e_{yz} = & 0 & 0 & 0 & s_{44} & 0 & 0, \\
e_{zx} = & 0 & 0 & 0 & 0 & s_{44} & 0, \\
e_{xy} = & 0 & 0 & 0 & 0 & 0 & s_{66}.
\end{array}$$

The z axis is here the axis of six-, three-, or four-fold symmetry. The x and y axes, at right angles to the z axis, are determined in a way which will be further specified when necessary under the description of the individual metals.

Physically the constants fall into three different groups. We may call the "rectangular" constants those whose subscripts are any combination of the numbers 1, 2, 3. These constants give the strains of elongation produced by stresses which act normally to their planes. The "shearing" constants are those whose subscripts are combinations of the numbers 4, 5, 6. Physically they determine the shearing strains produced by shearing stresses (stresses acting tangentially to their planes). The "cross" constants are the remaining ones, whose subscripts are chosen one from the group

1, 2, 3, and one from the group 4, 5, 6. These constants determine the elongation produced by shearing stresses, or the shearing strains produced by normal tractions. There is only one of these cross constants in the group of metals above, namely s_{14} of the trigonal system. Different methods are necessary in measuring the constants of the different groups.

In selecting the methods for determining these constants I have been guided by two desires; first, to make the method as direct as possible, measuring if possible the various constants directly instead of various combinations of them, and secondly, to obtain check measurements as abundantly as possible. Obtaining checks is of considerable importance if one is to have confidence in the results.

We first consider the rectangular constants. The arrangement of these is exactly the same for the three crystal systems, and the same methods of measurement apply to all. It is in the first place evident that the linear compressibilities give the two combinations $s_{11} + s_{12} + s_{13}$ and $2s_{13} + s_{33}$. The cubic compressibility is given by the combination $2(s_{11} + s_{12} + 2s_{13}) + s_{33}$. To completely determine the rectangular constants we need two more relations. The simplest method is to determine the constants directly. Thus it is evident that if a specimen is cut lying along the x axis and is subjected to a simple compression (stress X_x) the longitudinal contraction e_{xx} gives at once the constant s_{11}, and the two lateral expansions e_{yy} and e_{zz} the constants s_{12} and s_{13}. This was in fact the method adopted in many cases. Here the advantage of procuring large single crystals is evident, because it permits the use of specimens large enough to measure the lateral expansions. This has seldom been done previously, even by Voigt in measuring many non-metallic crystals. It is evident on examining the equations that for this kind of distortion there is rotational symmetry about the z axis, so that measurement of the three strains on any rod cut at right angles to the z axis gives directly the three constants, and it is not necessary to locate the x and y axes, but only the z axis. The remaining rectangular constant s_{33} may be obtained by a measurement of the longitudinal contraction under a compressive stress Z_z applied to a specimen whose length lies along the z axis. The lateral expansion of such a specimen in any direction gives again s_{13}; there is only one independent lateral expansion for such a specimen. Complete carrying out of the scheme outlined above would thus give eight measurements for the four rectangular constants (including the cubic compressibility of other observers). As a matter of fact this was not usually done because of the difficulty of preparing a specimen of sufficient size lying along the z axis. The reason for this is that the plane of easiest cleavage or slip usually lies perpendicular to the z axis, so that the specimen is very likely to break or be damaged in preparation. Usually therefore, I have obtained essentially only six measurements for these four rectangular constants, two linear compressibilities, one cubic compressibility (other observers) and three direct measurements under a simple compressive stress X_x. In many cases these last measurements were supplemented by measurements of the extension under tension of a rod lying along the x axis, this rod being the natural casting and unmachined and long enough to get rid of end effects. I did not measure the lateral contraction of these rods under tension. Neither did I attempt the measurement of any of the rectangular constants by the bending of rods. This has been the favorite method, but it is much less direct, and it seemed to me much less desirable in cases like the present where the direct measurements can be made.

The shearing constants involve a shearing stress. Unfortunately there seems no simple mechanical way of applying a simple shearing stress analogous to the simple normal tractions which may so easily be applied in measuring the rectangular constants, and a more indirect method has to be used. The best is without doubt the usual method of measuring the twist of a cylindrical rod under a torque. The method is more or less indirect because the state of strain is not uniform throughout the rod, but is greatest at the outside surface and least at the inside. The formulas are comparatively simple, however, and the results of measurements are not open to much question if the material is homogeneous, as it must be in the case of these crystals. The amount of twist of a rod of given dimensions under a given torque depends on the orientation of the rod with respect to the z axis, and in general involves all the elastic constants. The formulas are given in Voigt's book. The specimen is chosen by preference so that the combination of constants is as simple as possible

For the hexagonal and trigonal systems there is only one independent shearing constant s_{44}, and a single observation of the twist of a rod of known orientation suffices to determine it. The most convenient orientation is with the z axis perpendicular to the length of the rod, (principal cleavage plane parallel to the length) and in this case the torsion constant is given by $s_{11} - s_{12} + \frac{1}{2}s_{44}$. Since s_{11} and s_{12} are supposed already known, the constant s_{44} may at once be found. If it were possible to observe the torsion of rods whose length is parallel to the z axis, the constant s_{44} could be obtained directly without connection with the other constants, but it has already been explained that the preparation of such rods is usually difficult (except in the case of tellurium for which it is easiest). The first method does not lead to a bad value for s_{44}, however, because the constants s_{11} and s_{12} are usually considerably less than s_{44}, so that although s_{44} is obtained by difference, the difference involved is not the difference of two numbers closely equal to each other. A check on the value of s_{44} is given by torsion measurements on rods of other orientations or by repeating the measurements on several rods of the same orientation. A number of such check measurements were usually made, the details of which will be given under the separate metals.

The tetragonal system differs from the others in having two independent shearing constants. These must be determined by torsion observations on rods of two different orientations, and the check measurements by observations on still other orientations. There is an inconvenient complication here in that it is necessary to know the orientation of the x and y axes in addition to the relative position of the z axis and the axis of the rod. The details will be given later.

There is only one cross constant among the metals measured here, s_{14} of the trigonal system. Its measurement is complicated. Direct measurement on a specimen cut so as to lie along one of the crystallographic axes would involve the measurement of an extension under a shearing stress or a shear under a normal traction, neither of which can conveniently be made on small specimens. An indirect method is therefore necessary. The simplest method seems to be the measurement of the longitudinal extension under tension of a rod cut at an angle other than 0° or 90° with the z axis. The formulas given by Voigt show that in general under such conditions the extension involves the constant s_{14}. The orientation of the x and y axes also enters the result. For one direction of orientation the effect of s_{14} on the extension may be a maximum, while for a change of orientation of only 30° the effect disappears entirely. With a period so short one would expect the influence of s_{14} at best to be comparatively slight, and it is in fact difficult to determine accurately under the best conditions, being given by the difference of numbers nearly equal. Furthermore, the best conditions are rather exacting, and among a large number of castings I found only a few that were suitable for the attempt. A check on the values of s_{14} is given by observations on different specimens at different orientations.

Experimental Methods of Measuring Strain. There was nothing especially novel in the methods employed. The general method for measuring changes of length, either longitudinal or transverse, was some adaptation of the rocking knife edge with mirror and scale. High magnifications were usually necessary, which for the transverse expansions sometimes rose to 1.2×10^5. Various arrangements for loading, either direct or through a lever, were adopted, depending on the dimensions of the specimens. Transfer pieces of varying dimensions were used for transferring the change of length to the rocking lever. In measuring the change of length, the shortest effective length employed was 0.4 cm. and the longest 10 cm. The transverse expansions were usually measured on pieces 6 mm. or 1.2 cm. in diameter.

Considerable difficulty was experienced in measuring the longitudinal compression under a one-sided compressive force. Short specimens are necessary, as long ones will buckle. There is considerable difficulty with short specimens because of the necessity of perfect end conditions. I was not successful in compressing a specimen with flat ends between plungers with flat ends in spite of extreme care in machining both the specimens and the plungers. The method by which success was attained was to cement with a very small amount of soft cement to each end of the specimen a cylindrical steel block, accurately concentric with it, in the center of the other side of which was a depression for a small steel ball (1.6 mm. in diameter), which rested also in a depression in the center of the plunger. In this mounting the specimen is free to adjust itself for any irregularities, and consistent results were obtained. The plungers, of course, must be rigidly held to avoid side play.

The twist was measured with an instrument of obvious design. The torque was applied through an arm working practically without friction on knife edges, and the amount of twist was measured with two telescopes and scales set on mirrors attached to the two ends of the specimen.

In all cases the deformation was measured between actual points situated on the specimen. Early attempts to measure the deformation between points not on the specimen itself, as by measuring the the longitudinal compression by measuring the approach of the steel plungers with which the compression was produced, were a failure. Attempts were also a failure to utilize specimens built up of two pieces, although this is possible in measuring linear compressibility under hydrostatic pressure or thermal expansion, there being in these cases no canghe of stress across the opposing surfaces.

Due regard was paid to the question of end effects, and the measuring appliances were always situated far enough from the ends to avoid error from this effect.

In all these measurements the most easy source of error arises from exceeding the elastic limit of the crystal, which is often unbelievably low. Care must be taken that the measuring instrument itself, where applied to the specimen, does not locally exceed the elastic limit. This was a particular source of trouble in measuring the transverse expansion. In the early devices for this measurement, motion was transferred to the mirror by steel points pressed against the specimen. The points had to be replaced by flat bearing surfaces of steel, maintained in position parallel to themselves by arrangements which need not be described here in detail.

DETAILED DATA.

Tungsten. Tungsten belongs to the cubic system, and therefore properly does not belong within the range of material intended here. But since I was fortunate enough to obtain a good crystal and the measuring devices were already developed, the opportunity was not to be lost. Being cubic, the properties of tungsten are of special simplicity. The main questions are whether the properties of the single crystal differ appreciably from those of the drawn wire on which most measurements have been made, and what are the *three* elastic constants.

I owe the crystal to the kindness of Dr. Langmuir of the General Electric Co. who obtained it from the German General Electric Co. It was in the form of a rod 7 or 8 cm. long and 7 mm. in diameter. It had been recrystallized from the sintered rod by heating in vacuum for several hours to a temperature near the melting point. When heated in this way for a long time, recrystallization starts at some point and proceeds rapidly until the entire rod is converted into a single crystal grain, except the extreme ends which are in contact with the supports. After recrystallization, the surface of the rod was etched by the admission of a slight amount of water vapor to the vacuum chamber, thus bringing out the crystal structure. By the optical test the axes could be readily located. The reflection pattern is that of the dodecahedral faces corresponding to the edges of the fundamental cube. The orientation was such that ten of the twelve faces could be identified; the angles of reflection corresponding to the other two faces were unfavorable. For the measurements contemplated it was necessary to completely determine the location of all three cubic axes. The dodecahedral faces are designated as (1 1 1) and (1 1 0) faces. This fixes the x, y, and z axes with respect to the reflection pattern. The z' axis was chosen along the axis of the cylinder, and the x' and y' axes arbitrarily at right angles. The scheme of angles for this crystal is as shown.

	x	y	z
x'	90°.0	87°.0	3°.0
y'	29.0	61.0	89.0
z'	61.0	29.6	87.5

In deriving this scheme all nine angles were independently measured, and then such readjustments made as were necessary to satisfy the orthogonality conditions. Four of the nine angles were readjusted, the maximum change being 2.5° and the average change (for the four) 1.4°.

After determining the orientation, the crystal was ground to a geometrically perfect cylinder. This was necessary because the rod was bent somewhat, having sagged between the electrodes during the heating before recrystallization. Of course the crystal structure was linear, independent of the curvature of the rod. The grinding destroyed the reflection pattern, so that it was necessary to determine the pattern before grinding.

The linear compressibility was measured in the regular way with the apparatus which in my previous description I have designated as the lever apparatus for long specimens. The length of the finished cylinder was 6.3 cm. The measurements were entirely satisfactory, and show as much accuracy and consistency as any of the compressibility measurements by this method, and this is in general a tenth of a per cent or better. The linear compressibility was found at 30° and 75°, from which the volume compressibility is:

$$-\frac{\Delta V}{V_0} = 3.18 \times 10^{-7}p - 1.4 \times 10^{-12}p^2 \qquad \text{at } 30^\circ$$

$$= 3.18 \times 10^{-7}p - 1.5 \times 10^{-12}p^2 \qquad \text{at } 75^\circ,$$

the unit of pressure being the kg/cm².

Tungsten belonging to the cubic system has only one independent linear compressibility, so that this is all the information about the three elastic constants that can be obtained by measurements under hydrostatic pressure.

These results for compressibility may be compared with results previously found. The result for the single crystal agrees within 1% with that previously found for drawn wire of high density, the compressibility of the crystal being slightly greater. Not only is the initial compressibility the same as found before, but the second degree term is also nearly the same.

Other relations between the elastic constants were obtained by the following measurements: (1) Longitudinal compression under a one-sided compressive stress applied lengthwise of the rod. The constant for this deformation, that is the longitudinal compression per unit length per unit stress per unit area, is designated by Voigt by s_{33}', who gives the general formulas from which the special value for the cubic system in terms of the direction cosines and the three principal elastic constants may be deduced. (2) Lateral contraction under longitudinal compression along the x' and y' axes. These constants are designated by Voigt by s_{12}' and s_{13}', and the formulas are given. (3) Torsion about the axis of the cylinder. The constants for this deformation (twist per unit length, etc.) is designated by Voigt by $s_{44}' + s_{55}'$; the special value for the cubic system may be found from Voigt's formulas. Substituting now into the equations derived from Voigt the experimental values for the constants for the various directions and the numerical values of the direction cosines, and using the results found for the linear compressibility in dyne units, gives the following set of five equations to determine the three elastic constants.

$$s_{11} + 2s_{12} = 1.082 \times 10^{-12}, \tag{1}$$

$$.636s_{11} + .364s_{12} + .182s_{44} = 2.54 \times 10^{-3}, \tag{2}$$

$$.359s_{11} + .640s_{12} - .179s_{44} = -8.39 \times 10^{-14}, \tag{3}$$

$$.0038s_{11} + .977s_{12} - .002s_{44} = -7.14 \times 10^{-14}, \tag{4}$$

$$1.456(s_{11} - s_{12}) + 1.272s_{44} = 13.0 \times 10^{-13}. \tag{5}$$

These equations are not entirely consistent, and we have the problem of the best method of determining the most probable value of the constants. The method I have adopted is to solve equations 1 and 4 for s_{11} and s_{12} (1 is by far the most accurate of the equations and the s_{12} term in 4 greatly preponderates), and then to average the values which these two values give for s_{44} when substituted in the three remaining equations, weighing according to the coefficient. As a matter of fact, this method had to be applied by successive approximations, first neglecting the s_{11} and s_{44} terms in 4, carrying through the solution as suggested, and then correcting the value of s_{12} by the small correction terms which are given with sufficient accuracy by the first approximation.

The degree of consistency of the measurements, and the accuracy of the final result is to be judged by the closeness with which the equations 2, 3, and 5 are satisfied by the final values. These are the final values adopted:

$$s_{11} = 2.534 \times 10^{-12},$$

$$s_{12} = -.726 \times 10^{-12},$$

$$s_{44} = 6.55 \times 10^{-12}.$$

Substituted in 5 these give 8.34 on the left hand side against 8.25 on the right; into 2, 2.536 against 2.54; and into 3, —.727 against —.839.

This check is as close as could be expected in view of the accuracy of the readings. The sensitiveness of the readings for an individual determination of a deformation other than a linear compressibility did not correspond to an accuracy greater than 2 or 3%; the mean of ten readings may have possibly improved the accuracy three times.

It is interesting to see how nearly these constants satisfy certain significant relations. In the first place, if the material were isotropic, and characterized by only two elastic constants instead of three, the shearing constants would be connected with the two rectangular constants by the relation $s_{44} = 2(s_{11} - s_{12})$. Substituting the values above for s_{11} and s_{12} gives 6.52 against 6.55×10^{-12} for s_{44}. Hence for this substance it would appear that the isotropic relation is probably satisfied within the limits of error.

In the second place we have the relation known as Cauchy's relation, which has been of considerable historical importance, but is not now to be taken seriously, deduced on the hypothesis that the forces between atoms act in the line of centers. This relation, which is between the elastic moduli instead of between the constants, is $c_{12} = c_{44}$. The moduli may be computed from the constants above by well known equations, and are: $c_{11} = 5.13 \times 10^{12}$, $c_{12} = 2.05 \times 10^{12}$, and $c_{44} = 1.53 \times 10^{12}$. Cauchy's relation therefore fails within a margin considerably greater than the experimental error.

The only other constant determined for this tungsten was its specific electrical resistance. The potentiometer method was used. In order that the current flow might be in straight lines, which would not occur in a specimen of these dimensions if the usual point or single contact electrodes were used, the electrodes were made of flat pieces of copper, amalgamated, and pressed against the ends of the cylinder so as to be in contact over the entire plane ends. The specific resistance at 20.1° C. was found to be 5.48×10^{-6}. This is a trifle lower than the value 5.51 at 20° given by Langmuir for ordinary tungsten wire. The difference is in the direction one might expect, the conductivity of the crystal being better because of the more regular arrangement, but is in the opposite direction from that which might be expected from the difference of density.

Zinc. This metal is very easy to crystallize, and a large number of experiments were made on rods varying in size from 2.5 mm. to 2.2 cm. in diameter. The preferred manner of growth is with the basal plane parallel to the axis of the cylinder. This is in accord with what might be expected, because it is known that the separation of the atoms is greatest along the hexagonal axis, and therefore their density is greatest in the basal plane, so that they might pile themselves into the crystal most rapidly in this direction, the forces being greatest because of proximity. Practically all the castings of 1 cm. or more diameter had the basal or principal cleavage plane within a few degrees of parallel to the axis of the casting, but a number of specimens of the small size (2.5 mm.) were obtained in which the angle of inclination was sometimes as great as 83°. Because of the ease of growing these crystals I did not experiment much with variations in the rate of lowering the mold from the electrical furnace, but confined myself to the slower rates, between 1.6 and 0.4 cm. per hour. Crystals 1 cm. in diameter were readily produced by the more rapid rate, and the only attempt I made at the largest size was successful with the slowest rate.

Zinc from three different sources was used, all supposed to be of high chemical purity. Specially refined zinc from Eimer and Amend, melting point zinc from the Bureau of Standards, and the purest zinc from Kahlbaum. The Bureau of Standards zinc has been used in some of my previous work, and I have given the analysis in detail. The total purity claimed was 99.992 zinc, by difference. I have no special analysis for the Eimer and Amend zinc, but the purity claimed for it is as great as this. It is to be noticed, however, that this analysis does not include the oxide, and there was doubtless more or less present in both these zincs. I have not the analysis of the Kahlbaum material, but its electrical conductivity was consistently higher than that of the other two varieties, which were in close agreement, and is therefore presumably of higher purity, the difference doubtless being due to oxide. There was no difference evident between zinc from the different sources in other than the electrical properties, and they could be used interchangeably in the elastic measurements. There is however, an obvious difference in properties which were not specially investigated here, namely the phenomena of plastic flow. The Kahlbaum zinc was distinctly more plastic than the other varieties. It is evident that the highest purity is needed here, and that a comparatively slight amount of impurity by acting as " keys " between the slip planes (see Jeffries[7]) may very materially alter the flow phenomena.

The reflection pattern of zinc is that of a truncated double hexagonal pyramid, the planes which are usually developed being the (0 0 0 1), basal, and the (1 0 $\bar{1}$ 1) planes. The relative development of the latter depends somewhat on the rate of cooling. Because, however, of the very perfect cleavage of zinc it is possible in many cases to dispense with a complete optical examination. The examination by the cleavage planes is easily made as follows. At each end of the cylindrical casting, within a couple of mm. of the end, a saw cut is made girdling the casting and reaching half way to the center. The flange thus formed is broken through at various places by driving into it a steel point in the direction of the cylindrical axis. Rupture takes place along the cleavage plane. Similar orientation of the cleavage plane at all places gives the criterion for a unique crystal grain, and at the same time the plane locates the hexagonal axis, which is perpendicular to it. It is not necessary in order to determine those properties of hexagonal crystals investigated here to know further the location of the x and y axes with respect to the hexagonal axis, so that the location of the cleavage plane completely suffices.

A fresh metal cleavage surface is a strikingly beautiful thing, and it was possible to obtain them in great perfection from zinc.

Thermal Expansion. The samples used were the same as those whose linear compressibility was also measured. They were of Bureau of Standards zinc, cut from a casting 1.2 cm. in diameter. The specimen whose cleavage plane was parallel to the length was in one piece, but the one with length perpendicular to the cleavage planes was in two pieces. It has already been explained that this introduced no error in such cases as this.

At room temperature, over a mean range of 5° on each side, the linear expansion was found to be:

$$\alpha_{\perp} = 12.6 \times 10^{-6}$$

$$\alpha_{\parallel} = 57.4 \times 10^{-6}.$$

Both of these values are less than those found recently by Grüneisen and Goens,[2] which are 14.2 and 63 respectively. Their diagram is not altogether clear, however, and would seem to indicate that for another specimen than that for which they made complete measurements the constants may be less.

The extreme difference of the values in the two directions is the feature of chief interest here.

Electrical Resistance. Measurements of the specific resistance were made by the three methods described in the introduction. Of course those on the unmachined castings were the best. Measurements on about forty different specimens were made in all. As already mentioned, the specific resistance of the Eimer and Amend and the Bureau of Standards zinc was consistently higher by about 3% than that of Kahlbaum. The results obtained with Kahlbaum's zinc are the only ones retained in the final results, but the other measurements confirm these in giving the same ratio for the resistance parallel and perpendicular to the axis.

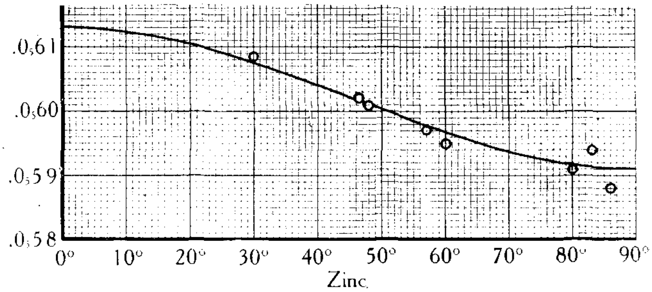

FIGURE 2. The specific resistance at 20° C. of zinc as a function of the angle between the hexagonal axis and the direction of flow.

The specific resistances of rods of Kahlbaum's zinc at various inclinations to the axis are shown in Figure 2. These rods were all about 6 mm. in diameter. None were obtained with the cleavage plane more inclined to the cylindrical axis than 55°, but this is sufficient to give the resistance for the 90° inclination by formula A of the introduction.

From this curve the following values may be deduced for the specific resistances at 20° C.:

$$\rho_\perp = 5.91 \times 10^{-6}$$

$$\rho_{11} = 6.13 \times 10^{-6}$$

The ratio of the two resistances is 1.036.

The agreement with the values of Grüneisen and Goens is not as close as might be desired. They find for $\rho_\perp$ at 20° 5.84×10^{-6}, which does not differ greatly, but for the ratio of the two resistances 1.080. The average resistance for all directions is 5.98×10^{-6} according to my measurements, and 5.99 according to theirs. The difference is small, but is not in that direction which would be accounted for by greater impurity of my zinc. Grüneisen and Goens do not give the details of the sources of their zinc, but at least part of it was Kahlbaum, the same as mine. Neither do they give the details of their measurements of specific resistance, so that we have no means of estimating the probable accuracy of their results.

The temperature coefficient of resistance of three pieces was measured between 0° and 90°, with the following results:

Angle of hexagonal axis with axis of specimen	Average temperature coefficient 0°–100°
7.°	0.00422
28.5	.00409
90.	.00412

The source of the 28°.5 specimen was the Bureau of Standards, whereas that of the other two was Eimer and Amend. Discarding the 28°.5 observation, the coefficient parallel to the axis appears to be probably about 0.24% greater than at right angles. Unfortunately I have no measurements on the temperature coefficient of the Kahlbaum zinc. The results found above agree well with those of Grüneisen and Goens. who find a coefficient greater by 0.28% along the axis.

The effect of pressure on the resistance was measured on three samples of varying orientation at 0° and 95° C. Two of them were Eimer and Amend's zinc, and the third (28°.5) was from the Bureau of Standards. They were rods 2.5 mm. in diameter, and were used without machining. The conventional potentiometer method was used; the terminals were soldered to the specimens. The total length between potential terminals was from 4 to 6 cm. The measurements were in every respect as good as those which I have previously obtained with non-crystalline samples. The average deviation from a smooth curve of the individual readings with the 7° sample was 0.17% of the total change, with the 28°.5 sample 0.20%, and with the 90° sample 0.64%. The following results were obtained:

Angle of Hexagonal axis with axis of Rod	Measured Changes of Resistance 0° C. $-\Delta R/R_0$	95° C. $-\Delta R/R_{95}$
7°	$(11.68p - 11.72 \times 10^{-5}p^2) \times 10^{-6}$,	$(11.58p - 12.65 \times 10^{-5}p^2) \times 10^{-6}$
28°.5	$(9.965p - 9.41 \times 10^{-5}p^2) \times 10^{-6}$,	$(10.127p - 10.86 \times 10^{-5}p^2) \times 10^{-6}$
90°.	$(5.258p - 5.97 \times 10^{-5}p^2) \times 10^{-6}$,	$(5.316p - 6.19 \times 10^{-5}p^2) \times 10^{-6}$

The unit of pressure is here 1 kg/cm². The variations of pressure coefficient with direction are seen to be much larger than those of temperature coefficient of resistance or of resistance itself. The decrease under pressure is greatest parallel to the hexagonal axis, which is also the direction of greatest resistance.

The results above are for "measured" resistance. We may convert these to changes of specific resistance by the formulas of the introduction. Correcting the results at 0° C., we have for the initial pressure coefficients (the second degree term in the pressure coefficient cannot be corrected with the data at hand because we do not know the variation of the elastic constants with pressure):

Hexagonal axis 90° with length	-6.55×10^{-6}
" " 28.5 " "	-9.54
" " 7 " "	-10.80

Using these coefficients of specific resistance at 90° and 7° we find with the aid of formula C in the introduction that the initial pressure coefficient of specific resistance at 0° C. parallel to the hexagonal axis is -10.87×10^{-6}.

Correcting the temperature coefficients of measured resistance, we obtain the following results:

Temp. coeff. of sp. resis. parallel to hex. axis	0.00419
" " " " " perpendicular " "	0.00418

Within the limits of error, the effect of temperature on specific resistance is probably the same in every direction.

Elastic Constants. The linear compressibility was measured in the regular way with the lever apparatus for short specimens. Measurements were made perpendicular and parallel to the hexagonal axis. Both specimens were machined carefully from a large casting, the diameter of each being about 6 mm. and the length 2.6 cm. The perpendicular specimen was in one piece, but the parallel specimen was in two. It was found that the compressibility perpendicular to the axis is very nearly the same as that of steel. Since the method is a differential method, this means comparatively large irregularities in the measured value of the difference of compressibility, but the compressibility itself should not have any greater percentage inaccuracy than usual for this reason.

Measurements on each of these specimens to 12000 kg. at 30° and 75° C. gave the following results:

Length parallel hex. axis

$$-\frac{\Delta l}{l_0} = 12.98 \times 10^{-7}p - 5.32 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$
$$= 13.55 \times 10^{-7}p - 7.82 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

Length perpendicular hex. axis

$$-\frac{\Delta l}{l_0} = 1.946 \times 10^{-7}p - 1.11 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$
$$= 2.025 \times 10^{-7}p - 1.47 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

From these we obtain for the cubic compressibility

$$-\frac{\Delta V}{V_0} = 16.87 \times 10^{-7}p - 8.08 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$
$$= 17.60 \times 10^{-7}p - 11.35 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

This value for the cubic compressibility is to be compared with that of Richards[8] (17×10^{-7}), and of Adams, Williamson, and Johnston[9]

(17.1 × 10^{-7}), the latter value corrected using my recent value for the compressibility of iron. The agreement is to be considered rather good.

We have also to comment on the agreement between these results and the preliminary values which I have given in my paper on the compressibility of 30 metals. These preliminary measurements were only rough, and entirely satisfied their main purpose of bringing out the very great differences of linear compressibility in different directions. The specimens were not precisely oriented with respect to the crystal, so that it was not possible from the preliminary measurements to calculate the compressibility along the axes. However, since the measurements were made in three mutually perpendicular directions, the sum of the changes of length previously found should check with the change of volume given by the present measurements. The changes found were respectively 1.66 × 10^{-7}, 5.30, and 7.13. The sum of these is 14.90 × 10^{-7}, very appreciably less than the value found above. The explanation of the discrepancy I have found to be that the former specimen was not composed of a single grain, but was two, inclined at different angles. X-ray analysis had shown only one grain, but this analysis was made by the reflection method on one face, and was not able to show the second grain in the back part of the specimen. It is interesting that the lowest value found previously for the linear compressibility is less than the minimum for any direction in the single crystal; this is evidently a result of the constraint exerted on the one grain by the other.

The linear compressibilities give two relations between the five elastic constants. These follow in equations 1 and 2 in Abs. C. G. S. units. To obtain the other constants the following measurements were made. On one machined specimen 1.27 cm. in diameter the transverse expansion under a one-sided compression was measured, giving direct values for s_{13} and s_{12}. (Equations 3 and 4.) On another machined specimen, also 1.27 cm. in diameter, measurements were made of the longitudinal compression under a one-sided compressive stress and of the transverse expansion in two directions, giving direct values for s_{11}, s_{12}, and s_{13} (equations 5, 6, and 7). On so large a diameter as 1.27 cm. no appreciable effect from the machining is to be feared. Torsion measurements were made on 12 different rods, approximately 6 mm. in diameter, and inclined at various angles. These rods fall into three groups; in one group the hexagonal axis is nearly at 90° with the axis of the rod, in another the angle is nearly 50°, and in a third group, consisting of a single specimen, the angle is 7°. The variation of angle within the groups is so little that the average torsion coefficient within the group may be correlated with the average angle of inclination. These three groups, when averaged in this way, give three equations each containing the shearing constant s_{44}, equations 8, 9, and 10 respectively. In taking the averages, one result in the 50° group was discarded. In the 90° group the extreme range of the observed values was from 40 to 50, and in the 50° group from 62 to 69. Finally there are measurements of the change of length under tension of a number of rods 2.5 mm. in diameter inclined at various angles. Three of these are at angles so close to 90° that their results may be averaged, giving a direct value for s_{11} (equation 11). The extreme variation in this group is from 8.34 to 8.81. The other rods were inclined, one at 47° and the other at 66° (equations 12 and 13), and involve all the constants. It is to be remembered that one source of error with the unmachined rods is geometrical imperfections. The equations follow. The fundamental forms due to Voigt were used, and the special values of the direction cosines were substituted into them.

$$s_{11} + s_{12} + s_{13} = 1.932 \times 10^{-13}, \tag{1}$$

$$2s_{13} + s_{33} = 13.11 \times 10^{-13}, \tag{2}$$

$$s_{13} = -6.64 \times 10^{-13}, \tag{3}$$

$$s_{12} = +.31 \times 10^{-13}, \tag{4}$$

$$s_{11} = 8.23 \times 10^{-13}, \tag{5}$$

$$s_{13} = -6.74 \times 10^{-13}, \tag{6}$$

$$s_{12} = +.48 \times 10^{-13}, \tag{7}$$

$$s_{11} - s_{12} + \tfrac{1}{2}s_{44} = 23.1 \times 10^{-13}, \tag{8}$$

$$1.304s_{11} + .489s_{33} - .815s_{12} - .978s_{13} + .224s_{44} = 33.4 \times 10^{-13}, \tag{9}$$

$$s_{44} = 26.0 \times 10^{-13}, \tag{10}$$

$$s_{11} = 8.57 \times 10^{-13}, \tag{11}$$

$$.535s_{11} + .217s_{33} + .249[2s_{13} + s_{44}] = 11.95 \times 10^{-13}, \tag{12}$$

$$.697s_{11} + .027s_{33} + .138[2s_{13} + s_{44}] = 7.92 \times 10^{-13}. \tag{13}$$

The following are the final values adopted for the constants:

$$s_{11} = 8.23 \times 10^{-13},$$

$$s_{12} = +\ .34 \times 10^{-13},$$

$$s_{13} = -\ 6.64 \times 10^{-13},$$

$$s_{33} = 26.38 \times 10^{-13},$$

$$s_{44} = 25.0 \times 10^{-13}.$$

As already mentioned, the linear compressibilities are much the most accurate. In choosing the best values for the constants, the first condition demanded was that the linear compressibilities should check exactly. In getting the rectangular constants by far the greatest weight was given to equations 3–7, as the specimens used there were much the best. In selecting the best value for s_{44} a more devious course of compromise was adopted, giving much weight to the direct value of equation 10. The success with which these values meet the required conditions can in many cases be told by inspection; the less obvious cases are equation 8, in which substitution of the values adopted gives 21.0 on the left hand side against 23.1 on the right, equation 9 with 30.4 against 33.4, equation 12 with 13.17 against 11.95, and equation 13 with 8.15 against 7.92.

The check afforded by this considerable number of independent measurements must, I think, be considered satisfactory, and gives considerable confidence in the final results. Particularly gratifying is the agreement between the cubic compressibility calculated from the linear compressibilities and that directly observed by other observers.

If now we compare these values with those of Grüneisen and Goens,[2] we note that their computed cubic compressibility is 19.0 against 17.1 experimental. Their s_{12} has the opposite sign from mine. The positive sign found here constitutes a distinct abnormality of zinc, but I believe that the experimental evidence for the positive sign is very strong (two direct measurements above). The values of Grüneisen and Goens were obtained from only two different sorts of measurements, extension and torsion measurements on rods of different orientations, and the elastic constants were computed so as to give as well as possible the observed variation with direction of the effective constants. It seems to me that the constants above, determined by more direct methods, are without doubt to be preferred.

The most striking feature of the elastic constants is the very great difference of the extensibility along and at right angles to the axis. The extensibility is greatest along the axis, which is what might be expected because the separation of the atoms is greatest in this direction.

Cadmium. The results which I have so far obtained on cadmium are incomplete and unsatisfactory in several particulars, and they must be considered as preliminary in character. There are two difficulties, neither of which I sufficiently realized until a large number of measurements had been made. In the first place the single crystal of cadmium is extraordinarily deformable, so that it is questionable whether it is allowable to attempt to machine it at all. My measurements of linear compressibility and thermal expansion were made on machined specimens, and these had to be entirely discarded. In the second place, under high pressures there are two new modifications, with reversible transitions. If pressure is applied to a single crystal of cadmium, it will be transformed into another crystal modification above a certain pressure, and then on still further increasing pressure into another. If now pressure is released into the domain of stability of the ordinary modification, it will not return in the original orientation. Hence all measurements of the properties under pressure of single cadmium crystals must be made on virgin pieces, which have never experienced either of the reversible transitions. The importance of this I did not realize for some time, and it explains a number of very puzzling and inconsistent results. The essentially new results which I have to communicate here are with respect to the transition.

Like zinc, cadmium is rather easy to crystallize, but is perhaps a little more exacting. I found more difficulty due to the formation of new nuclei. This difficulty I largely met by a more positive temperature control, lowering the glass mold into an oil bath, instead of into the air, which was adequate for zinc. The same speed of lowering as with zinc produced good results. It is also necessary that solid particles of dirt which may act as nuclei be more rigorously excluded. I succeeded, however, in growing crystals of the largest diameter, 2.2 cm. The purity of the cadmium is a matter of even greater importance than of zinc; the very deformable crystals are not obtainable unless the purity is high. I did not find any cadmium of American origin which was satisfactory, but used Kahlbaum's cadmium in all my final measurements.

The reflection pattern of cadmium is the same as that of zinc, but with a stronger tendency to the development of the basal planes, and the nearly complete suppression of the other planes under some conditions. The pits, or negative crystals, of cadmium were larger than for any of the other metals, large enough so that the different individual faces could be readily studied under a magnification of 500.

Thermal Expansion. I obtained no results of value, the crystal structures of my specimens doubtless having been destroyed by the machining. However, the expansion has been measured by Grüneisen and Goens,[2] so that this lack is not of such importance. They find the same strikingly great relative expansion along the axis that is also shown by zinc.

Electrical Resistance. Measurements were made on a number of virgin pieces, and should therefore be satisfactory.

The specific resistances, obtained by the methods described, for a number of rods of Kahlbaum's cadmium, are shown in Figure 3 as a function of the orientation, together with the smooth curve of the theoretical resistances calculated according to Voigt's formula A. It appears that the best values for the specific resistances at 20° C. are 6.80×10^{-6} at right angles to the hexagonal axis, and 8.30 parallel to the axis. The ratio of these two resistances is 1.221. Grüneisen and Goens[2] find for the ratio 1.188. This is in closer agreement than our values for zinc, and the difference is in the opposite direction. However, their absolute values of specific resistance are much higher than mine, being (using their values for the temperature coefficient of resistance) 7.11×10^{-6} and 8.45 perpendicular and parallel respectively at 20° C. The difference is in the direction to be ac-

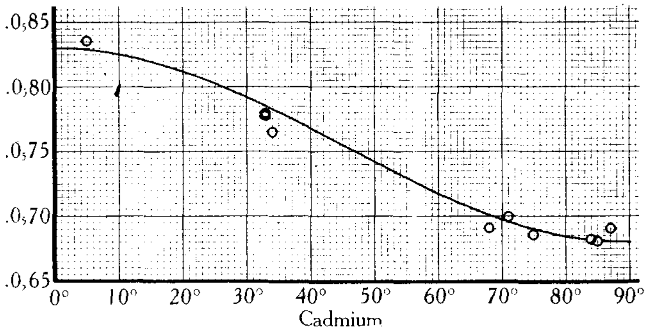

FIGURE 3. The specific resistance at 20° C. of cadmium as a function of the angle between the hexagonal axis and the direction of flow.

counted for by impurities in their sample, but the high value of their temperature coefficient would indicate high purity.

The specific resistance of the haphazard aggregate is 7.24×10^{-6} according to my measurements, and 7.51 according to Grüneisen and Goens. Jaeger and Diesselhorst[10] have given 7.54 at 18° C. for cast cadmium.

I have no satisfactory measurements of the temperature coefficient of resistance, my observations having been made on specimens which had experienced a transition under pressure. One of these gave the high value 0.00428, so that it is probable that there is some direction in the crystal in which the coefficient is as high as this. This is higher than the value of Grüneisen and Goens above, 0.00426, which again is a little higher than the best previous value 0.00424 obtained in my pressure work on extruded wire, and also by Holborn.[11]

The pressure coefficient was measured over the usual range of pressure, and at 0° and 95° C., but because of the polymorphic transitions produced by pressure, most of these readings are without significance for the effect of pressure on specimens of known orientation. However, it was fortunate that on the first application of pressure a seasoning application of 2000 kg. was made, releasing the pressure again to atmospheric after the application of 2000, so that the data are at hand by which the average pressure coefficient over the first 2000 may be found at 0° C. The results are as follows:

Angle of hexagonal axis with axis of specimen	Average pressure coefficient of measured resistance at 0° C. to 2000 kg/cm².
5°	-1.450×10^{-5}
87°	-0.693×10^{-5}

On the 5° specimen, readings were made at three intermediate pressures between 0 and 2000 and it was established that the effect is linear within the limits of error, but on the 87° specimen readings were made only at 0, 2000, and 0 again. The value of the coefficient which I have previously found for extruded cadmium wire was -1.063×10^{-5}. The results above on the single crystal give for the pressure coefficient of the haphazard aggregate, formula H of the introduction, -0.912×10^{-5}, less than the coefficient of the extruded wire. It is probable, therefore, that the orientation of the wire was not really haphazard, but that grains preponderated with the hexagonal axis parallel to the length of the wire.

Let us now consider what part of the difference of these coefficients is to be ascribed to unequal changes of dimensions under pressure. Using the values of the elastic constants which will be presently given, the following approximate values are found:

	5°	87°
Pressure coefficient of specific resistance	-1.311×10^{-5},	-0.871×10^{-5}.

To a sufficient degree of approximation these may also be taken as the coefficients of specific resistance respectively parallel and perpendicular to the hexagonal axis.

The effect of the change from measured to specific resistance is the same that it was with zinc, namely the coefficients in the two directions become more nearly equal, but as before, there is left a comparatively large outstanding difference, the specific resistance in the direction of the hexagonal axis (which is also the direction of greatest resistance) being decreased more rapidly than the resistance at right angles.

Elastic Constants. It has already been explained that many of the measurements of the elastic constants were vitiated either because the stresses producing the deformation to be measured exceeded the elastic limit, or else because the crystal structure had been destroyed by machining or by subjection to pressure with polymorphic transition. The measurements on compressibility had to be discarded entirely, the linear compressibilities failing entirely to check with known values of the cubic compressibility. This was true even for the initial compressibilities found before the polymorphic transition had been produced, and the only explanation seems to be a destruction of the crystal structure by machining. Among the other measurements of the constants were also some which were evidently impossible. It was possible, however, to select from the fifteen measurements enough to give fair values for the constants. In selecting the measurements to use I was guided by Grüneisen and Goens. This, however, was merely by way of saving time, because the inconsistent measurements discarded were inconsistent by large amounts, and were very different from those accepted.

The following sorts of measurements were used in obtaining the final results. The extension under tension of two rods nearly parallel and perpendicular respectively to the hexagonal axis gave direct values of s_{11} and s_{33}. The torsion was measured of a rod at 90° (average of two specimens), and the tension and torsion of a rod at 30°. There were thus five measurements available for the five constants, for which I have adopted the following values as best:

$$s_{11} = 12.9 \times 10^{-13},$$

$$s_{12} = -1.5 \times 10^{-13},$$

$$s_{13} = -9.3 \times 10^{-13},$$

$$s_{33} = 36.9 \times 10^{-13},$$

$$s_{44} = 64.0 \times 10^{-13}.$$

In addition to my five measurements there is the condition imposed by the cubic compressibility of other observers. I chose the constants so as to check almost exactly with the cubic compressibility of Adams, Williamson and Johnston[9] (22.5×10^{-13} against 22.4), and also used the directly measured values of s_{11} and s_{33}. The other constants were chosen so as to give as good a compromise as possible when substituted in the three remaining equations. In these equations the agreement was as follows: 46.4 against 43.5, 28.6 against 29.8, and 64.1 against 65.0. The agreement is not bad, but it must nevertheless be recognized that these constants are not nearly as secure as those of zinc, or for that matter most of the other metals measured here.

Compared with the values of Grüneisen and Goens, the most important difference is in the larger value which I find for s_{44}, 64 against 54. It is to be noticed that I have given negative values to both s_{12} and s_{13}, unlike zinc. In doing this I was doubtless influenced by the values of Grüneisen and Goens; I do not believe that this result should be accepted as final or made the basis of theoretical considerations without further verification, preferably a direct measurement, such as was possible for zinc.

Transitions of Cadmium under Pressure. These results are preliminary, and have to do chiefly with proving the existence of the transitions, and establishing their approximate locations. Most of the results were obtained in the course of attempts to measure the linear compressibility. These measurements were made first on specimens cut at right angles to the axis, and the first suggestion of anything wrong was in the unusually large permanent changes of zero after an excursion to the maximum pressure and back again. The curve itself was fairly regular, but careful study showed irregularities greater than could be explained by experimental error. Measurements were now made on the specimen parallel to the axis, and the effects found were much greater. At two places on the curve there were discontinuities by amounts rising in the extreme case to as much as 2% of the total change of length under 12000 kg. These discontinuities were reversible in the sense that they could be obtained in approximately the same location with increasing or decreasing pressure, but there were peculiarities which made them unlike the volume discontinuities ordinarily found in measuring transitions. It must be remembered that these were discontinuities of length. The discontinuities were not always of the same sign, but the change of length might be either a decrease or an increase.

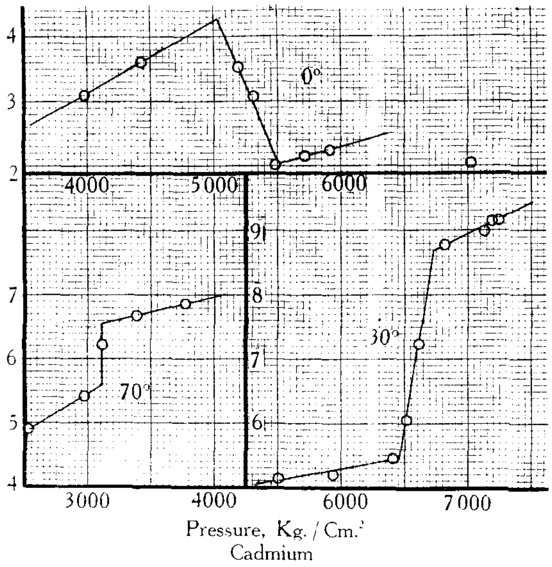

FIGURE 4. Shows the discontinuity of length on an arbitrary scale observed at the transition points of cadmium under pressure. For the high pressure transition an example is shown of both an increase and a decrease of length.

Furthermore, after several changes of pressure back and forth, the discontinuity usually became less and less pronounced and ultimately practically disappeared. In one case, after several applications of pressure, the discontinuity, which at first was an increase, reversed and became a decrease of length. The most valuable measurements were always obtained on virgin specimens. In Figure 4 are shown the best examples of the two discontinuities; at the higher pressure an example is shown of both an increase and a decrease of length. In some cases it was possible to obtain readings within the discontinuity itself, as shown in the figure, and in others it was only possible to shut the discontinuity within limits.

Having established the existence of the effect by compressibility measurements, I verified its existence by finding a discontinuity in the electrical resistance of a rod whose hexagonal axis was parallel to the length, which is the most favorable orientation.

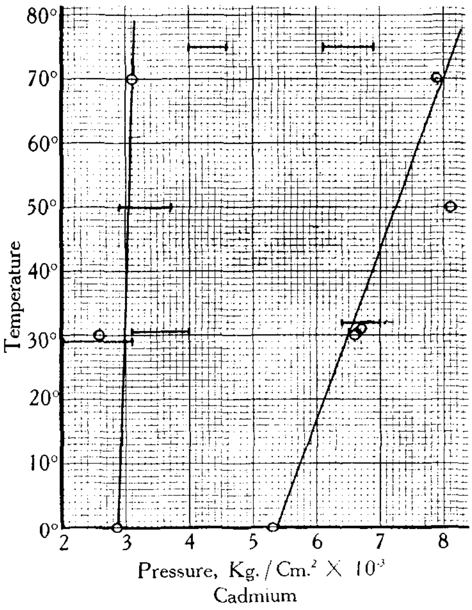

FIGURE 5. The observed transition pressures of cadmium as a function of temperature. The circles indicate readings obtained within the discontinuity, the two phases being simultaneously present, while the lines show the range within which the transition was shut by other readings obtained with only one phase present and therefore not under equilibrium conditions.

The collected results are shown in Figure 5, where the points indicate the readings determined within the actual discontinuity, and the lines indicate the range within which the transition was included when it was not more exactly located. The existence of the transition must be considered to be beyond doubt, and also the location approximately as given, but the reason for the discrepancies between readings obtained at different times is not altogether clear. It is possible that it may be an effect of internal stress. If the new modification does not appear as a single crystal, but as several grains of different orientations, there may well be a stress due to the unequal change of dimensions, which may modify the transition pressure. It is well known that the effects of a non-hydrostatic stress on a transition point may in some cases be very large, particularly when the heat of transition is low.

Whatever the other properties of the new modifications, it is evident that they cannot differ appreciably in volume from the ordinary modification, but that the spacing of the atoms in different directions must differ, being in some directions extended and in others contracted. This is demanded not only by the facts found here, but also by the fact that I have never before found any suggestion of such a discontinuity. I have previously measured the pressure coefficient of resistance, the compressibility, and the thermal e.m.f. under pressure, but always on specimens of many grains arranged approximately at haphazard. It follows that the electrical properties also of the new modifications can, on the average, differ only slightly from the ordinary.

One is at once reminded of the polymorphic transitions claimed by Cohen[12] for cadmium, but it is probable that there is no connection, the transitions found here being reversible, and occurring only at pressures of several thousand kilograms.

Bismuth. The source of most of the lead used in these measurements was the U. S. S. Lead Refining Co., to whose courtesy I owe some especially refined electrolytic bismuth. Some of this material has been used in my previous work,[13] and some of the identical lot was left for my present work; I also procured a fresh lot made by the same method. In my previous work will be found a chemical analysis; the chief impurity is less than 0.03% Ag. The purity is also vouched for by the high value of the temperature coefficient of resistance. Furthermore, it is to be remembered that the process of crystallization acts as a very efficient purification, at least from those metals which depress the freezing point. I have found that the lower end of a casting of commercial bismuth yields metal of as high a temperature coefficient as the best electrolytic material.

Bismuth is one of the most strongly crystalline of metals, and until the recent work of Grüneisen and Goens is practically the only metal for which any attempt has been made to obtain data for the individual crystal. The previous material has been usually obtained from large single crystals found in the interior of large masses of slowly solidified bismuth. There have, however, probably been large errors introduced by the methods of manipulation, as will be explained later. It was a surprise, in view of this previous experience with bismuth, to find that it is not one of the easiest to grow in large single crystals. My usual method of growth readily produces rods which to the first casual inspection are one grain, either by the reflection or the cleavage test, but on closer examination it will be found that what appears as a single cleavage plane is really not one but several inclined at very slight angles to each other. Instead of a single grain, it is therefore much more usual for the casting to consist of several, all oriented very nearly alike, and growing together for the entire length of the casting. Not only is the orientation of the cleavage plane, and therefore of the trigonal axis, nearly the same in these different grains, but the orientation of the other axes about the axis of three-fold symmetry is also approximately the same, because the cleavage on the rhombohedral planes, which is well developed in bismuth although not nearly as perfect as that on the basal plane, runs across from one grain to another with very slight change of direction. I did not succeed in obtaining any of the largest castings (2.2 cm. diameter) in one grain, and only one as large as 1.2 cm. The smaller castings, 2 to 6 mm. in diameter, came much more readily in one grain. The presence of several grains has no effect on the determination of certain properties, such as the linear compressibility under hydrostatic pressure, but it may introduce rather large errors into the determination of certain other combinations of the elastic constants, such for instance as depend on the deformation under a one-sided compression. Considerable trouble from this cause was experienced before the reason was located. Toward the end of my manipulations of bismuth I found that the best results are obtained with a high rate of cooling, instead of a low rate as for zinc and cadmium, and I have no doubt that with a little pains it would now be possible to obtain the larger castings.

The preferred manner of growth, as also for zinc and cadmium, is with the basal plane parallel to the axis of the casting, but variations from this direction take place more readily than they do for zinc. In the small sizes it is possible to obtain large departures from parallelism, but because of the extreme ease of cleavage along the basal plane, I did not find it feasible to attempt measurements on any long slender rods with an angle of inclination greater than 45°, although measurements were made on machined pieces short compared with their length in which the basal plane was at right angles to the length.

A particular source of trouble with bismuth was the great amount of occluded gas set free on solidifying, which makes special trouble with the long slender castings. The gas must be removed by a prolonged preliminary heating in vacuum, as already explained.

Thermal Expansion. This was measured in the regular way on the same two specimens whose linear compressibility was determined. The specimens were machined from a large casting, were 6 mm. in diameter, and 2.8 cm. long perpendicular to the trigonal axis, and 2.1 cm. parallel, this latter being in two pieces. The following results were found at 20° C.:

$$\alpha_{\perp} = 10.36 \times 10^{-6},$$

$$\alpha_{\parallel} = 13.96 \times 10^{-6}.$$

The expansion is thus greatest along the axis of rotational symmetry, as it has always been found to be, but the difference is very much less accentuated than it is for zinc and cadmium. This is perhaps surprising when the strongly crystalline character of bismuth is considered.

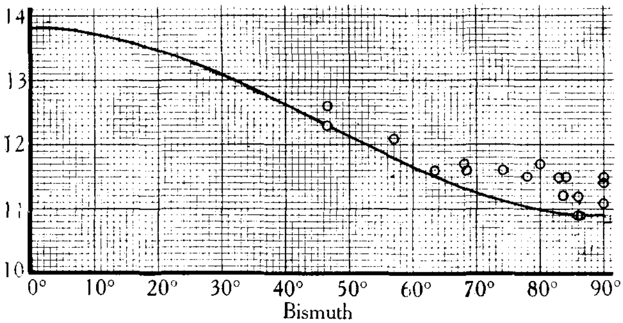

FIGURE 6. The specific resistance at 20° C. multiplied by 10^5 of bismuth as a function of the angle between the trigonal axis and the direction of flow.

The thermal expansion at 40° C. has been found by Fizeau to be 16.2 parallel and 12.1×10^{-6} perpendicular to the axis, both greater than the values above. The reason for the discrepancy is not clear, unless there were flaws in Fizeau's specimen, which seems to have been the case with many of the early specimens because of their method of preparation.

Electrical Resistance. Measurements were made on the specific resistance at room temperature of about 20 specimens, all unmachined rods, inclined at various angles up to 45°. These are shown in Figure 6 (two have been discarded, lying impossibly high), and I have also drawn the curve which seems to represent the most probable value of the true resistance. The points scatter a great deal, and lie above the curve. The reason for this is to be sought perhaps partly in fissures introduced into the bismuth by handling, the cleavage being very easy, and also by geometrical imperfections, especially minute bubbles on the sides of the casting, which I have never succeeded in completely eliminating.

The most probable values of the specific resistance at 20° C. may be deduced from the diagram to be

$$\rho_{\perp} = 109 \times 10^{-6},$$

$$\rho_{\parallel} = 138 \times 10^{-6}.$$

The ratio of the resistances in the two principal directions is thus 1.27.

These results differ considerably from those of previous observers. Thus Matteuci[14] finds for the ratio of the two principal resistances 1.6, Borelius and Lindh[15] 1.57, and van Everdingen[16] 1.68. The difference is in the direction to be expected. It has been brought out by Borelius that most resistance measurements have been faulty because of the existence of flaws in the metal. Their existence may be shown very strikingly by subjecting the metal to a compression at right angles to the basal plane, when the resistance experiences a sudden and large decrease. The effect is so large that Borelius has even gone so far as to suggest that if there were no fissures the resistance would be the same in all directions. These fissures are an almost certain result of the method of preparation which many previous experimenters have adopted. Thus Borelius describes slowly cooling a dish of bismuth 3 cm. high and 2 cm. in diameter with every precaution to ensure the slow growth of the crystal, then breaking the casting in two, judging from the continuity of the cleavage plane that there was really only one grain, and then using the two halves of the broken casting as the source of further material. Such treatment would be expected to produce fissures on the basal plane, and to give exactly the larger value for the resistance along the axis (across the planes) which was actually found. That my values above do not suffer from this source of error is made probable by the extreme care with which the glass tubes were broken from the castings, and more especially by the measurements of the pressure coefficient of resistance. If there are minute fissures, these will be closed by a small initial pressure, so that there will be an initial sharp decrease of resistance at a different rate from the subsequent rate. No such effect was found.

Whatever the uncertainties shown in Figure 6, it is evident that the ratio of the resistances in the two directions must be much less than 1.6. The highest ratio which seems at all consistent with the data is 1.31.

From the results above it may be calculated that the resistance of a haphazard collection of grains in all orientations is 117×10^{-6}, which agrees closely with the value 119 at 18° C. found by Jaeger and Diesselhorst,[10] and is confirmatory evidence of the correctness of the small ratio of the principal resistances found above.

Temperature Coefficient of Resistance. This was determined for two samples incidentally during the measurements of pressure coefficient of resistance, the angle of inclination of the axis to the length of these two pieces being respectively 90° and 57°. The measurements on the 90° sample were a little more regular, and gave the following results for the resistance as a function of temperature:

Temperature	Relative Resistance
0°	1.000
25	1.102
50	1.209
75	1.322
100	1.444

The departure from linearity is in the normal direction. Within the limits of error, which were not more than 1% on the coefficient, the measurements on the 57° sample agreed with the above, so that there appears to be no appreciable variation of temperature coefficient with direction. Corrected for thermal expansion, the mean coefficient between 0° and 100° C. of specific resistance is 0.00445, the same in all directions.

The average coefficient of the measured resistance given by the above is 0.00444, which is a trifle higher than the best value which I have previously found for electrolytic bismuth (0.00441) and is evidence of high purity. Holborn,[11] however, has found a value 0.00446.

Pressure Coefficient of Resistance. The effect of pressure to 12000 kg. at several temperatures was measured on the two samples described above in connection with the temperature coefficient. The results are not linear with the pressure, and are not conveniently given by formulas, so I have reproduced them in a Table I. There was no especial incident in making the measurements; the regularity was that to be expected from a specimen of small resistance. With the 90° specimen the average deviation from a smooth curve varied with the temperature from 1.1 to 0.6% of the effect, and with the 57° specimen the variation was from 0.06 to 0.5%.

TABLE I.

Relative Resistances of Bismuth Under Pressure.

Trigonal axis perpendicular to length				
Pressure kg/cm^2	Relative Resistance			
	0° C.	30° C.	62° C.	95° C.
0	1.0000	1.0000	1.0000	1.0090
1000	1.0090	1.0092	1.0082	1.0005
2000	1.0188	1.0187	1.0188	1.0199
3000	1.0294	1.0289	1.0292	1.0305
4000	1.0404	1.0395	1.0403	1.0416
5000	1.0518	1.0511	1.0518	1.0534
6000	1.0637	1.0627	1.0635	1.0656
7000	1.0758	1.0750	1.0758	1.0782
8000	1.0884	1.0876	1.0885	1.0914
9000	1.1010	1.1003	1.1013	1.1046
10000	1.1143	1.1133	1.1144	1.1182
11000	1.1279	1.1263	1.1276	1.1320
12000	1.1418	1.1393	1.1409	1.1463

Trigonal axis 57° to length			
Pressure kg/cm^2	Relative Resistance		
	0° C.	30° C.	95° C.
0	1.0000	1.0000	1.0000
1000	1.0155	1.0149	1.0126
2000	1.0319	1.0305	1.0270
3000	1.0490	1.0475	1.0423
4000	1.0671	1.0648	1.0574
5000	1.0873	1.0831	1.0755
6000	1.1065	1.1024	1.0929
7000	1.1278	1.1218	1.1110
8000	1.1501	1.1422	1.1300
9000	1.1733	1.1637	1.1499
10000	1.1971	1.1854	1.1706
11000	1.2213	1.2081	1.1917
12000	1.2471	1.2318	1.2129

As has already been found for extruded wire, the effect of pressure is to increase the resistance, the rate of increase itself becoming greater at the higher pressures. We now find that the increase under pressure is more rapid across the cleavage planes than at right angles. This is the reverse of the effect to be expected if the cleavage planes are the seat of incipient fissures.

We have now to compute from the effects at 90° and 57° the effect at 0° (cleavage plane perpendicular to the length). In making this computation we have first to calculate the pressure coefficients of specific resistance at 90° and 57°. The formulas already given, together with the elastic constants to be given in the next section, give for the correction term -1.6×10^{-6} at 90° and -1.16×10^{-6} at 57°, of which latter -0.11×10^{-6} is the angle correction. The data given above for resistance as a function of pressure yield for the initial pressure coefficients of measured resistance 9.1×10^{-6} and 14.6×10^{-6} at 90° and 57° respectively. Applying the corrections, we get for the pressure coefficients of specific resistance $+ 7.5 \times 10^{-6}$ and $+ 13.4 \times 10^{-6}$ at 90° and 57°. Substituting now these values in formula C for the pressure coefficient of specific resistance as a function of direction, we obtain for the pressure coefficient of specific resistance along the axis $+ 24.5 \times 10^{-6}$, which is thus three times larger than the coefficient in the perpendicular direction.

Elastic Constants. Bismuth is characterized by six elastic constants; five of these enter in the same way as the five constants of zinc and cadmium and may be determined in the same way, the sixth is the so-called cross constant s_{14} and requires different sorts of measurement. In getting these constants a number of measurements had to be discarded, because it was found that the specimens were not a single grain, but several of almost exactly the same orientation. Some of the constants determined with these multigrained rods were of the order of twice the correct values.

The measurements from which the constants were finally deduced were as follows. First and most accurate, the linear compressibility of specimens parallel and perpendicular to the trigonal axis, giving equations 1 and 2. Only the initial values of the compressibilities from these pressure measurements are to be used in combination with the other elastic constants; the behavior of the linear compressibility over the pressure range will be described later. Second, direct measurements of the longitudinal compression and two transverse expansions of a machined specimen 1.27 cm. in diameter perpendicular to the trigonal axis subjected to a simple compressive stress, giving equations 3, 4, and 5. These measurements give direct values for s_{11}, s_{12}, and s_{13}. Thirdly, measurements were made of the torsion of two rods with the cleavage plane parallel to the length, giving by the average equation 6. The two constants for the two rods differed by 1.7%. Finally to determine s_{14}, measurements were made of the extension under tension of three rods inclined at various angles from 46.5° to 74°, giving equations 7, 8, and 9. In making these determinations it is necessary to know the location of the rhombohedral cleavage planes as well as the basal cleavage plane. Each of these tension measurements gave a value for s_{14}, using in the equation the values for the other constants which seemed best from the previous equations. Each of the values of s_{14} is the difference of two quantities of nearly the same magnitude, so that it is not to be expected that the result will have any great accuracy. The three values so obtained were averaged to obtain the final value, weighting the different values according to the ratio of the difference to the larger of the two terms of the equation.

The equations in detail are as follows:

$$s_{11} + s_{12} + s_{13} = 6.66 \times 10^{-13}, \tag{1}$$

$$2s_{13} + s_{33} = 16.28 \times 10^{-13}, \tag{2}$$

$$s_{12} = -13.8 \times 10^{-13}, \tag{3}$$

$$s_{13} = -6.1 \times 10^{-13}, \tag{4}$$

$$s_{11} = 27.3 \times 10^{-13}, \tag{5}$$

$$s_{11} - s_{12} + \tfrac{1}{2}s_{44} = 95.3 \times 10^{-13}, \tag{6}$$

$$s_{14} = (60.4 - 44.2) \times 10^{-13} = 16.2 \times 10^{-13}, \tag{7}$$

$$s_{14} = (53.7 - 36.0) \times 10^{-13} = 17.7 \times 10^{-13}, \tag{8}$$

$$s_{14} = (90.8 - 80.0) \times 10^{-13} = 10.8 \times 10^{-13}. \tag{9}$$

Equations 1 and 2 for the linear compressibility may be checked against the cubic compressibility. The value found by adding 2 to twice 1 is 29.60×10^{-13} for the cubic compressibility; the experimental values are 29.7(5) by Adams, Williamson, and Johnston,[9] and 29×10^{-13} by Richards.[8] The agreement is probably within the limits of error. The first two equations were therefore accepted as correct. The rectangular constants were now chosen approximately, as given by equations 3 to 5, but with slight readjustments in s_{11}, s_{12}, and s_{13} so as to satisfy equation 1. s_{33} is then given immediately by equation 2, but there is no check on it. The magnitude of the adjustments of the first three constants gives an idea of the probable accuracy. The shearing constant s_{44} was then obtained directly from 6 by substitution. In equations 7 to 9 the values of the first five constants thus found were substituted, giving the equations as they stand for s_{14}. The weighting process described gave the final value for s_{14}. The final values for the constants follow in Abs. C.G.S. units.

$$s_{11} = 26.9 \times 10^{-13},$$

$$s_{12} = -14.0 \times 10^{-13},$$

$$s_{13} = -6.2 \times 10^{-13},$$

$$s_{33} = 28.7 \times 10^{-13},$$

$$s^{44} = 104.8 \times 10^{-13},$$

$$s_{14} = 16.0 \times 10^{-13}.$$

In contrast with zinc and cadmium, the two constants s_{11} and s_{33} have nearly the same values. The exceedingly large value of the shearing constant s_{44} is the particularly interesting feature for bismuth.

So far as I know there are no previous values for the individual elastic constants of bismuth. This is surprising in view of the fact that others of its physical constants have been determined, and that large crystals are obtained so readily. It is probable that minute fissures lead to larger inconsistencies in the elastic constants than in some of the others; witness the discrepancies found above when there were several grains.

Finally we have the linear compressibility as a function of pressure. These measurements ran as smoothly as any of the linear compressibility measurements, and gave the following results, pressure in kg/cm^2:

Length perpendicular to hexagonal axis

$$-\frac{\Delta l}{l} = 6.624 \times 10^{-7}p - 4.39 \times 10^{-12}p^2, \qquad \text{at } 30°,$$
$$= 7.044 \times 10^{-7}p - 8.40 \times 10^{-12}p^2, \qquad \text{at } 75°.$$

Length parallel to hexagonal axis

$$-\frac{\Delta l}{l} = 15.92 \times 10^{-7}p - 11.1 \times 10^{-12}p^2, \qquad \text{at } 30°,$$
$$= 15.80 \times 10^{-7}p - 11.6 \times 10^{-12}p^2, \qquad \text{at } 75°.$$

These measurements were on electrolytic bismuth of high purity Measurements were also made on a specimen of commercial bismuth perpendicular to the axis, giving

$$-\frac{\Delta l}{l} = 6.450 \times 10^{-7}p - 4.60 \times 10^{-12}p^2, \qquad \text{at } 30°,$$
$$= 6.423 \times 10^{-7}p - 4.57 \times 10^{-12}p^2, \qquad \text{at } 75°.$$

The difference, due to impurity, is about 2.7%.

Combining the above results for the change of length gives for the change of volume of electrolytic bismuth:

$$-\frac{\Delta V}{V_0} = 29.17 \times 10^{-7}p - 22.43 \times 10^{-12}p^2, \qquad \text{at } 30°,$$
$$= 29.89 \times 10^{-7}p - 31.13 \times 10^{-12}p^2, \qquad \text{at } 75°.$$

Antimony. It is astonishingly easy to obtain antimony in single grain castings; it does not have the tendency to form several grains of nearly the same orientation that bismuth does. Because of the high melting point, it is necessary to use a mold of quartz or of high melting combustion tubing; pyrex softens below the melting point. When using combustion tubing, care is necessary not to exceed the melting point too much; I obtained good results by running the furnace 50° above the melting point. I used the slowest rate of cooling, 4 mm. an hour for the largest size casting, but I have no doubt that considerably more rapid rates would be equally satisfactory. Remarkably clean castings were obtained, with practically none of the trouble from occluded gas shown by bismuth. The preferred manner of growth is with the trigonal axis perpendicular to the axis of the casting, that is, with the principal cleavage plane parallel to the length. The larger castings, 2.2 cm. down to 6 mm., invariably came out with this orientation, but the smaller castings of 2 mm. diameter gave a number of oblique incidences up to 65°.

Thermal Expansion. The regular apparatus and procedure were used. The specimens were cut from one of the large castings, and were the same as those used for the linear compressibility. There is no difficulty in preparing the specimen with the cleavage plane parallel to the length, but because of the excessively easy cleavage there was considerable difficulty in preparing the perpendicular specimen. This specimen was built up of three short pieces resting against each other on natural cleavage faces, which are as flat and parallel as it would be possible to obtain by a machining operation. These short pieces, approximately cubical, were cut from a slab of the requisite thickness by grinding with a thin slitting wheel. A special brass clamp was made with which the specimens were firmly compressed across the cleavage planes, and in addition they were imbedded in a matrix of cement. In cutting, the clamp and the specimens were ground across together; this proved to be necessary because the specimen must be always firmly clamped on both sides of the cut.

The readings with the specimens so prepared were as self consistent as those obtained with less cleavable metals. The following results were found for the expansion in a range of 10° about room temperature:

$$\alpha_{11} = 15.56 \times 10^{-6},$$

$$\alpha_{\perp} = 7.96 \times 10^{-6}.$$

The expansion across the cleavage planes is thus nearly twice as great as that at right angles; this is the normal type of behavior, and is that found for all the metals examined here.

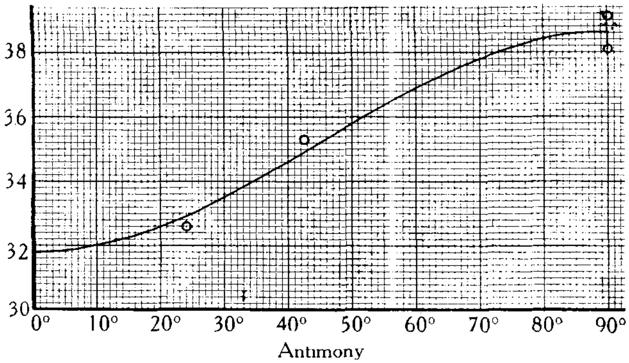

FIGURE 7. The specific resistance of 0° C. multiplied by 10^4 of antimony as a function of the angle between the trigonal axis and the direction of flow.

At 40° C. Fizeau has found for the expansions 16.92 and 8.82×10^{-6} respectively, both larger than my corresponding values. The difference is in the same direction as for bismuth. At least part of the difference is to be ascribed to the difference of temperature.

Resistance. The specific resistance of four small castings was measured; at angles varying from 24° to 90°. The specific resistances at 0° C. as a function of orientation, and the most probable curve, are shown in Figure 7. The specific resistance parallel to the axis (across the cleavage planes) is 31.8 and at right angles 38.6×10^{-6}. This is the first time that we have found a resistance across the cleavage plane less than in the perpendicular direction, and is not what we might expect. It is evident that this sort of result cannot be explained by the presence of minute fissures.

There are no previous determinations for comparison. We may obtain a sort of check by computing the resistance to be expected in an aggregate of crystals of haphazard orientation. This turns out to be 33.8×10^{-6} at 0° C. This may be compared with 38.7 at 0° given in Bureau of Standards Circular No. 74, 1918. The difference may be ascribed to differences of purity, the Bureau having used American antimony, which is always less pure, judging by the melting point, and may also be in part due to the microscopic fissures which would be expected in a haphazard casting because of the unequal thermal expansions in different directions of the different grains.

The temperature coefficient of resistance of three of the above four samples was determined between 0° and 95°; the specimen at 24° to the axis was damaged by an application of pressure before the temperature coefficient could be determined. These temperature determinations were made with especial care, the temperature being run through a complete cycle, from 0° to 95° back to 0° with intermediate readings, and in one case back to 95° again. In all cases the readings were all that could be asked, there was no hysteresis and no permanent change, but the readings were in all cases single valued functions of the temperature within the sensitiveness of the readings. The two specimens perpendicular to the axis gave results indistinguishable from each other. The following values were found for the resistance as a function of temperature.

Temp. °C.	Length 90° to axis	Length 42.5° to axis
0	1.000	1.000
25	1.120	1.126
50	1.245	1.260
75	1.375	1.401
100	1.510	1.550

There is thus a very perceptible difference between the temperature coefficients in different directions.

We may now calculate the temperature coefficient across the cleavage planes. We first find the temperature coefficient of specific resistance to be 0.00511 for the 90° direction and 0.00552 for the 42.5°

direction. Substituting these values and the resistances in formula C yields 0.00595 for the 0° direction for the specific resistance. The variation with direction is the largest yet found.

The temperature coefficient of the haphazard aggregate may be calculated from these results to be 0.00510.

Resistance of Antimony under Pressure. It is already known that antimony in the haphazard aggregate is unusual in that the resistance increases under pressure. The behavior is that normal to this type of conductors in that the rate of increase under pressure itself increases as the pressure increases. The measurements on single crystals have now disclosed further abnormalities, which make antimony unique. These new abnormalities are associated with conduction perpendicular to the trigonal axis; parallel the behavior is more normal. Perpendicular to the axis at 0° C. the resistance increases under pressure, as it does in the haphazard aggregate, and at first the rate of increase increases with rising pressure, but presently there is a point of inflection near 6000 kg., and above this pressure the rate of increase decreases with rising pressure. At 95° C. on the other hand, the resistance decreases under pressure, which is the normal behavior for most metals, but the curvature is anomalous; at low pressures the rate of decrease itself decreases with rising pressure, but there is a point of inflection, and above this the rate increases. At the intermediate temperature at 50° C. the behavior is a compromise between that at 0° and 95°; at low pressures the resistance increases, but passes through a maximum and from here on decreases. This is the first example of a maximum resistance as far as I know; it is to be set by the side of the minimum resistance which I have recently found for caesium.

This unusual behavior was verified on two samples. On the repetition, more care was taken in the preparation of the sample, especially in being sure that the current leads made soldered connections across the entire section of the specimen, so that any fissures developed between the cleavage planes would not have the effect of insulating part of the sample from the rest and so changing the resistance. The results found with the two samples agreed at the maximum pressure within 0.2% of the total resistance at 0° C., and 0.5% at 95°.

Measurements on the sample with the axis at 42.5° to the length gave normal results in that the resistance increases under pressure at all temperatures, and the curvature is normal, the rate of increase of resistance under pressure also increasing with pressure, but the temperature effect is unusual in that the rate of increase at the higher temperatures is relatively and absolutely less than at the lower temperatures. I made a further attempt at resistance measurements on

TABLE II

Resistance of Antimony under Pressure.

Trigonal axis 90° to length

Pressure kg/cm^2	Resistance		
	0°C.	50° C.	95° C.
0	1.0000	1.2450	1.4830
1000	1.0035	1.2460	1.4808
2000	1.0085	1.2481	1.4787
3000	1.0140	1.2506	1.4765
4000	1.0204	1.2533	1.4744
5000	1.0275	1.2561	1.4722
6000	1.0347	1.2587	1.4702
7000	1.0416	1.2610	1.4674
8000	1.0479	1.2628	1.4645
9000	1.0532	1.2640	1.4610
10000	1.0580	1.2646	1.4575
11000	1.0622	1.2645	1.4530
12000	1.0650	1.2632	1.4475

Trigonal axis 42°.5 to length

Pressure kg/cm^2	Resistance		
	0° C.	50° C.	95° C.
0	.9040	1.1391	1.3732
1000	.9136	1.1476	1.3825
2000	.9242	1.1571	1.3904
3000	.9351	1.1674	1.3992
4000	.9473	1.1782	1.4089
5000	.9607	1.1903	1.4195
6000	.9760	1.2030	1.4298
7000	.9913	1.2161	1.4411
8000	1.0064	1.2302	1.4523
9000	1.0218	1.2435	1.4638
10000	1.0370	1.2567	1.4755
11000	1.0523	1.2700	1.4872
12000	1.0676	1.2834	1.4987

a sample with the cleavage planes inclined at 66° to the length, but these were not successful, there being very large permanent changes of resistance after every application of pressure. This is doubtless due to the opening of fissures between the cleavage planes, for which the viscosity of the transmitting medium must have been responsible.

The results obtained with the 90° and the 42.5° specimens are shown in Table II and Figure 8. These give the measured resistance

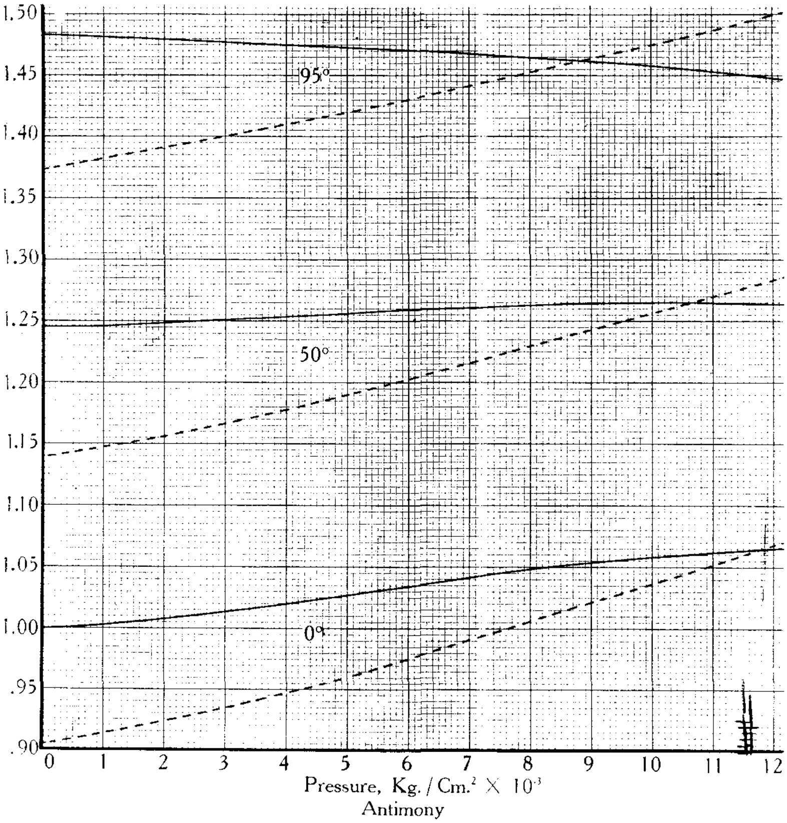

FIGURE 8. The relative resistances at 0°, 50°, and 95° C. of antimony as a function of pressure in two different directions in the crystal. The full lines show the resistance 90° to the trigonal axis, and the dotted lines 42.5° to the axis.

at intervals of 1000 kg. at 0°, 50°, and 95° C. in terms of the resistance at 0° C. for the 90° direction as unity. The figure brings out the very interesting result that the abnormalities under pressure are connected with the abnormal result found at atmospheric pressure that the resistance across the cleavage planes is less than that at right angles. At high pressures, this behavior is reversed, and the resistance across the cleavage planes becomes greater, as we have found it for all the other metals. In this respect the effect of pressure is to compel a return to the normal type of behavior. Furthermore, the return to normal appears to be more rapid at the higher temperatures, and is connected with the unusual temperature effects.

We may now obtain from the coefficients of measured resistance approximate values for the coefficients of specific resistance, using for the elastic constants at all temperatures the values found at room temperature (the error so introduced must be small). We find in the first place that the initial pressure coefficient for the 90° direction of the axis is $+ 10.5 \times 10^{-7}$ at 0° C. and $- 1.7 \times 10^{-7}$ at 95° C. For the 42.5° direction, the initial pressure coefficients of specific resistance are respectively $+ 90.4 \times 10^{-7}$ and $+ 66.4 \times 10^{-7}$. Using now formula C, we find for the 0° direction of the axis that the initial pressure coefficient of specific resistance at 0° C. is 172×10^{-7} and 131×10^{-7} at 95° C. These are of the same order of magnitude, but less, than found for bismuth.

Elastic Constants. The rectangular constants were obtained in the first place by measurements of the linear compressibility parallel and at right angles to the trigonal axis, equations 1 and 2. The specimens used for these measurements have already been described in connection with the thermal expansion measurements. Secondly, measurements were made of the longitudinal compression and two transverse expansions under one-sided compression of an unmachined casting with cleavage planes parallel to the length, giving direct values for s_{11}, s_{12}, and s_{13}, in equations 3, 4, and 5. Thirdly, a measurement of the extension under tension of a natural casting with the cleavage plane parallel to the length gives another independent direct value for s_{11}, equation 6. The shearing constant s_{44} was determined by a torsion measurement on a single specimen with cleavage plane parallel to the length, equation 7. The cross constant s_{14} I attempted to get from measurements of the extensions under tension of two inclined rods, but one of these had to be discarded, the casting being curved, and so receiving additional curvature under tension. The data retained for s_{14} are in equation 8. The equations follow:

(1) $$s_{11} + s_{12} + s_{13} = 5.40 \times 10^{-13},$$

(2) $$2s_{13} + s_{33} = 16.84 \times 10^{-13},$$

(3) $$s_{13} = -9.3 \times 10^{-13},$$

$$s_{11} = 17.6 \times 10^{-13}, \tag{4}$$

$$s_{12} = -4.2 \times 10^{-13}, \tag{5}$$

$$s_{11} = 17.8 \times 10^{-13}, \tag{6}$$

$$2(s_{11} - s_{12}) + s_{44} = 84.0 \times 10^{-13}, \tag{7}$$

$$.908s_{11} + .002s_{33} + .0445(2s_{13} + s_{44}) - 0.396s_{14} = 20.3 \times 10^{-13}. \tag{8}$$

We have thus 8 equations for the 6 constants. An additional check is given by the cubic compressibility to be calculated from equations 1 and 2. This is found to be 27.64×10^{-13} against 24 (given to only two significant figures) by Richards.[8] The agreement is probably within experimental error, and I have accepted equations 1 and 2 as correct. Equations 4 and 6 give two direct values of s_{11} differing by about 1%. I have taken the mean of these two values. This value of s_{11}, together with the direct values of s_{12} and s_{13}, should satisfy equation 1, but does not exactly. I have adjusted the values of s_{12} and s_{13} in equations 3 and 5 by equal percentage amounts to satisfy 1. Equation 2 now gives a value for s_{33} for which there is no check, but which must be fairly good, since it involves the sum of two quantities. Equation 7 gives a single value for s_{44}, which is also fairly good, the measurements being easy and the difference involved being that of two quantities one twice the other. Equation 8 now gives a single value without check for s_{14}; this value has much greater error than any of the others, since it was obtained from the difference of two quantities nearly equal (20.3 — 17.1). The final values for the constants obtained in this way are, in Abs. C. G. S. units:

$$s_{11} = 17.7 \times 10^{-13},$$

$$s_{12} = -3.8 \times 10^{-13},$$

$$s_{13} = -8.5 \times 10^{-13},$$

$$s_{33} = 33.8 \times 10^{-13},$$

$$s_{44} = 41.0 \times 10^{-13},$$

$$s_{14} = -8.0 \times 10^{-13}.$$

The sign of s_{14} is the opposite of that found for bismuth. There does not seem to be any normal sign for this constant, but among the minerals examples of both signs have been found by Voigt.

Finally we have the linear compressibility as a function of pressure. The measurements were made in the regular way with the lever apparatus for short specimens, and the consistency of the results was that usual for this kind of measurement. The results are as follows:
Parallel to trigonal axis

$$-\frac{\Delta l}{l_0} = 16.48 \times 10^{-7}p - 20.5 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$
$$= 16.37 \times 10^{-7}p - 18.0 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

Perpendicular to trigonal axis

$$-\frac{\Delta l}{l_0} = 5.256 \times 10^{-7}p - 4.56 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$
$$= 5.091 \times 10^{-7}p - 3.04 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

This gives for the cubic compressibility:

$$-\frac{\Delta V}{V_0} = 26.99 \times 10^{-7}p - 31.6 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$
$$= 26.55 \times 10^{-7}p - 25.3 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

The unit of pressure is here the kg/cm^2.

The decrease of compressibility with rising temperature, both in the first and the second degree terms, is an unusual feature of the behavior of antimony.

Tellurium. The material used in all the final measurements was obtained from the Raritan Copper Works, who were so kind as to especially refine it for me in order to " completely " remove the 1% of selenium which is the principal impurity of the ordinary commercial material. Professor Saunders was kind enough to make a spectroscopic examination, and could find no selenium, but the spectroscopic test is known not to be sensitive. He did find some Ag and Cu, but no Fe, and considers it on the whole very pure.

Tellurium shows the same difficulty as bismuth in growing single crystal grains, but in an enhanced degree, in that the casting is very likely to consist of a number of grains all oriented very much alike. Toward the end of this work I found that this could be avoided by rapid cooling, and I obtained several rods 6 mm. in diameter which were single grains. The preferred direction of growth is exactly the same as with the other metals, that is, with the plane of easiest cleavage parallel to the axis of the casting. The analogy with the other metals ceases here, however, for in tellurium the cleavage on the basal plane, perpendicular to the axis of three-fold symmetry, is not at all well developed, but there are three planes of principal cleavage, inclined at 60° to each other and intersecting along parallel lines parallel to the axis of three-fold symmetry. The preferred direction of growth is with all three planes parallel to the axis of the casting, that is, with the three-fold axis parallel to the length, whereas with the other metals the axis of rotational symmetry is usually perpendicular to the length. Inclination of the cleavage planes to the length is facilitated by rapid crystallization. The limiting velocity which I was able to use for the 6 mm. rods, with a wall thickness of the glass mold of about 1 mm. was about 1 cm. per minute, lowering into oil. More rapid cooling produces crystallization in quite a different way, numerous nuclei starting at the walls, and growth proceeding radially toward the center, so that a microscopically crystalline aggregate is obtained, with the axes of all the grains on the radii of the cylinder. In fact one casting of this character was obtained with the 1cm. rate.

I found that the practice sometimes adopted of melting tellurium in an iron receptacle is fatal to purity, the alloying with iron, particularly at the higher temperatures, being comparatively rapid.

There is one difficulty unique to tellurium among the metals I have tried. The thermal expansion along the axis is abnormal in being of the negative sign, so that when the mold is cooled the metal expands greatly relatively to the glass along the length, and sometimes pulls the glass apart. This is a disadvantage, because internal strains are thereby produced which in this very brittle metal may produce partial separation of the cleavage planes. When I have occasion to try this metal again, I shall make the mold of as thin glass as possible.

Thermal Expansion. This was measured in the regular way with the regular apparatus, using two specimens whose linear compressibility was also measured. One of these was parallel to the trigonal axis, and was a single piece 2.7 cm. long. The second was perpendicular to the trigonal axis (one set of cleavage planes perpendicular to the length) and was also one piece, but was only 1.1 cm. long. The thermal expansion for a small range in the neighborhood of room temperature was:

$$\alpha_{\perp} = +27.2 \times 10^{-6},$$

$$\alpha_{\parallel} = -1.6 \times 10^{-6}.$$

The expansion is positive, as is normal, across the cleavage planes, that is, the cleavage planes tend to separate at the higher temperatures, presumably resulting in easier cleavage.

The volume expansion given by the linear expansions above is 52.8×10^{-6}, which is normal in magnitude for a metal.

Electrical Resistance. The resistance measurements on tellurium were often inconsistent, doubtless because of the formation of fissures along the cleavage planes by the internal stresses incident to cooling. I have made no measurements of the temperature or pressure coefficients, but have made a careful examination of the specific resistance. Even the best castings were not uniformly good, but have bad places in them, so that it is necessary to make a very careful examination. Resistance was measured in the regular way with the potentiometer, but the potential terminals were only 1 cm. apart, and these were moved along the specimen by small amounts (a few mm. at a time) and a resistance exploration made of the whole rod. It was usually possible to find lengths of several cm. over which the resistance remained constant and a minimum. The specific resistance was calculated from the resistance of these localities. The results are shown in Figure 9, together with the curve of the most probable resistance.

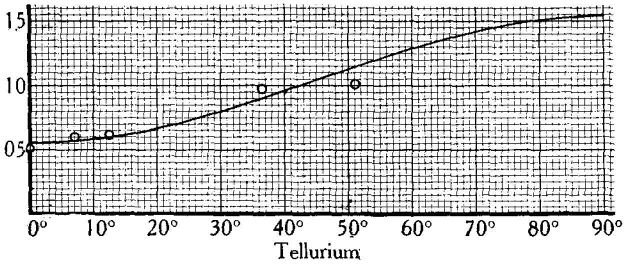

FIGURE 9. The specific resistance of tellurium at 20° C. as a function of the angle between the trigonal axis and the direction of flow.

I attempted to get the resistance perpendicular to the cleavage planes by the device of measuring the longitudinal resistance of one of the radially developed castings described above, but the values so found were impossibly high and had to be discarded. The explanation is without doubt to be found in fissures developed in the radial casting during cooling because of unequal thermal contractions in different directions.

The figure gives for the specific resistance at room temperature parallel to the trigonal axis 0.056 and perpendicular 0.154. The resistance is thus greatest across the cleavage planes, which is normal. The average resistance for the haphazard aggregate may be found to be 0.0973 ohms per cm. cube at 20° C.

This value for the resistance is much higher (30 times) than that which I have previously found for a specimen whose resistance under pressure I have measured,[4] and is doubtless to be explained by the much greater purity of the present specimen. So far as I know, this is the only example of the resistance being increased by increasing purity. It is perhaps not so surprising when one considers that tellurium is non-metallic in character in many respects, and that its specific resistance is of the order of 10000 times higher than that of its metallic impurities.

Comparisons of the value of the specific resistance found above with other values are of uncertain significance, because the higher resistance which we have seen to go with higher purity may equally well be explained by the presence of the fissures which are almost unavoidably formed in the crystalline aggregate. However it is worth while to record that Beckman[17] has found for the specific resistance values ranging from 0.05 to 0.62, and Matthiesen[18] gives the values 0.2. These are of the order of that given above.

Elastic Constants. Tellurium has six elastic constants, as do also bismuth and antimony, but I was not able to obtain a specimen of suitable orientation, and therefore have no value for the cross constant s_{14}. The other constants were obtained as follows. In the first place there are measurements of the linear compressibility in two directions, equations 1 and 2. Then there are direct determinations of s_{33} and s_{13} from the longitudinal compression under one-sided compression of an unmachined casting with length parallel to the trigonal axis, equations 3 and 4, a direct value of s_{12} from the lateral expansion under one-sided compression of one of the compressibility pieces, equation 5, and another direct value for s_{33} from the extension under tension of an unmachined rod, equation 6. For the shearing constant s_{44} there are torsion measurements on two rods with axis parallel to the length, giving directly (without the use of the other elastic constants) values of s_{44}, equations 7 and 8. The equations for the constants follow.

$$s_{11} + s_{12} + s_{13} = 28.0 \times 10^{-13}, \tag{1}$$

$$2s_{13} + s_{33} = -4.2 \times 10^{-13}, \tag{2}$$

$$s_{33} = 24.4 \times 10^{-13}, \tag{3}$$

$$s_{13} = -12.2 \times 10^{-13}, \tag{4}$$

(5) $$s_{12} = -6.85 \times 10^{-13},$$

(6) $$s_{33} = 23.7 \times 10^{-13},$$

(7) $$s_{44} = 56.8 \times 10^{-13},$$

(8) $$s_{44} = 59.4 \times 10^{-13}.$$

In selecting the compromise values to fit these equations, I assumed as always that the linear compressibility measurements are correct, although there is in this case no check by means of the cubic compressibility, which apparently has not been measured previously. For the two constants s_{33} and s_{13}, there are four equations to be satisfied. I made 2 exact, doing as little violence to the others as possible. The adjustment necessary was not serious, so that these constants must be regarded as fairly well established. For the two constants s_{11} and s_{12} we have only two equations 1 and 4. and there is no check, but nevertheless the results must be fairly good, because s_{12} is the result of a direct measurement, and s_{11} is the sum of positive terms. Finally for s_{44} we have the mean of equations 7 and 8, differing from each other by 4.5%. The final values for the constants in Abs. C. G. S. units are:

$$s_{11} = 48.7 \times 10^{-13},$$

$$s_{12} = -6.9 \times 10^{-13},$$

$$s_{13} = -13.8 \times 10^{-13},$$

$$s_{33} = 23.4 \times 10^{-13},$$

$$s_{44} = 58.1 \times 10^{+13},$$

$$s_{14} = \text{————————}.$$

For the linear compressibilities as a function of pressure we have measurements in the regular way and with the usual accuracy on specimens already described. The results in kg/cm² units for pressure are:

Perpendicular to trigonal axis,

$$-\frac{\Delta l}{l} = 27.48 \times 10^{-7}p - 52.7 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$

$$= 27.77 \times 10^{-7}p - 53.6 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

Parallel to trigonal axis,

$$+\frac{\Delta l}{l} = 4.137 \times 10^{-7}p - 9.6 \times 10^{-12}p^2, \qquad \text{at } 30° \text{ C.}$$

$$= 5.132 \times 10^{-7}p - 13.2 \times 10^{-12}p^2, \qquad \text{at } 75° \text{ C.}$$

Notice the astonishing fact that the compressibility along the trigonal axis is negative, that is, when the crystal is subjected to a hydrostatic pressure all over it elongates along the trigonal axis. This result I have already found for a casting made by slow cooling, but the precise numerical value was not obtained. Of course the cubic compressibility is of the normal sign, as may be seen from the very much larger compressibility of the normal sign in the perpendicular direction, and as it must be because of conditions of stability.

The cubic compressibility to be obtained from the above linear compressibilities is:

$$-\frac{\Delta V}{V_0} = 50.82 \times 10^{-7}p - 101.1 \times 10^{-12}p^2, \qquad \text{at } 30° \text{ C.}$$

$$= 50.41 \times 10^{-7}p - 85.6 \times 10^{-12}p^2, \qquad \text{at } 75° \text{ C.}$$

The cubic compressibility thus decreases at high temperatures. The compressibility is about twice that of antimony. If one tries to connect this value of the compressibility with the periodic properties of the elements, as Professor Richards[8] has done with his curves of compressibility, it will be found that when plotted as a function of atomic weight in the usual way it falls entirely off the curve, coming after iodine in weight and being much less compressible. But when plotted against atomic number, the relative positions of iodine and tellurium are reversed, and the compressibility now comes between that of antimony and iodine, as does also the atomic number.

Tin. The material used was sometimes melting point tin from the Bureau of Standards (the analysis has been published in connection with my previous work on compressibility and resistance, the total impurity being about 0.012%), and sometimes Kahlbaum's purest tin. There appeared no difference between tin from the two sources, the electrical resistance being the same, and they have been used indiscriminately in obtaining the final results.

Tin is among the more difficult, of the metals tried here, to produce in single grains, there often being a reversion from one grain to another so that the first part of the rod is one grain and the rest another of entirely different orientation. The slowest rates of crystallization produced the best results. It is also necessary to pay scrupulous attention to removing all specks of dirt and all occluded air. There seems no such unique preference as to manner of growth which some of the other metals show, but the axis of four-fold symmetry prefers to be either parallel or perpendicular to the length of the casting; I obtained no castings with angles between 20° and 74°.

More trouble was experienced in locating the axes of the crystal than with most other metals. Before the necessity of scrupulous freedom from all film of oxide on the surface was recognized I obtained several rods which appeared at first sight to be single crystals, but later were found to be multiple. This is probably connected in some way with the polymorphic transition which tin experiences on cooling below 160°. I found that the existence of several grains could be brought out by etching with dilute HCl with a few per cent chromic acid. Doubtless the reflection pattern shown before etching is that appropriate to the high temperature modification, whereas of course, the etching brings out the actual present structure of the crystal. With removal of all oxide and the obtaining of perfectly clean castings, this difficulty disappeared, however. It would seem that the surfaces of the microscopic pits undergo change also on passing through the transition point if there is no surface coating to hinder them.

The reflection pattern of the ordinary modification shows perfectly the tetragonal symmetry of tin, and affords a perfect method of identifying the crystallographic axes, but I nevertheless experienced considerable trouble at first because it appears that tin does not develop the basal plane perpendicular to the tetragonal axis as one of the surfaces of the reflection pattern, as do most of the other metals. The pattern which is usually developed is composed of the {1 1 1} and the {1 1 0} planes. Of course all these planes do not usually appear, because of unfavorable situation for reflection. I found that the construction of a card board model of the complete reflection pattern was of considerable assistance.

Examination of the reflection pattern may well be supplemented by examination of the mechanical flow pattern, particularly that obtained by compressing a short cylinder between polished steel plattens. For some orientations slip takes place discontinuously on the basal planes with a slight click, leaving a beautiful tracery of parallel slip lines on the flat faces. In other orientations the circular section of the cylinder changes into a perfect ellipse, the generating lines of the cylinder remaining straight. I tried to develop the possibility of cleavage in tin by violent deformation at liquid air temperatures, but the plasticity persists and no cleavage appears. A similar attempt with cadmium was also without success.

After the special details of growth and identification had been worked out, I was successful in obtaining castings of all sizes from 2 mm. to 2.2 cm. in diameter.

Thermal Expansion. The determinations were made in the regular way on the same specimens as those used for the linear compressibility. These were machined from a larger casting. The specimen perpendicular to the tetragonal axis was a single piece 2.76 cm. long; the parallel specimen was in two pieces of total length 2.44 cm.

The results for a small range of temperature in the neighborhood of room temperature are:

$$\alpha_{1} = 15.45 \times 10^{-6},$$

$$\alpha_{11} = 30.50 \times 10^{-6}.$$

The expansion is nearly twice as great along the axis as at right angles; this is the normal direction of difference.

The average thermal expansion for a haphazard aggregate given by the above is 20.4×10^{-6}. This may be compared with 22.3 at 40° C. by Fizeau, and 23.0, mean between 0° and 100°, by Matthiesen. The difference is in the direction and of approximately the magnitude to be accounted for by the difference of temperatures.

Electrical Resistance. The specific resistance at room temperature was measured by all three methods described in the introduction. The crystal structure of tin seems less likely to be upset by machining than that of some of the other metals (the deformability not being as extreme as it is for cadmium and there being no cleavage) and much more consistent results were obtained with the methods requiring machining than were obtained with the other metals.

The collected results for the specific resistance at 20° C. are shown in Figure 10. The circles are the results for the natural unmachined rods, the crosses for the machined 3 mm. rod between copper terminals, and the crosses within the circles the data obtained by the four point method. The results with the unmachined rods are to be given the greatest weight, and they all seem to lie on the theoretical curve with an error considerably less than 1%. The specific resistances to be deduced from the figure are 9.90×10^{-6} perpendicular to the tetragonal axis, and 14.30×10^{-6} parallel to the axis.

The average specific resistance for a collection of haphazard crystals calculated from the above is 11.06×10^{-6}. This is materially less than the value 11.5 at the same temperature given in the Bureau of Standards circular. The difference is in the direction to be accounted for by greater purity of my sample.

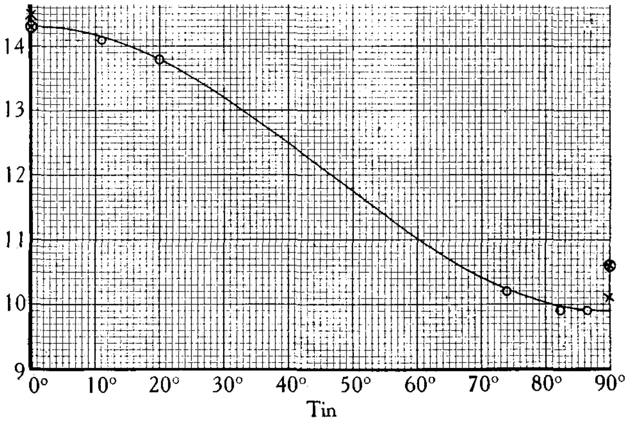

FIGURE 10. The specific resistance of tin at 20° C. as a function of the angle between the tetragonal axis and the direction of flow.

The variation of resistance with temperature at atmospheric pressure was measured for the two unmachined rods inclined at 11° and 74° with the length. The 11° rod was measured up and down the temperature range with a very small permanent change of resistance; the 74° rod was only measured up, and at the two extremes, but the permanent change of resistance after a pressure change of 12000 kg. was inappreciable, so that there is every reason to anticipate no trouble with the temperature effects.

Temperature	11° rod Resistance	74° rod Resistance
0°	1.000	1.000
25	1.107	
50	1.217	
75	1.330	
100	1.446	1.464

The temperature coefficient thus appears greatest perpendicular to the axis, which is the direction in which the resistance is least, so that the differences of resistance with direction become less accentuated at higher temperatures. The temperatures coefficient of specific resistance in these two directions may be found to be 0.00446 at 11° (correction inappreciable), and 0.00467 at 74°. Now to find the temperature coefficients of specific resistance along and at right angles to the axis we have two simultaneous equations in terms of the values for the 11° and the 74° directions. Solving the equations, we find for the average temperature coefficient of specific resistance between 0° and 100° C.:

Parallel to the tetragonal axis	0.00447,
Perpendicular to the tetragonal axis	0.00469.

I have previously found for the mean coefficient of measured resistance of extruded wire of Kahlbaum's tin the value 0.00447, essentially the value above for the direction parallel to the axis.

The effect of pressures to 12000 kg. on the resistance at 0° and 95° C. was measured on the same 11° and 74° specimens discussed above. The usual method was used with the usual accuracy, the deviations from a smooth curve of the average reading being of the order of a few tenths of a per cent. The resistance decreases under pressure, as is normal, and the rate of decrease itself becomes less at the higher pressures. At 0° C. the relation between pressure and resistance was representable by a second degree equation in the pressure within the limits of error, but at 95° the relation could not be given so simply. I have represented the results at this temperature in the way previously adopted, namely giving the average pressure coefficient between 0 and 12000 kg., and tabulating in addition as a function of pressure the correction term which must be added to the decrease of resistance calculated by a linear relation with the mean coefficient. The results follow.

Tetragonal axis 11° to length:

0° C. $$-\frac{\Delta R}{R_0} = 1.043 \times 10^{-5}p - 9.35 \times 10^{-11}p^2, \quad (p \text{ in kg/cm}^2).$$

95° Average coefficient to 12000 -9.576×10^{-6}.

The deviations from linearity are given in Table III.

Tetragonal axis 74° to length:

0° C. $$-\frac{\Delta R}{R_0} = 9.696 \times 10^{-6}p - 9.70 \times 10^{-11}p^2,$$

95° Average coefficient to 12000 -8.453×10^{-6}.

The deviations from linearity are given in Table III.

TABLE III

Pressure kg/cm²	Deviations from Linearity 11° rod	74° rod
0	.00000	.00000
1000	.00146	.00118
2000	257	202
3000	335	268
4000	398	309
5000	420	333
6000	409	325
7000	370	295
8000	325	258
9000	263	204
10000	191	148
11000	107	81
12000	.00000	.00000

This method of representing the results will be made clearer by an example. Thus suppose we wish to find the change of resistance at 95° C. of the 74° rod under 8000 kg/cm². The decrease calculated by the linear relation, using the average coefficient above, is $8000 \times 8.453 \times 10^{-6}$, which is equal to 0.06762. To this is to be added the deviation from linearity given in the table for 8000 kg, or 0.00258, making for the decrease 0.07020. That is, the resistance of this rod at 95° C. under 8000 kg. is 0.92980 of the resistance at atmospheric pressure at 95° C.

The above results show that the decrease of resistance under pressure is most rapid in the direction of the axis, that is, the direction in which the specific resistance is the greatest. The differences of resistance with direction in the crystal thus become less accentuated at high pressures. This is also the effect of increasing the temperature; it is more usual that the effect of an increasing pressure is similar to that of a decreasing temperature. The pressure coefficients of resistance in different directions vary much less than the specific resistances in those directions, so that the equalizing of resistance by pressure is a very slow process.

With the values of the elastic constants to be given presently we may calculate that the initial pressure coefficients of resistance at 0° C. are:

Tetragonal axis 11° to length -1.095×10^{-5},
" " 74° " " -1.035×10^{-5}.

The difference of measured coefficient with direction is hence nearly the same as the difference of specific coefficient with direction.

The formulas of the introduction now enable us to find the pressure coefficient of specific resistance perpendicular and parallel to the axis. We find at 0° C.:

Initial pressure coefficient of specific resistance parallel to the axis -10.96×10^{-6}.

Initial pressure coefficient of specific resistance perpendicular to the axis -10.28×10^{-6}.

Elastic Constants. Tin being tetragonal, has six elastic constants. It differs from the metals hitherto measured in that there are two independent shearing constants, so that it is necessary to somewhat modify the procedure. It is fortunate that castings could be obtained with the axis nearly parallel and nearly perpendicular to the length. The following equations were obtained for the elastic constants.

$$(1)\quad 2s_{13} + s_{33} = 6.83 \times 10^{-13},$$

$$(2)\quad s_{11} + s_{12} + s_{13} = 6.13 \times 10^{-13},$$

$$(3)\quad s_{11} = 17.3 \times 10^{-13},$$

$$(4)\quad .9876s_{11} + .0037(2s_{13} + s_{44}) + .0012(2s_{12} + s_{66}) = 20.3 \times 10^{-13},$$

$$(5)\quad .9876s_{11} + .0037(2s_{13} + s_{44}) + .0012(2s_{12} + s_{66}) = 15.2 \times 10^{-13},$$

$$(6)\quad .9540s_{11} + .0168(2s_{13} + s_{44}) + .0061(2s_{12} + s_{66}) = 22.2 \times 10^{-13},$$

$$(7)\quad .0012s_{11} + .9283s_{33} + .0351(2s_{13} + s_{44}) = 20.1 \times 10^{-13},$$

$$\begin{aligned}(8)\quad & .9662(s_{11} - s_{12}) + .4834s_{44} \\ & + .0167\{2(s_{11} + s_{33} - 2s_{13}) \\ & + \tfrac{1}{2}(s_{66} - s_{44})\} + .4709\{s_{66} - 2(s_{11} - s_{12})\} = 83.7 \times 10^{-13},\end{aligned}$$

$$\begin{aligned}(9)\quad & .9926(s_{11} - s_{12}) + .4963s_{44} \\ & + .0037\{2(s_{11} + s_{33} - 2s_{13}) \\ & + \tfrac{1}{2}(s_{66} - s_{44})\} + .4926\{s_{66} - 2(s_{11} - s_{12})\} = 106. \times 10^{-13},\end{aligned}$$

$$\begin{aligned}(10)\quad & .0013(s_{11} - s_{12}) + .9290s_{44} \\ & + .0352\{2s_{11} + 2s_{33} - 4s_{13} - \tfrac{1}{2}s_{44} + \tfrac{1}{2}s_{66}\} = 56.8 \times 10^{-13},\end{aligned}$$

The equations are more complicated than for the previous substances. The complication arises from the fact that when deformations are measured of rods inclined to the axis, it is necessary to know the position of the three crystallographic axes, instead of merely the axis of rotational symmetry, which sufficed in all previous cases except for the constant s_{14} of the trigonal system. The equations were obtained as follows. 1 and 2 are given by linear compressibility measurements of the samples already described under thermal expansion. These equations are taken as correct in the calculations. A check is afforded by the cubic compressibility calculated from them, which is 19.09×10^{-13}, against 18.7 by Adams, Williamson, and Johnston, and 19 by Richards. Equation 3 comes from the longitudinal compression under one-sided compression of one of the compressibility samples. 4 comes from the longitudinal compression under one-sided compression of an unmachined casting with axis at 86.5° to the length, 5 from the extension under tension of the same rod as used in 4, and 6 from the extension under tension of an unmachined rod with axis at 82.5° to the length. 7 is from the extension under tension of an untouched rod with axis at 11° to the length. 8 and 9 are from torsion measurements of the rods of equations 4 and 6, and 10 from torsion of the rod of equation 7.

It will be noticed that some of the equations contain predominantly only one of the constants, as 4, 5, and 6 contain predominantly s_{11}. Hence, in these, very rough values for the other constants will suffice, which may be obtained by a solution of a sufficient number of the other equations. In solving the equations I first obtained the value of s_{11} from 3, 4, 5, and 6 (using the approximate values of the other constants as already explained). The values so found were 17.3×10^{-13}, 20.2, 21.4, and 15.2 respectively. As the best value of s_{11} I took the average, 18.5×10^{-13}. Equation 7 gives a value of s_{33}, $11.8 = 10^{-13}$. These values now substituted into 1 and 2 give the values accepted as final for s_{12} and s_{13}. This completes the list of rectangular constants. For the shearing constants there are equations 8, 9, and 10. 8 and 9 are of approximately the same form, and I took the average of these, averaging the coefficients straight through. The average differs from the extremes by something of the order of 15%, so that the accuracy is not high. Equation 10 differs in form markedly from the average of 8 and 9. These two equations were now solved simultaneously for s_{44} and s_{66}.

Of the constants obtained by the procedure above it will be seen that there are no checks on s_{33}, s_{12}, and s_{13}. The first of these should be fairly good, since it is given essentially by direct measurement. The two others are less certain. The check on the shearing constants, which consists of three equations for two quantities, is not as close as desirable, but the essential result cannot be in doubt that s_{66} is much larger than s_{44}.

The final results are:

$$s_{11} = 18.5 \times 10^{-13},$$

$$s_{12} = -9.9 \times 10^{-13},$$

$$s_{13} = -2.5 \times 10^{-13},$$

$$s_{33} = 11.8 \times 10^{-13},$$

$$s_{44} = 57.0 \times 10^{-13},$$

$$s_{66} = 135. \times 10^{-13}.$$

The second shearing constant s_{66} differs greatly from the value which it would have in the hexagonal, trigonal, or cubic systems. In these systems we have the relation $s_{66} = 2(s_{11} - s_{12})$. Substituting the values above for tin, the right hand side of the equation becomes 56.8 against 135 experimental, or less than half.

We have finally the linear compressibility as a function of pressure to 12000 kg. The measurements were made in the small lever apparatus, using the specimens described under thermal expansion. The data were of the usual regularity, and gave the following results, pressure in kg/cm².

Length perpendicular to tetragonal axis

$$-\frac{\Delta l}{l_0} = 6.022 \times 10^{-7}p - 4.20 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$

$$= 6.144 \times 10^{-7}p - 4.26 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

Length parallel to tetragonal axis

$$-\frac{\Delta l}{l_0} = 6.719 \times 10^{-7}p - 4.07 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$

$$= 6.956 \times 10^{-7}p - 3.91 \times 10^{-12}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

The linear compressibility is thus greatest across the basal plane, which is normal. The difference of compressibility in different directions is, however, much less than normal.

The change of volume as a function of pressure is:

$$-\frac{\Delta V}{V_0} = 18.76 \times 10^{-7}p - 13.6 \times 10^{-12}p^2, \qquad \text{at } 30^\circ \text{ C.}$$

$$= 19.24 \times 10^{-7}p - 13.7 \times 10^{-22}p^2, \qquad \text{at } 75^\circ \text{ C.}$$

MAGNETIC PROPERTIES.

A very rough examination was made for differences of magnetic permeability in different directions. The single crystal, in the form of a cylinder about 1.25 cm. in diameter and 2.8 cm. long, was suspended by a comparatively heavy strand of silk with its axis vertical between the conical poles of an electro-magnet giving something of the order of 15000 Gauss. The principal axis of the crystal was perpendicular to the vertical axis of the cylinder. Any tendency to orientation in a definite direction was observed. Results were found only with bismuth and antimony, and here the effects were large, there being a very strong directive action. Bismuth sets itself with the trigonal axis parallel to the lines of force. That is, the magnetic permeability is algebraically greatest along the trigonal axis in bismuth. This observation was made a long time ago, perhaps first by Tyndall. But antimony acts oppositely, setting itself with the *cleavage plane* parallel to the lines of force. The action is, if anything, stronger than in bismuth. So far as I know, this observation is new.

No effect was found with the other metals. This may have been partially masked by the very sluggish action in the magnetic field due to eddy current damping. An interesting effect was found with tin. There is no effect on making the magnetic field, but if the field is destroyed when the tetragonal axis is inclined to the lines of force, there is a rotational kick, in such a direction as to allign the tetragonal axis and the magnetic lines. The kick vanishes if the axis originally lies along the lines of force. The effect is evidently due to a reaction between the eddy currents induced in the tin and the magnetic field, the eddy currents not being in the plane perpendicular to the lines of force when the axis is not parallel because of the inequality of specific resistance in different directions. The effect was not found with the other metals; the difference of resistance of tin in different directions is greater than of the other metals.

SUMMARY AND GENERAL SURVEY OF RESULTS.

In Table IV are collected some of the principal results, namely the linear compressibilities, the two elastic constants s_{11} and s_{33} which give the extensions under simple tensions in the principal directions, the thermal expansion, the specific resistance, the temperature coefficient of specific resistance, and the initial pressure coefficient of specific resistance. In the detailed presentation, all the elastic

TABLE IV.

SUMMARY OF RESULTS ON NON-CUBIC METALS.

Metal	Initial linear Compressibility at 30° Pressure unit, 1 kg/cm²	Elastic Constants Abs. C. G. S. s_{11} ⊥ s_{33} ∥	Linear Thermal Expansion at 20° C.	Specific Resistance at 20° C.	Average Temperature Coefficient of Specific Resistance 0°–100° C.	Initial Pressure Coefficient of Specific Resistance at 0° C. Pressure in kg.
Zn ∥	12.98×10^{-7}	26.38×10^{-13}	57.4×10^{-6}	6.13×10^{-6}	.00419	-10.87×10^{-6}
⊥	1.95	8.23	12.6	5.91	418	− 6.55
Cd ∥	18.3	36.9	52.5*	8.30	428(?)	−13.1
⊥	2.1	12.9	20.2*	6.80		− 8.7
Bi ∥	15.92	28.7	13.96	138.	445	+24.5
⊥	6.62	26.9	10.36	109.		+ 7.5
Sb ∥	16.48	33.8	15.56	35.6	595	+17.2
⊥	5.26	17.7	7.96	42.6	511	+ 1.05
Te ∥	−4.14	23.4	−1.6	56000.		
⊥	27.48	48.7	27.2	154000.		
Sn ∥	6.72	11.8	30.50	14.3	447	−10.96
⊥	6.02	18.5	15.45	9.9	469	−10.28

* From Grüneisen and Goens.

constants are given, and the effect of pressures to 12000 kg/cm^2 on the other properties. In addition to the results on non-cubic metals shown in the Table, the elastic constants and compressibility of a single tungsten crystal have been determined.

It is evident that the cleavage plane, or plane of easiest slip in case the crystal does not cleave (this plane is also the basal plane perpendicular to the axis of rotational symmetry for all the metals except tellurium), corresponds to some fundamental fact in the crystal structure which is reflected in the other properties. The atoms are connected more loosely across the cleavage plane, so that external forces produce greater effects in this direction than in others. The linear compressibility is always greatest across the cleavage plane, and in fact in the case of tellurium the linear compressibility is negative in the cleavage plane. The extensibility under tension (effective Young's modulus) is also greatest across the cleavage plane, with the exception of tin, in which the reversal is rather surprisingly large in magnitude. The thermal expansion is also greatest across the cleavage plane, and in the case of tellurium the expansion in the plane is actually negative. The specific resistance is also greatest for current flow across the cleavage plane, with the exception of antimony, which, however, constitutes only a temporary exception, as it becomes normal under high pressure. The temperature coefficients of resistance do not seem to show large variations with direction, with the exception of antimony, which again may be regarded as a temporary anomaly disappearing at high pressures. The effect of pressure on specific resistance is also greatest across the cleavage plane, whether that effect is an increase or a decrease of resistance (tellurium not measured).

A detailed examination does not show any striking quantitative uniformities, but in general the statement is justified that the various electrical properties show much less variation with direction than do the elastic properties or the thermal expansion. This is in general agreement with what we might expect, the elastic constants being intimately associated with a property of the atoms somewhat like that of a rigid boundary, which prevents them from telescoping into each other when pushed together, whereas the rigid boundaries do not so effectively exist for that migration of the electrons which constitutes the electrical current.

The most interesting special results are probably the negative expansion and compressibility of tellurium along the axis, the maximum of resistance and the reversal of the axial ratio at high pressures of antimony, and the two new high pressure modifications of cadmium.

We are not yet in a position to give a theoretical account of these results, so simple a thing as the shearing constant of rock salt being as yet unexplained. It is perhaps significant that the shearing constant of a crystal of tungsten has been found to be connected with the other two constants by nearly the same relation as in an isotropic body.

It is a pleasure to acknowledge the assistance received from Mr. Walter Koenig in making most of the measurements of this paper.

THE JEFFERSON PHYSICAL LABORATORY,
Harvard University, Cambridge, Mass.

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VARIOUS PHYSICAL PROPERTIES OF RUBIDIUM AND CAESIUM AND THE RESISTANCE OF POTASSIUM UNDER PRESSURE.

BY P. W. BRIDGMAN.

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Introduction.

The alkali metals are the most compressible of the metals and it is therefore probable that they will show the most interesting and significant variations of physical properties under high pressure. I have previously published the results of various high pressure measurements on lithium, sodium, and potassium,[1] but have not examined the two heaviest and most compressible members of the series, rubidium and caesium. The reason for this omission has been two fold: the manipulations offer certain technical difficulties, and it is not easy to obtain a sufficiently large quantity of these two metals in a state of high purity.

In this paper are given new results on rubidium and caesium concerning those same properties which have been previously measured for the three lighter metals. These properties include in the first place a determination of the effect of pressure on melting up to 100° C., which involves the determination of the effect of pressure on the melting temperature and on the difference of volume between liquid and solid (from which the latent heat of melting can be calculated as a function of pressure and temperature up to 100°), secondly the electrical resistance of both solid and liquid phases as a function of pressure up to 12000 kg./cm.[2] at temperatures between 0° and 100°, and thirdly the compressibility to 15000 kg./cm.[2]

I have already published preliminary results on caesium which now appear to be incorrect.[2] I had apparently found a new modification of caesium at high pressures which has the abnormal property that its resistance increases with increasing pressure. It now appears that the apparent transition was due to an impurity, which separated out of solid solution at sufficiently high pressure, or else delayed the ordinary freezing; actually there is no discontinuity, but instead caesium has the unique property that its electrical resistance passes through a minimum with increasing pressure. This property, particularly because Cs is the most compressible of the metals, should be of considerable theoretical significance. Since this incorrect preliminary result I have made a large number of measurements on caesium in order to be quite sure of the result. In my earlier work I followed the universal previous practice of measuring the electrical properties of the metal enclosed in a glass capillary. But various experimental irregularities showed that the constraining effect of the glass might be of considerable importance (for although the stresses which these metals can support without yield are small, nevertheless the variations with stress of the various properties are unusually high), so that eventually I was driven to making measurements on bare wires of rubidium and caesium. In my previous work I had used bare wires of lithium and sodium, but the mechanical dificulties in the case of the much softer metal potassium had led me to use the conventional glass capillaries for it. With the experience now gained with these still softer metals, I have now returned to potassium, and have repeated the measurements of the pressure effects on the bare wire. I find that the pressure coefficient is not very much affected, but the conclusions which I had previously drawn as to the effect of pressure on the temperature coefficient of resistance must now be essentially modified. These new results on the pressure coefficient of resistance of potassium are also given in this paper.

EXPERIMENTAL DETAILS.

Preparation of the Metals. The importance and the difficulty of obtaining these metals in a state of high purity is not usually realized, and many results have been published on material of insufficient purity. With care in the preparation it is not difficult to obtain material free from metallic impurities, but the removal of oxide is a matter of greater difficulty. The oxide dissolves in these two metals, and behaves in some respects like an ordinary metallic impurity, as in depressing the freezing point. The usual method of preparation is by heating the chloride in contact with metallic calcium. It is comparatively easy to get the chloride free from foreign metals, but it is just as important that the calcium be pure, and in particular it should not contain any of the other alkali metals, which are difficult to remove at any later stage. Prepared in this way, with moderate heating, there is little likelihood of the rubidium or caesium containing other metals, but it is almost certain to contain oxide. This must be removed by slow distillation at the lowest feasible temperature in very high vacuum. It is practically impossible to make a satisfactory distillation with a gas flame, but an electric oven should be used.

The caesium used in my preliminary experiments was obtained from several sources. I am indebted to Professor G. N. Lewis for a generous supply of the metal, to Professor G. P. Baxter for a large quantity of highly purified chloride, and to the Research Laboratory of the General Electric Company for extracting the metallic caesium from the salt with calcium. I also obtained several grams of the metal from the Foote Mineral Co. All of this preliminary material, however, did not give satisfactory results, the best method of manipulation not having been found. The final results were obtained with metallic caesium from Kahlbaum. As provided by them it was stated to be entirely free from any foreign metals, but was obviously not pure, as was evidenced by the long temperature range over which melting took place. The caesium was provided sealed into glass tubes under a heavy white mineral oil. From these tubes it was transferred under Nujol to a distilling arrangement of Pyrex glass, and the Nujol washed out with petroleum ether, leaving a little ether in the apparatus, so that the metal was at no time uncovered. The glass was then sealed and evacuated to as high a degree as possible with a diffusion pump, heating all parts of the apparatus. A preliminary distillation was made from the receiving chamber to the first bulb, in this way removing the coarse dirt. The glass container was then transferred to an electric oven, where the metal was distilled from one bulb to the next at a temperature so low that between one and two hours was required for a transfer. After each transfer the vacated bulb was sealed off from the remaining part of the apparatus. In all, five such distillations were made. At the farther end of the apparatus was the arrangement adapted to the special experiment in hand, and which will be described in detail later. This method of preparation was adopted for all the material used in the final measurements.

The best test of the purity of the material (the presence of oxide being difficult to establish by the ordinary methods of spectroscopic analysis) is the sharpness of the freezing point. The most convenient method is by measurements of the electrical resistance, but there is a danger here which must be especially mentioned, which has probably in the past given rise to illusory results. It is of course well known that the greater the purity of the metal the higher its temperature coefficient of electrical resistance, other things being equal, so that under ordinary conditions a high temperature coefficient is pretty good evidence of high purity. But near the melting point this is obviously no longer the case, because the resistance increases when the metal melts, and premature melting may be brought about by impurity, thus simulating an improperly high temperature coefficient. This source of error is important only for the low melting metals, and is of course especially important for rubidium and caesium. It is not sufficient, therefore, to measure the temperature coefficient of caesium between 0° and 20°, for example, and to infer from its high value the high purity, but the resistance must be measured all the way up to the melting point and into the liquid state, if the metal is in a glass capillary. The corners of the discontinuity on melting must not be rounded. In practice it is easy to distinguish a rounding of the corner due to premature melting from the upward curvature due to the normal accelerated increase of resistance with rising temperature, provided the purity is high, but it is more difficult or impossible if there is considerable impurity.

Measurements of the temperature dependence of resistance should preferably be made by the method of stationary temperatures, instead of by the method of continually varying temperatures which is so often used. This latter method of necessity introduces some rounding of the corners due to varying temperature lag when there are such thermal effects as found in melting, and can be made reliable only by varying the temperature so slowly as to become virtually a stationary temperature method. The stationary temperature method, on the other hand, suffers from the disadvantage of not giving the exact melting temperature, but only shuts it between limits, which may be made narrower by longer experiment or greater good fortune. In my experiments this disadvantage was avoided by a second method of studying the sharpness of melting, namely by varying the pressure at constant temperature, measuring the volume as a function of pressure. Since the pressure can be manipulated much more easily than the temperature, and furthermore since the melting pressure automatically establishes itself within a certain range, it is possible to find exactly the melting pressure at any temperature, or to find whether this pressure varies with the fractional part of the metal which is melted, and so to study the sharpness of freezing. By both these tests, the freezing of the rubidium and caesium was unusually sharp, and the melting point so found was unusually high and presumably is close to the value for the absolutely pure metal.

Rubidium was made in the same way as the caesium, except that I made no measurements with preliminary material, but the source of all my material was metallic rubidium sealed under oil into glass obtained from Kahlbaum. The distillation temperature is higher than for caesium, but not high enough to make necessary any change in the general method of manipulation. Because of its higher melting point it is more convenient to manipulate at room temperature.

Experimental Methods. No new methods were necessary in making the measurements, but those were employed which have already been described in full detail. The resistances to be measured were of the order of a small fraction of an ohm, so that the potentiometer method with four leads was used which has been previously employed in measuring the effect of pressure on small resistances.[3]

The melting curves were traced by the method of the discontinuity of volume already used in connection with many melting and transition curves.[4] The apparatus was smaller than used before, and was in one piece instead of two, which eliminated the necessity for some corrections, but was in principle the same as that used before. The temperature limit of these measurements, 100° C., was set by the particular form of apparatus used. It would be interesting to follow the melting curve to higher temperatures (I have been as high as 275° with bismuth), but this would have demanded the construction of special apparatus, and it did not seem to me that there were any questions likely to be answered by an extension of the range sufficient to justify the construction of special apparatus for only two substances. On one or two occasions I also obtained the melting pressure corresponding to a given temperature by measurements of the electrical resistance. These results agreed with the others.

The compressibility was measured by two methods. One was the method for measuring linear compressibility which I have previously applied to a number of metals.[5] There were difficulties because of the extreme softness of the metals, particularly with caesium. The second method is that for measuring the cubic compressibility by determining the position of the piston of the compressing apparatus, which I have applied to 12 liquids,[6] and which has later been applied at the Geophysical Laboratory to a number of solids.[7] The apparatus was on a smaller scale than that previously used, and was the same as that used recently in determining the compressibility of gases,[8] by a method in all essentials the same. The pressure range was also, as for gases, 15000 kg./cm.^{2}, instead of the more usual 12000. The measurements of the cubic compressibility and of the melting curve were made with the same apparatus and usually with the same filling of the apparatus. There were various difficulties in the compressibility measurements which make these the least satisfactory of this paper. It was a disappointment that these were not accurate enough to permit more than very rough statements about the behavior of the thermal expansion under high pressures.

NUMERICAL DATA.

The numerical data for the various effects, together with such detailed description as seems necessary, now follow.

Potassium.

Resistance. These measurements on potassium were made after the measurements on rubidium and caesium, and were suggested by the apprehension that the previous results found for potassium in a glass capillary might be in error because of a restraining action of the glass. The point of particular interest is the temperature coefficient of resistance at high pressures; I had found that beyond 6000 kg. the temperature coefficient of potassium decreases greatly.[9] This was a unique phenomenon, the temperature coefficient of all the other metals measured being approximately independent of the pressure. I connected the unusual decrease with the unusual compressibility of potassium, and expected that the same effect would be found in other metals at a pressure sufficiently high to produce in them a comparable volume compression. This supposed decrease of the temperature coefficient of potassium and the ideas suggested by it now turns out to be incorrect, the effect found being, as feared, due to the constraining effect of the capillary.

The potassium used in this investigation was from the same batch as that whose resistance, melting, and compressibility to high pressures has been previously measured. It has been kept since the former measurements sealed under Nujol in a small glass tube. For these measurements it was extruded under Nujol to a bare wire of about 1.5 mm. diameter. The four connections, two for current, and two for potential, were made by piercing through the potassium with very fine copper wires, looped into place. The potassium wire was then folded at the middle to the shape of a hairpin and placed in a small glass test tube filled with Nujol with the connections folded over the edge. Connections were now made to the three terminal plug in the regular way, so that everything could be screwed as one self-contained unit into the pressure apparatus. Before this was done the Nujol in the test tube was replaced with petroleum ether by repeatedly flushing out the tube with ether. This is necessary to avoid distortion effects at high pressures due to viscosity of the oil.

Measurements were made in the first place of the temperature coefficient of resistance at atmospheric pressure from 0° to 35° and back again. It is to be emphasized that no measurement, either of a pressure or a temperature effect, on these soft and chemically active metals is trustworthy unless a complete cycle is made, returning to the starting point, because of the great liability to permanent changes of resistance after a change of pressure or temperature. After the temperature cycle above, 0° to 35° to 0° again, there was a permanent increase of resistance of 5% of the total change produced by 35°. The mean of readings with increasing and decreasing temperature was taken as giving the true temperature coefficient. The relation between resistance and temperature is linear, and the average coefficient between 0° and 35° in terms of the resistance at 0° as unity was 0.00541. I previously found for potassium in glass between 0° and 51° a linear relation, and for the numerical value 0.00512, corrected for the volume expansion of the glass to give the temperature coefficient of specific resistance. The value 0.00541 found above for bare wire is the coefficient directly measured with terminals attached to the wire. The correction term to reduce from measured to specific resistance is the linear expansion. Assuming for this the value

0.00008, the best value for the temperature coefficient of specific resistance is 0.00549, about 7% higher than the value previously found on the same material. The difference is to be ascribed to the restraining action of the glass walls, which, as temperature is raised, exert a pressure which lowers the resistance. It is to be noticed that measurements in a glass capillary may have no internal evidence of being incorrect, but the measurements with increasing and decreasing temperature usually agree, with no sign of hysteresis. This is understandable over a moderate temperature range, but over a wider range it would be expected that the thermal stresses set up in the metal would produce flow, and so permanent alterations, with hysteresis.

The pressure measurements were made in the regular way with the regular apparatus up to 12000 kg. at 0°, after a preliminary seasoning application of pressure which produced a permanent alteration of resistance of 6% of the change due to pressure. The temperature coefficient at 5000 kg. and 11300 kg. was measured by raising temperature at each of these pressures from 0° to 30° and then lowering again to 0°. A readily determined correction must be applied for the small change of pressure accompanying the change of temperature. This procedure is much better than that previously adopted of determining the temperature coefficient from the results of complete pressure runs at different temperatures. The return to the initial resistance at 0° in the new measurements was complete within 0.3%, and considerable confidence may be felt in the temperature coefficients so found.

The results are shown in Table I, which gives the relative measured resistance as a function of pressure to 12000 kg. at 0°, and the temperature coefficient at 1, 5075, and 11300 kg.

TABLE I.

RESISTANCE OF POTASSIUM.

Pressure kg./cm.	Relative Resistance at 0° C.
0	1.000
1000	.844
2000	.729
3000	.638
4000	.564
5000	.502
6000	.451
7000	.408

TABLE I.—*Continued.*

Pressure kg./cm.	Relative Resistance at 0° C.
8000	.372
9000	.340
10000	.313
11000	.291
12000	.271

TEMPERATURE COEFFICIENT OF RESISTANCE BETWEEN 0° AND 30° C.

At 1 kg./cm.°	0.00541
5075	.00459
11300	.00454

When compared with the previous results, these new pressure data show the effect of the constraint by the glass, as did also the temperature coefficient at atmospheric pressure. The pure pressure effect is much less affected than the temperature coefficient. At 12000 kg. the ratio of measured resistance to that at atmospheric pressure at 0° is 0.271, which corrects to 0.246 for the ratio of specific resistances. Previously for potassium in glass, the ratio 0.275 was found at 25° and 0.252 at 60°. At the higher temperature the value previously found agrees better with the new value than the ratio at the lower temperature. This is consistent with the explanation as due to the effect of the constraint, because at the higher temperature the internal viscosity is less and the internal stresses will be less. The difference between the old and the new results is of the order of 10% as far as the pressure effect goes. At lower pressures the agreement is closer, as would be expected because of the smaller viscosity of the metal at lower pressures.

The new result for the temperature coefficient of resistance at high pressures when compared with the previous results amounts however, to a result of a different order of magnitude. Previously I found at 12000 kg. a decrease of the temperature coefficient by a factor of 2.5 fold. The decrease now found is only from 54 to 45, and the very marked decrease formerly found beyond 6000 is now seen not to exist.

The final result of these new measurements on potassium is therefore to leave unchanged the generalization which I made after my first series of pressure measurements,[10], namely that the temperature coefficient is little affected by pressure. This is what would be expected in view of the fact that the temperature coefficient of all pure metals is nearly alike, because the same metal at two different pressures may be regarded as a special case of two different metals.

Rubidium.

Melting Data. Little need be added in the way of description of experimental detail to that already given, except the method of filling the apparatus. The chief difficulty is in determining the amount of rubidium, since it must be kept continually immersed in oil to avoid oxidation. Connected to the glass bulb in which the final distillation was made was a small test tube of about 6 mm. inside diameter. The rubidium was run into this, and then the top and bottom of the test tube broken off under Nujol. The metal was now extruded from the tube with a closely fitting plunger, so that a rod of rubidium 6 mm. in diameter was obtained. This was cut to an appropriate length, and the amount determined by weighing under Nujol. The density of the Nujol was independently determined and for the density of rubidium the value 1.532 of Richards and Brink[11] was assumed. From these data the amount of rubidium may be calculated. The rubidium was then placed inside a steel cup of test tube shape, and the total weight in air determined, from which the amount of oil included in the steel cup with the rubidium could be found (this last being necessary for the compressibility measurements to be made with the same set-up, but not for the melting determinations), and then it was mounted in the regular way in the high pressure cylinder with a known amount of kerosene. As already mentioned, the small apparatus used previously for the compressibility of gases was used for this, the amount of metallic rubidium and caesium available not being sufficient to allow the use of the larger apparatus, which would otherwise have been desirable because somewhat more accurate results may be found with it. The area of the piston of the small apparatus was 0.43 cm.^{2}

Nine points on the melting curve were found, ranging from 250 to 3600 kg., and from 44° to 96°. The melting was very sharp and affords gratifying evidence of the purity. Thus at the lowest pressure, 250 kg., there was no perceptible difference in melting pressure between 0.1 and 0.9 melted. The sensitiveness was such that a difference of 4 kg./cm.^{2} could have been detected, corresponding to 0.08°.

Doubtless the best value for the melting point at atmospheric pressure is to be obtained by an extrapolation of the results found at high pressures. The extrapolation is short, a matter of five degrees, and may be made with considerable certainty by graphical methods, because the curvature of the melting curve is not important. The melting temperature at atmospheric pressure found in this way is 38.7°. This is 0.25° higher than the result found from measurements of electrical resistance in glass capillaries (temperature at which the metal is half melted), and differs from the electrical result in the direction to be expected, since any internal stress not a hydrostatic pressure equally distributed between solid and liquid results in a lowering of the melting point. The melting point given above is somewhat lower than the highest previously recorded,[12] 39.00.°

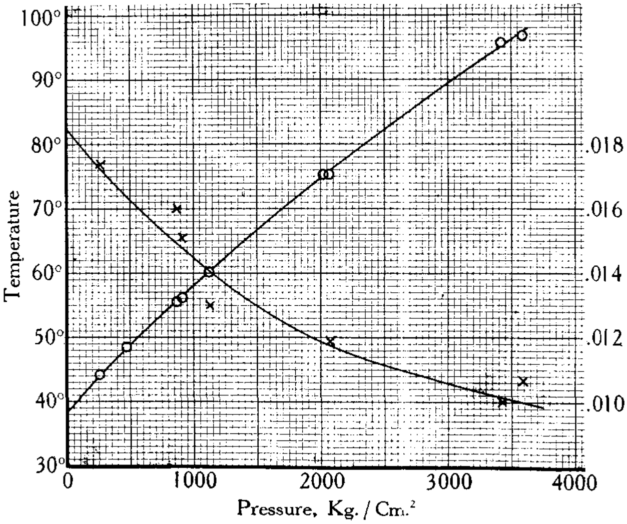

FIGURE 1. Melting data for Rubidium. The circles show the melting temperatures (scale on the left) as a function of pressure. The crosses show the observed changes of volume (scale on the right) in cm.³ per gm. as a function of pressure.

The changes of volume on melting were determined at the same time as the coordinates of the melting curve. The change of volume is small, so that the percentage accuracy is not high. However the main facts could be established that the change of volume becomes less at the higher pressures, and the direction of curvature is the normal one, namely convexity toward the pressure axis.

In Figure 1 are shown the experimentally determined coordinates of the melting curve, and in the same diagram the experimental changes of volume; the method of representation is the same as that previously used in my melting and transition determinations. In Table II are given the smoothed results obtained from these curves

TABLE II.

Melting Data for Rubidium.

Pressure kg./cm.^{2}	Temperature Centigrade	dT/dp	ΔV cm.^{3}/gm.	Latent Heat kg. m/gm.
0	38.7°	.0209	.0185	2.76
500	48.7	192	163	2.73
1000	57.9	178	145	2.70
1500	66.5	166	130	2.66
2000	74.5	155	119	2.67
2500	82.0	146	112	2.72
3000	89.1	139	106	2.76
3500	95.9	134	101	2.78

for the coordinates of the melting curve, the change of volume, and computed from these the slope of the melting curve and the latent heat of melting. The latent heat varies along the melting curve only by small amounts, which is the behavior found for most substances.

The change of volume on fusion and the latent heat do not seem to have been previously determined, so that there are no values for comparison.

Resistance. The data to be covered by the resistance measurements are the effect of pressure at various temperatures on the resistance of the solid and liquid metal, and the change of resistance when the solid melts to the liquid at various points of the melting curve. For those measurements in which the solid alone is concerned the bare wire must be used, but for those which involve the liquid, a glass capillary was necessary.

The manipulation of the bare wire was essentially the same as that of potassium. It was extruded through a steel die under Nujol; the electrical connections were fine wires of silver, instead of copper, pierced through the rubidium wire. Measurements were made as follows. First the temperature coefficient of resistance at atmospheric pressure was determined. Then pressure measurements were made to 12000 kg. and back at 20°. This was the first application of pressure, and there was a comparatively large permanent increase of resistance. Then a pressure run was made to the maximum and back at 35°, which showed only a small permanent change, and then a pressure run at 0°, which showed the smallest set of all. Then measurements of the temperature coefficient under pressure were made at several high pressures by the cyclic method already described for potassium. Finally the bare wire was measured again for the temperature coefficient at atmospheric pressure, this time raising the temperature until the wire melted. A reading was obtained with the bare wire at 38.4°, but at 38.8° it melted and open circuited, indicating a melting temperature somewhere between these limits.

The results obtained with the bare wire were used in calculating the finally tabulated values of resistance at atmospheric pressure up to the melting point, and the resistance as a function of pressure to 12000 kg. at 0° and 35°. There is a small inconsistency in the finally tabulated results which I have not attempted to remove in that the temperature coefficients determined from the difference of the pressure data at 0° and 35° do not agree precisely with the directly determined coefficients by the cyclic method. The difference is partly perhaps due to the difference of temperature range, but there is doubtless also some outstanding experimental error. The results are best used as given; the pressure values in discussing the effect of pressure at 0° or 35°, and the directly measured temperature coefficients in discussing temperature effects at various pressures.

The measurements in the glass capillary followed the same general outline as previously. The capillary was small, perhaps 0.5 mm. inside diameter and 4 or 5 cm. long. It was filled while directly attached to the purifying and distilling apparatus, sealed off, and then the upper end broken under Nujol. Measurements were made in the first place of the resistance at atmospheric pressure as a function of temperature, beginning at 0° on the solid, and running to 95° on the liquid. The temperature coefficient in glass to 27° was 0.00477, somewhat less than the value found for the bare wire. The difference was in the same direction as was also found for potassium, but is not as great, corresponding perhaps to the greater mechanical softness of rubidium. At a temperature of 36.6° the resistance measurements indicate that 1% of the metal is melted; I did not attempt to determine the melting point more accurately by this method. The resistance of the liquid in glass is a linear function of temperature from the melting point up to 95°. The correction by which the measured resistances in the glass are reduced to specific resistances of the liquid are so small as to be almost negligible, amounting to an increase of only 1 in the last place at the highest temperature. From the measurements at atmospheric pressure the ratio of specific resistance of solid to liquid at atmospheric pressure was obtained, correcting for the very slight rounding of the corner by a graphical extrapolation.

With the capillary, pressure runs were made at 65° and 95° up to and just beyond the melting point, so that in addition to the pressure coefficient of the liquid we have the ratio of resistance of liquid to solid at the equilibrium pressure at these two temperatures. Finally, pressure runs were made on the frozen solid to 10000 kg. at 95°, and the temperature coefficient determined by a cyclic change of temperature between 22° and 95° at 5700 kg. The coefficient so found was 0.00427 in terms of the resistance at 0° (linear extrapolation), which is higher than the value found directly with the bare wire. At atmospheric pressure the bare wire has yielded higher coefficients; the difference may well be due to the difference of temperature range, 0° to 17° against 22° to 95°.

The final results are given in Table III. Here are given the relative resistances at different temperatures and pressures in terms of the resistance of the solid at 0° and atmospheric pressure as unity. Up to the melting point at atmospheric pressure the results tabulated are the measured results for the bare wire, with no correction for volume distortion. The tabulated value is to be increased by the linear thermal expansion in order to obtain the specific resistance. This correction will probably amount to 2% of the change due to change of temperature, but it is not known accurately enough to justify giving final values in terms of it. The values given for the resistance of the liquid have, on the other hand, been corrected for the changes of form of the glass envelope, so that these give the relative specific resistances starting from the melting point as the fiducial point. The corrections for the glass are small, and amount at the highest temperature at atmospheric pressure to only 1 in the last place, and at the highest pressure on the liquid (95° and 3500 kg.) to only 4 in the last place. At 95° the pressure measurements on the solid, having been made with the glass capillary, are of the specific resistance. The corrections by which the measured resistances on the bare wire may be reduced to specific resistances are large, because of the high compressibility of rubidium, and at 12000 kg. amount to a decrease of the tabulated resistance by 10%. The precise amount may be found from the values given later for the volume.

The ratio of resistance of liquid to solid is also given in Table III for 1, 1470, and 3425 kg. Since there is no change of volume of the container during melting at constant pressure and temperature, these values are the ratios of the specific resistances.

Cubic Compressibility. Two methods were used. The first was the method of determining linear compressibility already used for a number of metals. The apparatus and the methods of measurement were the same as previously used. The rubidium was pressed into the shape of a regular cylinder about 7 mm. long, and mounted in the apparatus for direct measurement without lever magnification.

TABLE III.

Resistance of Rubidium.

At atmospheric pressure		Behavior under Pressure				
Temp.	Rel. Resis.	Pressure kg./cm.^{2}	Solid, bare wire 0° C.	Solid, bare wire 35°	Liquid, in glass 65°	Liquid, in glass 95°
0° C.	1.000 Solid	0	1.000	1.205	2.195	2.417
10	1.046	500			1.983	2.173
20	1.099	1000	.845	.982	1.823	1.984
30	1.165	1500			1.679	1.827
38.7	1.235	2000	.733	.840		1.695
38.7	1.990 Liquid	2500				1.578
40	2.000	3000	.648	.740		1.472
60	2.148	3425				1.381*
80	2.295					.879†
100	2.443	3500				.871
		4000	.583	.663		.820
		5000	.531	.602		.732
		6000	.490	.553		.663
		7000	.456	.514		.609
		8000	.428	.481		.565
		9000	.406	.455		.529
		10000	.387	.434		.499
		11000	.372	.418	* Liquid	
		12000	.360	.406	† Solid.	

Ratio, $\frac{\text{Resis. Liquid}}{\text{Resis. Solid}}$	0 kg.	1470 kg.	3425 kg
	1.612	1.608	1.571

Temperature Coefficient of Solid

Bare Wire,	0°–17°,	0.00481,	0 kg.	In Glass,
		.00385,	5700 kg.	22°–95°, 0.00427
		.00365,	11000 kg.	at 5730 kg.

Measurements were made to 12000 kg. at 0°. The difficulty with the method is a mechanical one due to the softness of the rubidium. There is danger of mechanical deformation during the assembling of the apparatus or during the application of pressure because of the viscosity of the transmitting medium. Because of the extreme compressibility of rubidium it is necessary to use specimens short compared with their length, and this may be responsible for a special sort of error due to deformation. The cylinder of rubidium is compressed between flat plattens of steel. Between the steel platten and the rubidium there is a thin film of oil. When pressure is applied the rubidium shrinks transversely more than the steel platten, so that there is relative slip tangentially to the platten. This slip is resisted by the viscosity of the oil. At high pressures, where the viscosity is great, this may result in an actual deformation of the rubidium, which becomes stretched transversely with respect to its natural figure, so that the accompanying decrease of length is too great, resulting in a too great measured compressibility. This is what was actually found, the compressibility determined by the linear method being greater above 8000 kg. than that determined by the other method.

The second method was by measuring the motion of the piston of the compression cylinder, as already explained. Two different fillings of the apparatus were used. The first employed about 4.5 gm. of rubidium, and was the same filling with which the melting curve data were obtained. This quantity of rubidium was so great, however, that the maximum pressure obtainable was 11000 kg. A second filling was therefore made, removing about 1 gm. of rubidium and replacing it by a steel core. This made it possible to reach 15000 kg. Runs with these two fillings were made at 50° and 95° especially to determine compressibility, in addition to the runs made with the first filling especially to determine the melting data. For some reason not definitely discovered the changes of volume obtained with the first filling were about 10% greater at the maximum than those obtained with the second. Since those obtained with the second filling agreed essentially with the results obtained by the other method, they were retained, and the first discarded. Attempts were made to obtain the thermal expansion as a function of pressure, but the measurements were not sufficiently accurate to give good values. The difficulties with the manganin measuring gauge are increased with the small apparatus and it was chiefly the uncertainty due to the shift of zero with changing temperature that was responsible for the lack of accuracy. Very likely this increased difficulty found with the small apparatus is not at all inherent in the size of the apparatus, but is due to the fact that the pressure range was 15000 and often 16000 kg. against the former 12000. This produces strains in the wire due to the viscosity of the transmitting medium which may result in a capricious shift of the zero. The greater pressure range is also responsible for a greatly increased elastic hysteresis in the deformation of the steel cylinder, which is not conducive to accuracy.

The final results for decrease of volume as a function of pressure to 15000 kg. at 50° C. are given in Table IV. Only three significant figures can be given, against the four which have been possible in most of the previous work. The volume at 0 pressure was obtained by extrapolation.

TABLE IV.

VOLUME OF RUBIDIUM UNDER PRESSURE.

Pressure kg./cm.^{2}	Relative Volume at 50° C.
0	1.000
1000	.953
2000	.914
3000	.883
4000	.857
5000	.836
6000	.819
7000	.803
8000	.789
9000	.777
10000	.766
11000	.756
12000	.746
13000	.737
14000	.729
15000	.721

With regard to thermal expansion under pressure, one may make the following rough statement with some assurance; up to 7000 or 8000 kg. the thermal expansion is fairly constant, but beyond this it drops rather abruptly, falling to one-half or one-third its initial value at 15000.

The initial compressibility of rubidium to be deduced from the Table is 0.000052, greater than that of water. This compressibility is much higher than the only previously published value, 0.000040 by Richards.[13] The difference is doubtless due to impurity. Richards' rubidium contained so large an amount of potassium that it was liquid at room temperature, so that the compressibility measured was of the liquid alloy. From the densities, assuming no contraction on mixing, Richards estimated that one-third of the total was potassium, and the probable error seemed to him so high that in his summary he tabulated only one significant figure for the result.

Attempts were also made to measure the compressibility of the liquid, but the wandering of the zero prevented accuracy. From the difference of slope of the volume curves above and below the melting point it is possible to say that at 56° the liquid is about 0.000006 more compressible than the solid, and at 96° 0.0000045.

CAESIUM.

Melting Data. The general method of handling this substance was the same as for rubidium. In determining the quantity of caesium by weighing under Nujol, the density of caesium at 0° was assumed to be 1.88. This seems to be the most probable value to be deduced from the measurements of Richards [11] and others, but the accuracy is not as great as in the case of rubidium.

Four points on the melting curve were determined, ranging from 250 kg. and 35° to 3700 kg. and 95°. In addition, a point was determined from the resistance measurements at 96° which coincides within the sensitiveness of the readings with the value found by the method of changing volume. The melting was not quite as sharp as for rubidium, but the melting pressure at 35°, for example, varies by 40 kg. between 0.1 and 0.9 melted, corresponding to a temperature shift of 1.0°. Of course the temperature of beginning of freezing or end of melting is to be taken as the correct melting temperature. The melting point at atmospheric pressure, extrapolated from the high pressure readings as in the case of rubidium, was 29.7°. This is considerably higher than previous values, and is evidence of high purity.

The change of volume is of the same order of magnitude as for rubidium, and values even more satisfactory were obtained.

In Figure 2 are shown the observed coordinates of the melting curve and the changes of volume, plotted in the usual way. The smoothed results taken from these curves are shown in Table V,

TABLE V.

Melting Data for Caesium.

Pressure kg./cm.^{2}	Temperature Centigrade	dT/dp	ΔV cm.^{3}/gm.	Latent Heat kg m./gm.
0	29.7°	.0250	.0136	1.65
500	41.4	221	118	1.68
1000	51.9	199	105	1.72
1500	61.4	183	96	1.76
2000	70.2	168	88	1.80
2500	78.3	155	83	1.88
3000	85.7	141	78	1.99
3500	92.4	128	75	2.14
4000	98.5	117	72	2.29

together with the values computed from them for the slope of the curve and the latent heat of melting. This latter shows perhaps

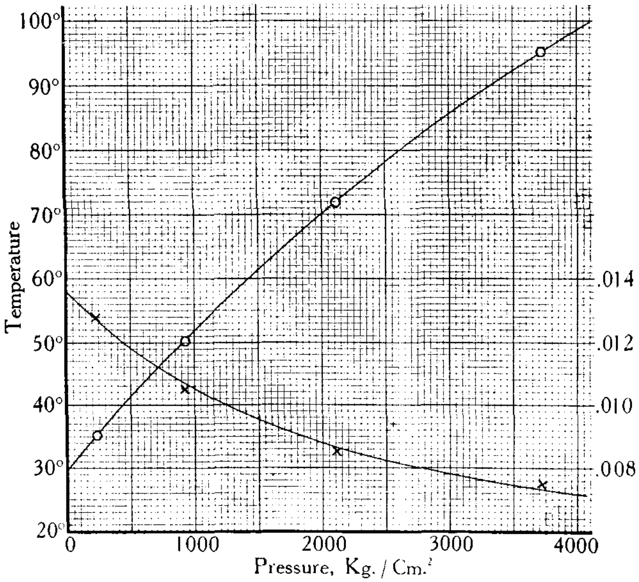

Figure 2. Melting data for Caesium. The circles show the melting temperatures (scale on the left) as a function of pressure. The crosses show the observed changes of volume (scale on the right) in cm.^{3} per gm. as a function of pressure.

more tendency than usual to change with pressure, becoming greater at the higher pressures.

Resistance. More time was spent in making the resistance measurements of caesium than all the other measurements of this paper. The measurements extended over three years, and were made on material from a variety of sources. The reasons for this have already been outlined and need not be reiterated. The final results to be given here were obtained with the purest material. There are no inconsistencies between the final and the preliminary results which are not explicable by impurities or improper manipulation.

The bare wire was obtained by extrusion out of the glass tube into which it was distilled after the final purification. One end of this tube was drawn out to the internal diameter desired for the wire (about 0.15 cm.), both ends of the tube were opened under Nujol, and the caesium was extruded by means of a plunger fitted with a closely fitting leather washer, so that the pressure which compelled the extrusion was transmitted to the caesium through an intermediate layer of oil. The metal is so soft that the extrusion was easily made holding the glass tube and the plunger in the hands. The extreme softness of course makes the various manipulations of assembly difficult, but ordinary care, without the use of special mechanical devices, is sufficient to surmount them. The leads were fine silver wires, as in the case of rubidium, and the subsequent manipulation was similar.

With this bare wire the temperature coefficient at atmospheric pressure was first determined from 0° to 10° and back to 0°. A pressure run was then made at 0°, locating carefully the position of the minimum of resistance, which was shown to be reversible by releasing the pressure a thousand kilograms or so after locating the minimum, and then proceeding to 12000 kg. with readings back to zero. Between 4000 and 6000 on the first increase of pressure there was a permanent increase of resistance of about 30%, due without question to distortion of the wire by the viscous transmitting medium. When corrected by this constant factor the initial results below the minimum of resistance agreed with the final results obtained with decreasing pressure. However, the pressure at which the minimum occurred was not the same with decreasing as with increasing pressure, but was about 400 kg. higher. I was never able to get entirely satisfactory values for the pressure of the minimum, which always varied somewhat. Doubtless the effect is due to internal strains set up by the transmitting medium. It is curious that more consistent values for the pressure of minimum resistance were found when the caesium was enclosed in a glass capillary than with the bare wire, although there were other fluctuations which were greater. The values listed as final in the following for the resistance of the solid under pressure are based essentially on the measurements obtained with decreasing pressure at 0° on the sample just described.

With this same bare wire the temperature coefficient of resistance was obtained at several pressures up to 11000 kg. by cyclic change of temperature from 0° to 10° to 0° again. The return to zero resistance was always exceedingly good, and these temperature coefficients may be accepted with considerable confidence.

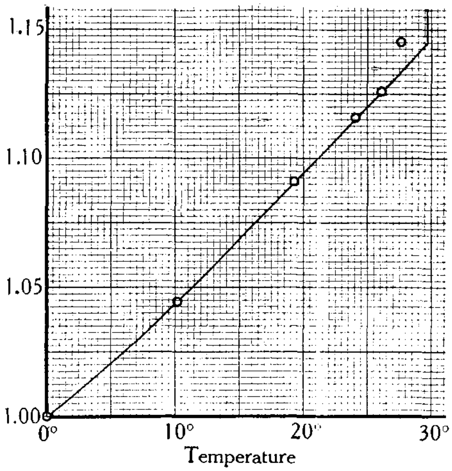

FIGURE 3. The resistance of solid caesium as a function of temperature at atmospheric pressure. Observe that up to 26° no premature melting is detectable.

The measurements on this bare wire were terminated by an attempt at a pressure run at higher temperatures, during which the wire was accidentally allowed to melt.

A number of readings were now obtained with a filling of a glass capillary, manipulated like the capillary of rubidium. First the temperature coefficient at atmospheric pressure was found below and through the melting point up to 95°. The values of the resistance up to the melting point shown in Figure 3 gave an idea of the sharpness of the melting. As before, the coefficient of the solid in glass was found to be less at atmospheric pressure than the coefficient of the bare wire; the bare wire over the common range, 10°, yielded a coefficient 10% higher. The results given in the final table for the resistance of the solid between 10° and the melting point are based on the values obtained with the glass capillary, corrected by a factor of 10%. The melting was sharp, as shown by the resistance measurements, at least judged by the usual standards. At 26.2° there was no premonition of melting, all the points lying on a smooth curve, but at 27.6° there was a jump in resistance corresponding to a premature melting of 1/60 part of the whole. Melting was completed somewhere between this temperature and 29.9°. These values are consistent with those found under pressure by the change of volume method. Above the melting point the change of resistance of the liquid in glass is linear with temperature up to 95° at least, with an error not more than 0.2%. The pressure coefficient of resistance of the liquid in glass was determined up to the melting point at 63°, and at 96° through the melting point and up to 11000 kg. The expected minimum of resistance of the solid shortly beyond the melting point was found. The temperature coefficient of the solid in glass was also found by the cyclic method between 20° and 95° at 9100 kg., and was considerably greater than found for the bare wire. This also agrees with the behavior of the temperature coefficient found for rubidium. Of course the value for the bare wire is to be preferred.

The ratio of resistance of liquid to solid at the normal melting point and at 96° was found to be 1.660 and 1.695 respectively. Again the variation of this ratio along the melting curve is slight compared with the variation of other properties.

The final results are shown in Table VI and Figure 4. The resistance of the solid at atmospheric pressure up to the melting point is first given. This is the measured resistance. To convert to specific resistance correction must be made for the linear thermal expansion, which is about 0.00011. Above the melting point the results tabulated are the specific resistance of the liquid, the directly observed values not differing appreciably from the corrected values. The pressure measurements on the solid at 0° are measured results for the bare wire. These results may be converted to specific resistances by correcting by the linear compressibility, to be given in the next section. At 12000 kg. this correction will produce a decrease of resistance of about 11%. The effect of the correction is to make the pressure of the minimum of specific resistance higher than the pressure of minimum of observed resistance.

In the table are also given the resistances of the liquid (specific) as a function of pressure up to the melting point at 63° and 96°. The correction for changing from observed to specific resistance is 0.08% per 1000 kg.

TABLE VI.

Resistance of Caesium.

At Atmospheric Pressure		Behavior under Pressure			
Temp.	Rel. Resis.	Pressure kg./cm.^{2}	Solid, Bare Wire at 0° C.	Liquid, in Glass 63.4°	Liquid, in Glass 95.8°
0° C.	1.0000	0	1.000	2.099	2.280
5	1.0247	500		1.934	2.094
10	1.0496	1000	.863	1.815	1.955
15	1.0748	1500		1.721	1.851
20	1.1002	2000	.779		1.770
25	1.1258	2500			1.721
29.7	1.1507 Solid	3000	.729		1.661
	1.910 Liquid	3500			1.625
30	1.912	4000	.709		
65	2.108	5000	.713		
100	2.304	6000	.728		
		7000	.756		
		8000	.794		
		9000	.833		
		10000	.879		
		11000	.931		
		12000	.994		

	0 kg.	3780 kg.
Ratio, $\frac{\text{Resis. Liquid}}{\text{Resis. Solid}}$	1.660	1.695

Temperature Coefficient of Solid.

Pressure kg./cm.^{2}	Bare Wire, 0°-10°	Solid in Glass
0	0.00496	Mean coefficient 0° to 96° at 9100 kg., extrapolated from results between 21° and 96° is 0.00463.
2000	434	
4000	388	
6000	366	
8000	367	
10000	386	
12000	418	

The temperature coefficient of the solid bare wire between 0° and 10° is given as a function of pressure, and in Figure 5 are plotted the experimental values on which these are based. It will be seen that, like the resistance, the temperature coefficient passes through a minimum, but the pressure of the minimum is higher than the pressure of the minimum of resistance.

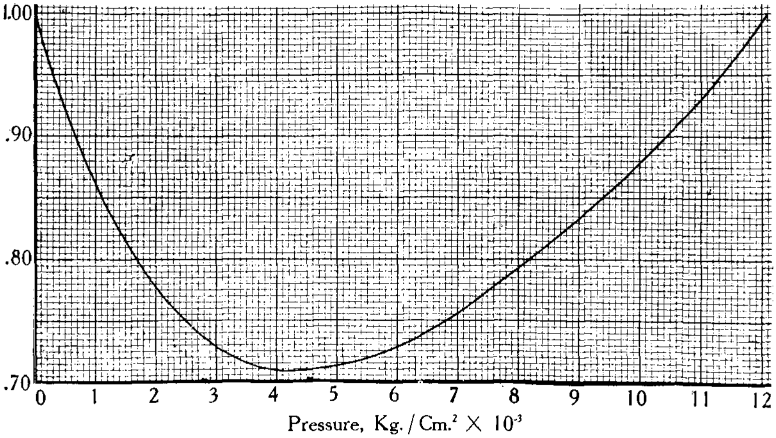

FIGURE 4. The relative resistance of solid caesium (bare wire) as a function of pressure at 0° C.

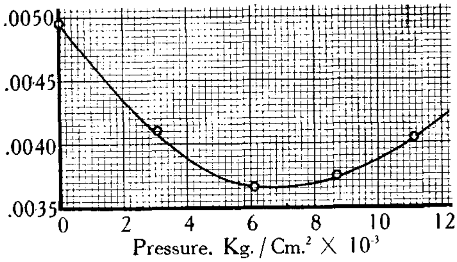

FIGURE 5. Mean temperature coefficient of resistance between 0° and 10° of bare caesium wire.

The ratio of resistance of liquid to solid is given at two points on the melting curve.

In Figure 6 is given the pressure of minimum resistance as a function of temperature. In this diagram are included some points obtained in some of the preliminary runs with caesium in glass. The agreement of the results so obtained with increasing and decreasing pressure seems to warrant their retention, although no other of the readings obtained with this material have been incorporated in the final results. It will be seen that these results show considerable spread; the probable explanation in terms of internal strains has already been given. In view of these irregularities it is probable that the values listed here for the resistance of the solid as a function of pressure have not a high degree of accuracy; there should be no such error in the resistances of the liquid.

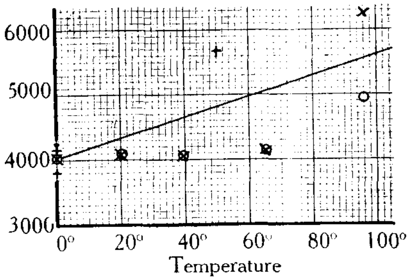

FIGURE 6. The pressure of the minimum resistance of caesium in kg/cm^2 plotted as ordinate against temperature. The open circles were obtained with decreasing pressure and the diagonal crosses with increasing pressure on caesium in glass. The rectangular crosses were obtained with increasing pressure and the dot with decreasing pressure on bare wire.

Compressibility. As in the case of rubidium, the compressibility was determined by the two methods of linear compressibility and cubic compressibility by piston displacement.

The difficulty found with the rubidium due to distortion in using the method of linear compressibility was very much accentuated in the case of caesium because of its greater softness, and the high pressure results had to be entirely discarded. Below 2000 kg., however, the results found by this method seem to be reliable, and they were used to supplement the results by the other method, which could not be extended below 1000 kg.

Measurements by the piston method were made with one filling of the apparatus, using 4.3 gm. of caesium, to a maximum pressure of nearly 16000 kg. at 50° and 95°, in addition to the low pressure runs made to obtain the melting data. The results finally given for the changes of volume in Table VII are based on the data obtained at 50°, these being somewhat more regular than the data at 95°. It was possible also to obtain readings on the compressibility of the liquid at 75°, which are also given in the Table. The liquid is not very much more compressible than the solid.

The values listed in the Table are changes of volume in cm.^{3} for 1.88 gm. of metal. The results given in this way were directly obtained from the experimental results without using any hypothetical values for the thermal expansion. 1.88 gm. is very approximately the amount of caesium that occupies 1 cm.^{3} at 0° C. at atmospheric pressure, so that the values listed in the Table are approximately fractional changes of volume.

TABLE VII.

Volume Decrements of Caesium under Pressure.

Pressure kg./cm.^{2}	Volume Decrement in cc. per 1.88 gm.	
	50° C., solid	75° C., liquid
0	.000	.000
500		.035
1000	.059	.063
1500		.086
2000	.102	.107
3000	.134	
4000	.161	
5000	.184	
6000	.204	
7000	.223	
8000	.240	
9000	.255	
10000	.269	
11000	.282	
12000	.294	
13000	.306	
14000	.317	
15000	.328	

The initial compressibility at 50° to be deduced from the Table is 0.0000070. Richards [13] gives for the average compressibility between 100 and 500 kg. 0.000060. He finds a more rapid decrease of compressibility with pressure than I do, giving for the average compressibility between 100 and 300 kg. 0.000067, and between 300 and

500 0.000053. A linear extrapolation would give 0.000081 for the initial compressibility. However, a detailed examination of Richards' data will show that his pressure coefficient of compressibility is in great doubt, and the agreement of our results is probably as close as could be expected.

Satisfactory numerical results could not be obtained for the variation of thermal expansion of the solid with pressure, but roughly, the behavior is of the same nature as that found for rubidium, the rapid decrease in expansion beyond 8000 or 9000 kg. being even more pronounced.

Discussion of Results.

We now have the data in hand for all the alkali metals, so that this discussion may well be concerned with a comparison of the properties of all of them.

The volume relations are the simplest, and first engage our attention. The volumes of all the alkali metals are shown in Figure 7 as a function of pressure. I have chosen somewhat unusual units for the volume, namely a volume proportional to the average space occupied by each extra-nuclear electron. This volume is obtained from the atomic volume (volume in cm.^{3} of a number of grams equal to the atomic weight) by dividing by the atomic number, or number of extra-nuclear electrons. This quantity I shall denote by the abbreviated expression "electronic" volume. The electronic volume seems somewhat better adapted to bring out certain relationships than the atomic volume. The values adopted for the electronic volumes at 0° C. at atmospheric pressure for Li, Na, K, Rb, and Cs are respectively 4.37, 2.15, 2.39, 1.507, and 1.284 cm.^{3}

The figure brings out an abnormality in the volume of potassium, which is between that of Na and Li, instead of between Na and Rb. This abnormality disappears, however, above 10000 kg., where the curves for Na and K cross. That the abnormality is to be ascribed to K and not to Na is made evident by some of the later curves.

The initial compressibilities at 0° of the 5 metals in order are $0.0_5 87$, $0.0_4 157$, $0.0_4 355$, $0.0_4 52$, and $0.0_4 70$. By multiplying these by the electronic volumes we obtain numbers proportional to the actual loss under an external pressure of 1 kg./cm.^{2} of the average space occupied by 1 electron. The "electronic" compressibilities found in this way are $0.0_4 38$, $0.0_4 34$, $0.0_4 85$, $0.0_4 78$, and $0.0_4 90$ respectively. It is interesting that the electronic compressibilities of the three heavier metals, in which the electrons are much more closely packed, is much larger than that of the two lighter metals, which have a much more open structure. This difference is the opposite of what one might expect. The difference is very much exaggerated if one calculates the atomic compressibility (ordinary compressibility multiplied by atomic volume) in place of the electronic compressibility, the factor of variation through the series of five metals being 40 instead of 3.

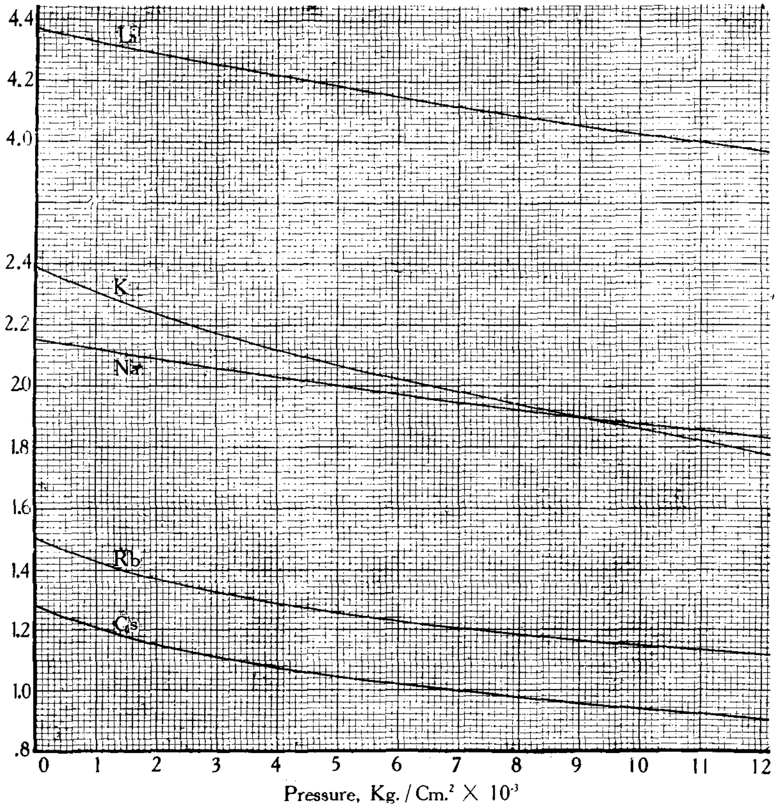

FIGURE 7. The electronic volumes (atomic volume divided by atomic number) in cm.^{3} at 0° C. of the five alkali metals as a function of pressure.

Under high pressures, the anomaly disappears. The electronic compressibilities are respectively 0.0_430, 0.0_425, 0.0_450, 0.0_418, and

0.0_419 at 12000 kg. Potassium occupies a highly anomalous position; except for this, the behavior under high pressure is as we would expect, the loss of volume per electron for a given increment of external pressure being less in the more complex and closely packed structure. The reversal of the effect at high pressures may be roughly described as a removal by the pressure of the "slack" which initially exists in the more complicated structures. Because the electronic compressibilities are much more nearly constant than the atomic compressibilities (both at 1 and 12000 kg.), it suggests itself that a more fruitful line of attack on the problem of the constitution of solids may be found by neglecting the atomic structure and considering the solid merely as an aggregate of electrons (and nuclei). In fact, these volume relations show at high pressures a very rough similarity to the gas law, particularly those of Rb and Cs.

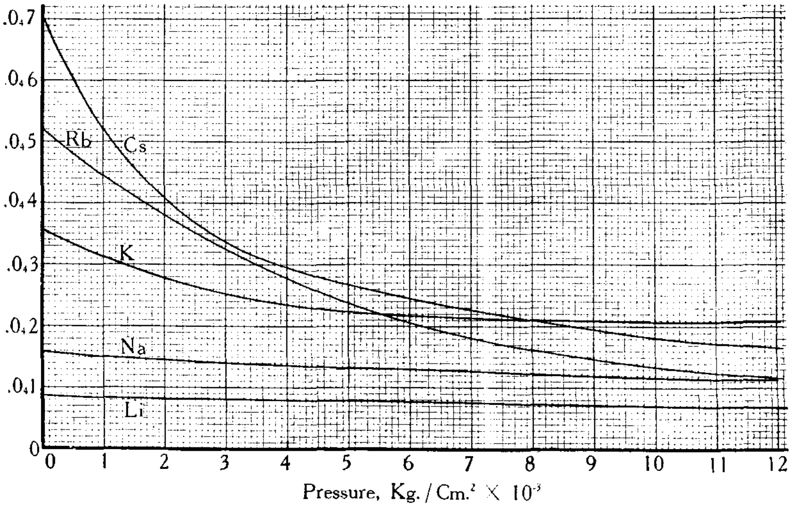

FIGURE 8. The instantaneous compressibilities $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t$ at 0° C. of the five alkali metals as a function of pressure.

The instantaneous compressibility, $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t$, at 0° C. as a function of pressure is shown in Figure 8. This brings out strikingly again the anomalous position of K; it is evident from the figure that the anomaly is not to be ascribed to Na. Figure 8 has an important bearing on a suggestion of Professor Richards[14] with regard to compressibility at high pressures. His conception of internal pressure has led him to the view that a relatively incompressible metal at low pressures behaves like a more compressible metal at higher pressures. He applied this conception quantitatively to my data for Na and K, which were then available. An extrapolation equation for the volume of 1 gm. of K was found such that it fitted on smoothly at 18000 kg. with the volume of a certain weight of Na, starting from atmospheric pressure. The question now before us is whether this is a general relation between all the alkali metals. It is evident that if this were the case it should be possible to join smoothly together the curves of $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$ for the different metals, merely by sliding them along the pressure axis, *without change of ordinate.* Now the figure makes it perfectly obvious that this is not the case, because the curves cross, whereas there would be no crossing if the supposed relation held. We conclude, therefore, that there are very important individual differences between the alkalis which become accentuated as the electronic structure becomes more tightly packed. The figure shows a crossing of the $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$ curve of K by that of Rb and Cs, indicates that in another 1000 kg. the curve for Rb will cross that of Na, in another 5000 kg. Cs will probably cross, and it is not unlikely that Cs will cross Rb, and eventually both of these cross Li. This is all as one might expect; the surprising and inexplicable result is the persistent compressibility of K. This again lends color to the surmise that I have already expressed several times that K may have another polymorphic modification at high pressures.

The melting phenomena of the five metals next engage us. In Figure 9 are shown the melting curves. At atmospheric pressure the order of the melting points is the inverse of that of atomic weights (or atomic number) but again under pressure K is anomalous, and above 10000 kg. its melting point is higher than that of Na. Rb and Cs fall naturally in the sequence; the initial rise of melting point with pressure is greater than for the other metals, as one would expect. The fractional changes of volume on melting do not show the same regularity as some of the other properties, and I do not give a figure for them. At atmospheric pressure the order of the fractional changes is:

Metal	Fractional Change of Volume on Melting
Li	0.0060
Ka	.0231
Cs	.0256
Na	.0271
Rb	.0284

At higher pressures there is a complicated interchange of positions; the curve for Rb crosses that of Na and practically coincides with that of K from 2000 to 4000 kg., Cs drops below K but acts as though it might rise above it again, and the curve for K drops off at high pressures with abnormal rapidity, which is one of the pieces of evidence for a new polymorphic modification. With the exception of Li, the approximate equality of the fractional changes of volume on melting is interesting, and suggests a fundamental similarity.

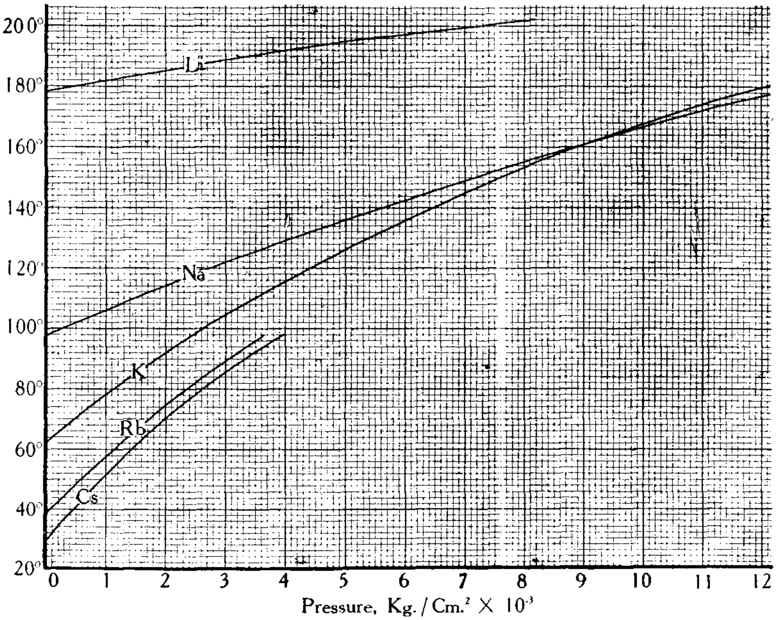

FIGURE 9. The melting temperatures as function of pressure of the five alkali metals.

The latent heats of melting are also interesting. At atmospheric pressure these are as shown.

Metal	Latent Heat of Melting, in kg. m. per gm. atom
Li	49.
K	215.
Cs	219.
Rb	236.
Na	271.

Again, except for Li, the approximate equality is striking. There is no connection here with the sequence of atomic weights, any more than there was for the change of volume. At high pressures, the behavior is complicated. The latent heat of Na is roughly constant, Cs rises rapidly, crossing Rb and Na, Rb is approximately constant but falls to a slight minimum at 2000 kg, while K again shows pronounced variation, rising to a maximum near 4000 and then falling rapidly to 18.9 at 12000 kg.

One may say in general that the melting data in the alkali group do not show as regular variations as do the compressibilities.

Finally we have to consider the electrical resistance under pressure. In Figure 10 are shown the relative resistances of the five metals at 0° C. as a function of pressure. In spite of their apparent complexity, there is an underlying regularity in the curves. There is a turning point for the electrical properties somewhere between K and Rb. This may be seen by plotting at any fixed pressure the decrement of resistance for the various metals as ordinate against atomic number (for example) as abscissa. The same sort of turning point is also shown on plotting the "conductivity per atom" at a definite reduced temperature against atomic number. This is described more in detail in my report to the fourth Solvay Conference on Metallic Conduction.

The diagram suggests the query whether all the alkali metals will ultimately, at pressure high enough, increase in resistance. This involves an extrapolation of the curves. Such an extrapolation is not easy; we may obtain a more satisfactory indication by extrapolating the instantaneous pressure coefficient $\frac{1}{R}\left(\frac{\partial R}{\partial p}\right)_T$. If the resistance has a minimum, the curve of the coefficient must cross the axis to become negative. It will be seen in Figure 11 that it is not improbable that Rb may cross the axis, and so have a minimum resistance, but it is much more uncertain in the case of K. The reversal of curvature shown by Cs on the negative side of the axis is, however, of great significance. If there should be a similar reversal for the other metals, they would all, of course, have minima.

It is an interesting question whether the minimum resistance of Cs is a function of the crystal structure of the solid, or whether it would be shown by the liquid also if pressure could be raised high enough without freezing. Figure 12 is an attempt to answer this question. Here are plotted the resistances under pressure of solid and liquid Cs at 95°, the resistance of the liquid being multiplied by 1/1.704 to remove the discontinuity at the melting point. The curves for solid and liquid are seen to join together continuously without change of slope at the melting point. It is therefore probable that the resistance of the liquid would also pass through a minimum if the

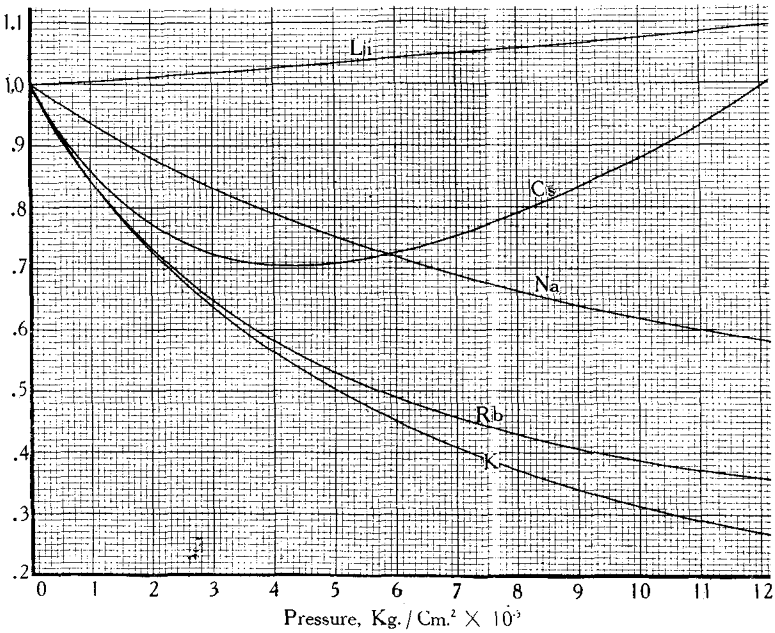

FIGURE 10. The relative resistances at 0° C. of the five alkali metals as a function of pressure.

pressure could be raised high enough without freezing, and we may draw the conclusion that the minimum does not involve the particular mechanism of the solid crystal.

The minimum resistance of Cs cannot but be of great significance for theories of conduction. It indicates that the conduction mechanism is not simple, but that there are several factors involved, which may be affected by pressure in different ways. With regard to my own theory, which has been developed in several papers,[15] it merely drives me more strongly to a view which other phenomena have been more and more insistently suggesting. This is that the conduction electron, in its way through the metal is effectively con-

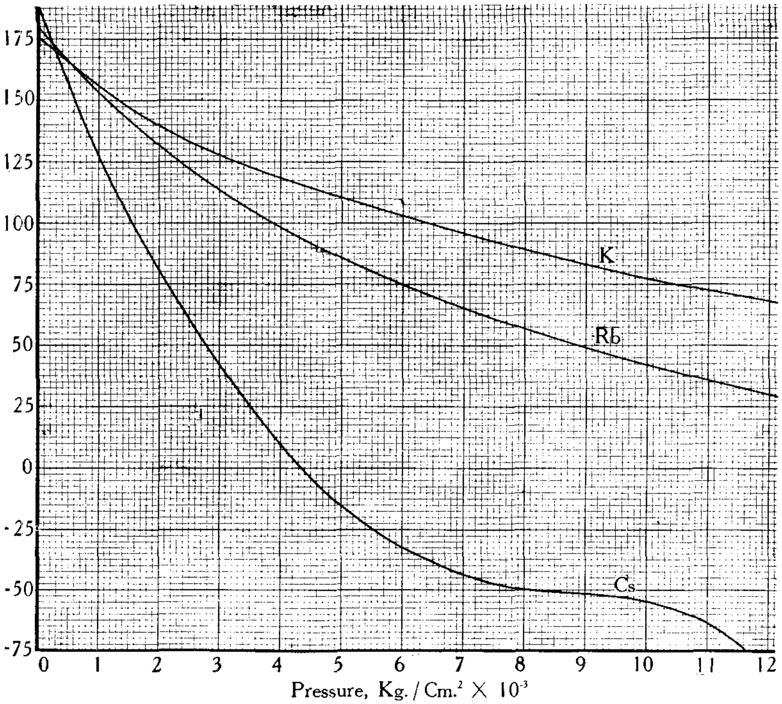

FIGURE 11. The instantaneous pressure coefficient of resistance $\frac{1}{R}\left(\frac{\partial R}{\partial p}\right)_t$ at 0° C. × 10^6 as a function of pressure for the three most compressible alkali metals.

fined to certain tracks through the atoms (probably in some cases between them) and that passage from atom to atom cannot be affected unless the tracks in adjacent atoms are in alignment. Under high pressures the atoms may in some cases, depending on the detailed structure of the atom, be so distorted or rotated that the tracks get out of alignment, and the resistance increases. Such a condition would be especially anticipated in caesium because of its high compressibility and complicated structure. The tracks themselves, it is natural to think, are the result of some sort of quantum phenomenon. It is possible that the track within the atom should itself be unfavorably modified by the distortion of the atom due to high pressure, but this would be expected to occur at considerably higher pressures than the peripheral effect, and may probably be disregarded in a first discussion.

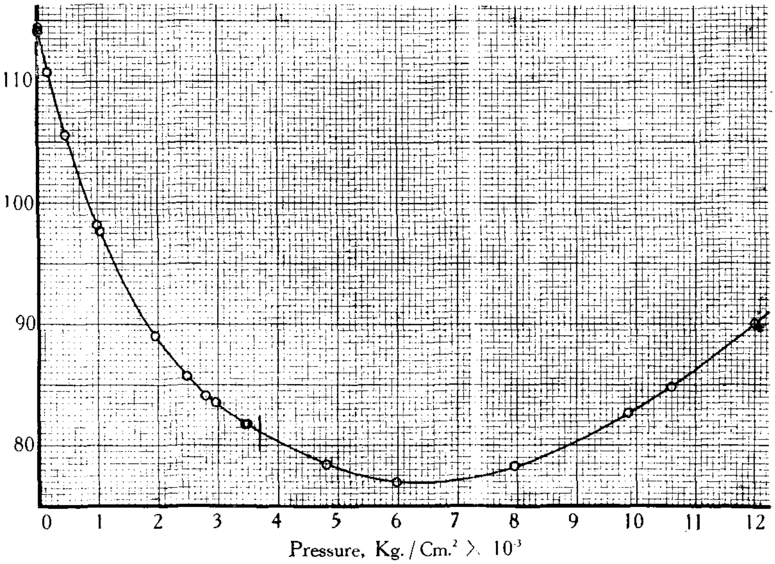

FIGURE 12. The resistance on an arbitrary scale as a function of pressure of liquid and solid caesium at 95°. The resistance of the solid has been multiplied by such a factor as to remove the discontinuity at the melting point, which takes place at the pressure shown by the vertical line. The smoothness of the curve indicates that the crystal arrangement plays a minor part in the resistance phenomena of this metal.

SUMMARY.

The following new data are presented for rubidium and caesium: the cubic compressibility to 15000 kg./cm.^{2}, the melting data (variation of melting temperature with pressure and the change of volume, permitting a calculation of the latent heats) to 100°, and the electrical resistance of both solid and liquid as a function of pressure between 0° and 100° to 12000 kg./cm.^{2}

In addition to the new data for rubidium and caesium, the electrical resistance of potassium under pressure has been redetermined; the new measurements were made with bare wire. The old measurements were made on potassium in a glass capillary, which exerted a constraining action. The new results do not show the great decrease of temperature coefficient at high pressures formerly found.

In the discussion, the properties of all the alkali metals are compared. The "electronic" volumes (atomic volume divided by the number of extra-nuclear electrons) is plotted as a function of pressure. Potassium is anomalous at low pressures, its volume being between that of lithium and sodium, but the anomaly is wiped out at high pressures. The compressibility of potassium is also anomalous, the rate of decrease at high pressures being much less than for the other metals. In general, the pressure volume relations of a compressible alkali at high pressures are *not* similar to those of a less compressible alkali at lower pressures. The melting of potassium is also anomalous; at high pressures its melting point becomes higher than that of sodium. Attention is directed to the fact that with the exception of lithium the fractional changes of volume on melting of all the alkali metals are nearly the same, as are also the latent heats per gm. atom. The small scale variations of these quantities with pressure are irregular.

The resistance of caesium is found to pass through a minimum with increasing pressure. Comparison of measurements on solid and liquid suggest that this minimum is probably not connected with the crystalline structure of the solid. Comparison with the resistance of the other alkalies under pressure suggests that at sufficiently high pressures the resistance of rubidium may also pass through a minimum, and if the reversal of curvature shown by caesium at high pressures should be a property of the other metals, all of them may have minima at sufficiently high pressures. The general significance of this for a theory of electronic conduction is discussed.

It is a pleasure to acknowledge the assistance received from Mr. Walter Koenig in making many of the readings of this paper.

The Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.

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THE EFFECT OF TENSION ON THE TRANSVERSE AND LONGITUDINAL RESISTANCE OF METALS.

By P. W. Bridgman.

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INTRODUCTION.

ALTHOUGH the effect of tension on the resistance of a metal when the current flows in the direction of the tension has been measured by a number of observers,[1] apparently the resistance to current flow at right angles to the direction of the tension has been measured only by Tomlinson,[2] and for only two metals. The matter is one of considerable interest for theories of metallic conduction, and deserves further attention. In the following I give the results of measurements of the transverse coefficient for several metals. I have also measured the longitudinal tension coefficient of the same samples; this is desirable because the longitudinal tension coefficient varies somewhat from sample to sample. In addition to the two tension coefficients, we have available the hydrostatic pressure coefficient[2] (not determined on the identical samples, but not nearly so variable with the specimen) so that the experimental basis for a discussion of the effect of changes of dimensions on resistance is now fairly well laid (at least for a few metals). It would be desirable if the effect of a pure shear could also be determined, but this is a matter of much greater experimental difficulty.

METHOD.

The experimental determination of the transverse coefficient is a matter of considerable difficulty. The method employed by Tomlinson[2] apparently leaves much to be desired. His specimen was in the form of a long narrow strip clamped by the long edges so that a tension could be applied crosswise of the strip. The current passed lengthwise of the strip, being led in and out by screw clamps, and the resistance was measured with a Wheatstone's bridge. The state of stress and strain in such a strip is evidently far from simple, because the lateral contraction is hindered, and there are complicated effects under the clamps, which may well constitute a considerable fraction of the whole. It would seem to be a desideratum of a good method that no part of the current which takes part in the measurements should be allowed to flow under clamps.

For the two metals (iron and zinc) which Tomlinson measured, he found that the transverse resistance decreases with tension, the opposite of the longitudinal effect.

In attempting to avoid the difficulties of Tomlinson's method, I adopted a method which demands two different sorts of measurement. The first is the geometrical mean of the longitudinal and transverse tension coefficients, while the second is the longitudinal coefficient alone. From these two data the transverse coefficient may be calculated.

Method of Measuring the Geometrical Mean of the Two Coefficients. In measuring the geometrical mean of the two coefficients, the metal in the form of a thin sheet (from 0.0075 to 0.015 cm. thick) was cut to a rectangle 5 by 10 cm., the two ends gripped by screw clamps, and tension applied by a simple lever arrangement lengthwise of the strip. By way of precaution the strip was insulated from the clamps by strips of mica, although special measurements showed that this precaution was not necessary. The electrical measurements were made on the central portion of the strip. Two sets of measurements were made, one transverse and the other longitudinal. To make either of these sets, a group of 4 contact points was employed, pressed against the strip by springs. These are shown in Figure 1. In making the transverse measurements current was led into the

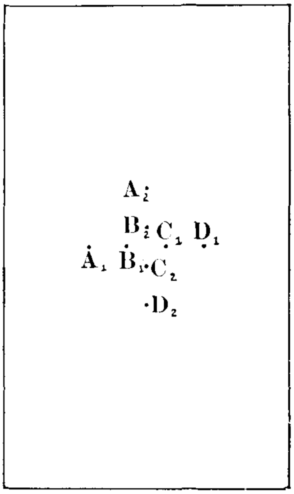

FIGURE 1. Shows the plate with the current and potential terminals. Either the horizontal or the vertical set was used. The current terminals are *A* and *D* and the potential terminals *B* and *C*. Tension is applied lengthwise of the plate.

plate at A_1 and out at D_1. The difference of potential between the two intermediate contact points B_1 and C_1 was determined on a potentiometer; in this way the specific resistance of the metal per unit of surface may be determined. An exactly similar set of measurements was made with the four vertically disposed contact points. Measurements made with and without tension give the effect of tension.

Without analysis, one might be inclined to think that the measurements with the transversely arranged points would give the transverse coefficient, and those with the longitudinal points the longitudinal coefficient, but detailed mathematical discussion, which will be given later, shows that this is not the case. Except for various corrections, both measurements should give exactly the same thing, namely the geometrical mean of the conductivity per unit surface longitudinally and transversely. At best, then, these two sets of measurements give merely a check on each other, which is not without value. As a matter of fact, however, the various corrections turn out to be so much larger with the longitudinal arrangement, that in making the final computations only those results were employed which were obtained with the transverse arrangement.

Some of the details of construction require comment. The contact points were large needles pressed against the metal strip with helical springs. The strip was backed with a piece of hard rubber to prevent its bending under the pressure of the contacts. The contact points, with the springs and the backing piece of rubber were constructed as one self-contained unit, which was independently supported, so as to float freely at the center of the metal sheet and follow, without introducing any stresses, the slight displacements due to applying and removing tension. A special holder was used in assembling the apparatus to avoid bending the thin metal sheet. The current and potential leads were flexible copper wire, about 0.015 cm. in diameter, soldered to the needles close to the points so as to minimize thermal e.m.f.'s. Because of the smallness of the effect it was necessary to use rather large currents, of the order of half an ampere. Since the resulting heating effects might be very serious, it was necessary to put the metal sheet with the contact points in a kerosene bath rapidly stirred. Measurements were attempted only at room temperature.

The spacing of the four points was maintained always the same by the use of a special jig; with this, holes were made in a piece of thin cardboard, by which the points were located, and then the cardboard was removed by tearing it apart along cuts previously made in it with a razor blade. Some such use of a jig was necessary, because the needles themselves had to be given some freedom of motion to follow the displacements due to tension. The points were equispaced at approximately 6 mm., making the distance between A_1 and D_1 1.8 cm., and similarly for the longitudinal points.

In terms of the potentiometer measurements the absolute value of the mean surface resistance may be found with and without tension. However, the chief interest of this work is not in values of absolute resistance, but in the percentage changes produced by tension, so that a somewhat simplified and more sensitive procedure in making most of the electrical measurements was adopted. The potential terminals B_1 and C_1 were connected to a senstive moving coil galvanometer (5 cm. deflection per micro-volt). Another e.m.f. was introduced into the same circuit and could be varied by a simple arrangement of variable shunts so that the e.m.f. due to B_1C_1 could be neutralized. The galvanometer zero was noted. Tension is now applied to the specimen; the resistance changes, the e.m.f. due to B_1C_1 changes, and the galvanometer balance is upset, giving a displacement, which is noted. The load is removed, and again the galvanometer zero noted. This is repeated a number of times, and the mean of the displacements taken. The change of resistance of the specimen corresponding to the galvanometer deflection was determined by finding the deflection caused by known changes of resistance in the balancing circuit. These resistances were so chosen that simple proportionality holds between the deflections and the changes of resistance.

This method would not be applicable unless equilibrium were reached very rapidly. Due to the thinness of the metal sheet and the rapid stirring, all temperature effects arising from the application of tension are dissipated in a time less than that required for the galvanometer to reach equilibrium, so that it is possible to make readings every half minute or less.

It will be seen that the method assumes that the current between the current leads A_1 and D_1 is unaffected by the tension. This was assured by the relative values of the various resistances. The current leads were fed by a storage battery of 12 volts through such a resistance as to cut the current to an ampere or less. The actual resistance between A_1 and D_1 is of the order of a few ten thousandths of an ohm, and the changes in resistance due to tension are of the order of a small fraction of a per cent of this, so that any changes in the feeding current may be entirely neglected.

In addition to these relative measurements of the effect of tension, absolute measurements with the potentiometer were made of the resistance under no load, from which the specific resistance of the metal was computed, and used as a check by comparing with the accepted values for the metal.

In applying tension to the specimen, considerable care is necessary to avoid any shock which might result in permanent set. This was accomplished by attaching the scale pan to the lever by a long helical spring. An arrangement of levers permitted the application and removal of the load by the observer at the galvanometer without changing his position.

Mathematical Theory of the Measurements of the Geometrical Mean Coefficient. The mathematical problem is one of flow in two dimensions in an anisotropic homogeneous substance, the anisotropy consisting of different conductivities in two perpendicular directions. Taking the x and y axes as the axes of principal conductivity (lengthwise and crosswise of our sheet under tension), we have

$$u_x = k_1^2 \frac{\partial \varphi}{\partial x}, \qquad u_y = k_2^2 \frac{\partial \varphi}{\partial y},$$

where u_x and u_y are the components of current, φ the potential, and k_1^2 and k_2^2 the principal conductivities. Since the condition is a steady one we also have the equation of continuity

$$\text{Div } u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0,$$

which, on substituting from above, becomes

$$k_1^2 \frac{\partial^2 \varphi}{\partial x^2} + k_2^2 \frac{\partial^2 \varphi}{\partial y^2} = 0.$$

This may be reduced to the equation for an isotropic medium, the solution of which is known, by the change of variable

$$x = k_1 \xi, \qquad y = k_2 \eta$$

giving

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial \eta^2} = 0.$$

The case in which we are interested is that of a source and a sink symmetrically situated on one of the median lines of a rectangle. We simplify our discussion at first by replacing the rectangle by an infinite plane. The solution may obviously be built up from two point singularities of the type

$$\varphi = -\log r,$$

where

$$r = \sqrt{(\xi - \xi_0)^2 + (\eta - \eta_0)^2}.$$

Consider the auxiliary ξ, η plane, with one point singularity at the origin ($\xi_0 = \eta_0 = 0$), the specific conductivity of the ξ, η plane being unity. Then the solution $\varphi = -\log r$ corresponds to a source at the origin of the auxiliary plane, current flowing uniformly from the source in radial lines to infinity, the total current flowing from the source being 2π. Changing back the variables now gives us a possible solution in the x, y plane, namely

$$\varphi = -\log \sqrt{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}}.$$

We have to ask what is the corresponding current (I) flowing from the source in the x, y plane. The result may be obtained by integrating u_n over unit circle about the origin.

$$u_n = u_x \cos xn + u_y \cos yn$$

$$= u_x \frac{x}{r} + u_y \frac{y}{r}.$$

Now differentiating φ and substituting above,

$$u_x = \frac{x}{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}}, \qquad u_y = \frac{y}{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}},$$

$$u_n = \frac{r}{\frac{x^2}{k_1^2} + \frac{y^2}{k_2^2}}.$$

We have to form $I = \int u_n ds$, where ds is the element of arc of unit circle $(x^2 + y^2 = 1)$

$$ds^2 = dx^2 + dy^2, \qquad dy = -\frac{x}{y} dx,$$

$$ds = \frac{r}{y} dx,$$

$$I = 4\int_0^1 u_n \frac{r}{y} dx = 4\int_0^1 \frac{dx}{\sqrt{1-x^2}\left[\frac{1}{k_2^2} + x^2\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right)\right]}.$$

This integral may be readily evaluated (See e.g. B. O. Pierce's Tables of Integrals, No. 158), giving

$$I = 2\pi k_1 k_2. \tag{1}$$

Now apply this to the special problem in hand, where we have a source and a sink of equal strength. The solution above applies, taking successively the origin at the source and sink, and using the minus sign for the sink. Our problem is to find, for an infinite sheet, the difference of potential between B_1 and C_1 due to a source at A_1

and a sink at D_1 (see figure 1). The distance A_1D_1 is denoted by 2β, and B_1C_1 by 2α. We have at once

Potential at B due to source at A is $-\log\dfrac{\beta-\alpha}{k_1}$.

Potential at B due to sink at D is $+\log\dfrac{\beta+\alpha}{k_1}$.

Total potential at B is $\log\dfrac{\beta+\alpha}{\beta-\alpha}$.

Potential at C due to source at A is $-\log\dfrac{\beta+\alpha}{k_1}$.

Potential at C due to sink at D is $+\log\dfrac{\beta-\alpha}{k_1}$.

Total potential at C is $\log\dfrac{\beta-\alpha}{\beta+\alpha}$.

Whence

$$\varphi_B-\varphi_C=2\log\frac{\beta+\alpha}{\beta-\alpha}. \tag{2}$$

Now the effective resistance between B and C is defined (and measured) as $(\varphi_B-\varphi_C)/I$, which becomes

$$\frac{1}{\pi k_1k_2}\log\frac{\beta+\alpha}{\beta-\alpha} \tag{3}$$

on substituting the values.

The essential feature of this solution is that k_1 and k_2 enter symmetrically, so that by this method only the geometrical mean of the conductivities in two mutually perpendicular directions can be determined.

In applying this mathematical analysis directly to the experimental arrangement two corrections are involved. In the first place, the actual three dimensional flow in the immediate neighborhood of the contact points by which current enters and leaves is replaced in the analysis by a current everywhere two dimensional. A precise mathematical discussion of the correction for this effect is complicated, and in view of the experimental irregularities there is probably little point in attempting it. The correction is probably beyond the limits of error, because of the thinness of the sheet compared with the distance between the current leads; this ratio was of the order of 1/150 for the metals used. The sign of the effect is to make the specific resistance calculated without correction too large. The effect of tension is to leave this (regarded as a per centage correction) unchanged for the transverse position of the points, but to decrease it slightly for the longitudinal position. The reason for this is that the ratio of the separation of the points to the thickness of the plate is unchanged by tension in the transverse position, but is changed in the longitudinal position.

The second correction is for the finite size of the plate, and is much more important. The rigorous solution for the finite plate may be obtained from the solution above for the infinite sheet by a simple application of the method of images. I owe this idea to a remark of Mr. B. O. Koopman. In applying the method, the plane is cut up into a set of rectangles similar to the original rectangle, with a source or sink in each rectangle in the same position as in the original rectangle. In the strip of rectangles laid off in the direction of the four points the signs alternate, the source being alternately to the right and the left of the sink, whereas in the strip running perpendicularly there is no alternation.

I have carried through an approximate evaluation of the correction for the transverse position of the points. The calculations are not too long with the actual dimensions of the metal sheet used in the experiment. The correction turns out to be approximately 12%, and in this particular case is contributed almost entirely by the phantom rectangles in the strip parallel to the direction of the points, the contributions of all the others nearly cancelling each other and being intrinsically smaller. As a matter of experiment, the mean of the specific resistance of all the metals measured, applying the formula for the infinite plane to the data obtained from the transverse position of the points, was 15% higher than the accepted values, sufficiently good agreement.

The calculation of the correction for the longitudinal position of the points is more difficult, because of slower convergence of the series, and I did not attempt it. It is evident from inspection, however, that the correction will be considerably larger than for the transverse position, and this agrees with the experimental fact that the uncorrected resistance from the longitudinal measurements was 25% high.

Besides determining the correction for the finite size of the sheet, we must also determine how the correction changes when tension is applied, thereby changing the dimensions of the plate, and also making the resistance anisotropic. Here the condition for the transverse position is very much simpler than for the longitudinal position. We have seen that the correction terms are contributed in the transverse position almost entirely by the phantom rectangles extending in the strip parallel to the four points. The terms in the potential arising from the images are logarithms of distances with plus and minus signs (sources and sinks), which may be written in terms of ratios of distances. Under tension, all the effective distances in the strip of phantom rectangles are changed uniformly because of the change of dimensions of the sample, and there is superposed another virtual change, also uniform, because of anisotropy. Hence the ratio of the distances (which enters the expression for the potential) are unchanged and hence the percentage correction on the absolute resistance is unchanged, so that the percentage change of resistance under tension may be calculated without correction from the formulas obtained for the infinite plane. For the longitudinal position, however, the contributions by the strips of phantoms running in both directions must be considered. The distortion is different in different directions, so that there is a correction. This is complicated to compute, and I have not attempted it. One may see by inspection, however, that the uncorrected change of resistance calculated from the longitudinal readings will be too large, and this agrees with the experimental facts.

In the actual calculation of the results, I have used only the data obtained with the transverse position of the points, demanding merely that the longitudinal readings should agree with what might be roughly expected.

Formula 3 yields the two dimensional specific resistance of the metal sheet. In calculating the ordinary resistance per cm. cube, the thickness of the sheet enters. The thickness is changed by tension, so that a correction has to be applied for it, the numerical magnitude of which is evidently σ/E, where σ is Poisson's ratio, and E Young's modulus. The correction is of such a sign that the actually measured proportional increase of resistance is greater than the proportional increase of specific resistance.

Method of Measuring the Longitudinal Coefficient. The measurements just described give the geometrical mean of the longitudinal and transverse coefficients. The straight longitudinal coefficient was measured in much the same way, using the same arrangement of levers, and kerosene bath, and electrical measuring apparatus. The specimen was a strip 1.25 cm. wide and 10 cm. long, cut lengthwise from the specimens the geometrical mean of which had just been measured. This strip was clamped at either end to screw clamps by means of which the tension was applied, and through which the current was led into and out of the specimen. The potential difference between two points 3 cm. apart in the center of the strip was taken off with needle points pressed against the strip. The dimensions of the strip ensure that at the center the lines of flow are straight and uniformly distributed, so that the change of resistance (measured in terms of galvanometer deflection as above) gives directly the longitudinal effect. The numerical magnitudes are much more favorable here than in the first case, so that it was possible to get the pure longitudinal coefficient with much greater accuracy than the geometrical mean.

In reducing the actually measured proportional changes of resistance to proportional changes of specific resistance, corrections must be applied for the increase of length due to tension and for the lateral contraction in both directions at right angles to the length. The magnitude of the correction is evidently $(1 + 2\sigma)/E$, and is in such a direction as to make the increase of specific resistance with tension less than the measured increase.

Experimental Details.

The experimental details were comparatively straightforward, but much care was necessary because the effects are so small. Greater sensitiveness is obtained by using a greater current, but an upper limit is soon reached because of the heating effects, which increase as the square of the current. The currents were chosen as near the upper limit as feasible. Much may be gained by properly directing the action of the stirrer by which the bath is kept at constant temperature. In applying load to the specimens much care must be taken to remain below the elastic limit, as the properties after even moderate overstrain may be greatly altered. This point is to be emphasized; I believe that in early work it has not been sufficiently regarded. The elastic limit of some of the softer metals in the thorougly annealed condition is surprisingly low. I found it impossible to obtain with this apparatus the effect on aluminum, although many attempts were made, because of the exceedingly low elastic limit of the annealed pure metal. The results obtained with gold and copper are also exceedingly uncertain from this cause. Silver gave less trouble. The harder metals, iron, nickel, palladium, platinum, gave entirely satisfactory readings, large enough not only to allow a determination of the average value of the coefficients, but also to allow an investigation of the linearity of the relation between stress and change of resistance. For theoretical use I believe it is safe to use the transverse coefficients of only the harder metals, but the longitudinal coefficients of all should be trustworthy.

Aluminum. This was of very unusual purity, containing not over 0.03% total impurity, and was obtained from the same source as the specimens for which I have previously measured the compressibility and the effect of tension on thermal conductivity.[4] It was annealed for several hours at 300°. It was so soft and the elastic limit so low that no measurements of the transverse effect with the four point apparatus could be obtained. A number of attempts were made to obtain the transverse effect, but in all cases loads had to be applied beyond the elastic limit to obtain anything measurable, and the attempt was abandoned. The exceeding of the elastic limit was shown not only by permanent change of dimensions, but also by failure of linearity between stress and change of resistance.

I was able, however, to get better measurements than hitherto of the pure longitudinal effect. This turns out to be very sensitive indeed to slight permanent stretch. The coefficient of a specimen slightly stretched beyond the elastic limit was only one third as great as the highest value obtained on a specimen whose elastic limit had not been exceeded. The maximum stress applied to the best specimen was only 130 kg./cm^2. Within the limits of error, which were somewhat large, the effect is proportional to the load. This is not the case, however, for aluminum which has been strained beyond the elastic limit, the effect departing markedly from linearity and becoming less at the higher stresses.

Two different specimens of aluminum which had apparently not been permanently stretched gave values of the pure longitudinal coefficient not very different from each other, and both were larger than the values which I have obtained before, and larger than values by other observers. The new values are doubtless to be preferred.

Gold. This was sheet 0.012 cm. thick, obtained from Baker and Co. and stated to be of the highest possible purity (guaranteed better than 99.9%). It was annealed by Baker, and also one piece was further annealed by me, but with no perceptible change in the coefficient. Measurements of the purely longitudinal effect were made on three different specimens with fairly concordant results. I cannot find that the pure longitudinal effect has been previously measured for this metal. The value obtained now is interesting because it is so high. The longitudinal effect is linear with tension with an error not more than 3 or 4% over the range of stress applied here, which was not more than 120 kg./cm^2. The transverse effect for gold was very much less than the longitudinal effect, but is so uncertain that I have not tried to assign any probable value to it.

Copper. Ordinary commercial sheet copper, annealed, was used. There were two different specimens, of thickness 0.012 and 0.20 cm. The measurements of the purely longitudinal effect on the two specimens agreed within 4%, and also agreed with a result which I have previously found for copper of unusually high purity, so that evidently the amount of impurity in commercial copper does not introduce any perceptible error here. Two measurements were also made of the geometrical mean of the two effects; these both agreed in giving a measured effect so small as to be zero within the limits of error. The check measurements with the four points in the longitudinal position behaved as they should for one of the samples, but for the other gave an effect of the wrong sign, for some reason which I have not been able to find. This is the only case of such a discrepancy in all this work, and throws considerable suspicion on the results obtained with the transverse position.

Silver. This was obtained from Baker and Co. of the highest purity, in the form of annealed sheets 0.012 cm. thick. Two samples were used. The behavior when stretched beyond the elastic limit is the same as was found for aluminum, namely the purely longitudinal effect becomes small; in this case about 65% of the effect when the limit is not exceeded. Measurements of the purely longitudinal effect on two different pieces gave coefficients agreeing within less than 1%. The effect is linear within 2% over a tension range of 250 kg./cm^2. The geometrical mean effect was measured on one specimen. It is linear with tension up to 300 kg./cm^2. within an experimental error of 10%.

Platinum. This was annealed sheet, 0.012 cm. thick, of the highest purity (99.9% or better) from Baker and Co. Only one specimen was used. The pure longitudinal effect agreed within about 3.5% with the value which I have previously found for platinum rod. It is linear to better than 1% to 400 kg./cm.^{2}, the highest tension used. The geometrical mean is linear within 4% up to 420 kg./cm.^{2}, and was large enough to give very comfortable measurements. The behavior for the longitudinal position of the points is as would be expected.

Palladium. This was similar in dimensions to platinum, was obtained from the same source, and was stated to be of the same purity (99.9% or better). There seems, however, for some reason to be considerable difficulty in getting pure palladium, so that I tried to get a check on the purity by measuring the temperature coefficient of resistance of a piece of palladium wire drawn by Baker from metal from the same lot. The coefficient was very low, only 0.0030, so that it would seem that the purity is probably not as high as estimated.

The purely longitudinal coefficient agreed within 4% with the value which I have previously found. The relation is linear with the stress to better than 1% up to 400 kg./cm.^{2}, the highest tension used. The geometrical mean coefficient was measured for three different loads to a maximum of 440 kg./cm.^{2} The irregularities were greater than usual, the maximum departure from the linear relation being 15%, but there is no reason to think that this is other than an effect of accidental errors, or that the effect is not actually linear.

Iron. I was not able to obtain pure iron in sheets thin enough for the purpose, and had to content myself with a commercial mild steel of low carbon content. The thickness was 0.010 cm. The dimensions of the original sheet were large enough to allow two samples to be cut, parallel and at right angles to the direction of rolling. This was not possible with the other metals.

The purely longitudinal effect of the two specimens agreed to three significant figures. The value is about 20% lower than I have previously found for iron of very high purity, and agrees essentially with the value found by Tomlinson for iron of no especial purity. The difference is evidently to be ascribed to the carbon content. It is interesting to note that in every case examined so far the effect of impurity is to lower the longitudinal tension coefficient of resistance (if there is any effect at all). Impurity also lowers the temperature coefficient and the pressure coefficient.

The geometrical mean effects for the two specimens did not agree, but was less by 25% for the specimen to which tension was applied in the direction of rolling.

Both effects for both specimens were linear with tension. Because of the largeness of the effect it was possible to make a rather careful examination of this point. The greatest departure of any reading, of which there were 12 in all, from linearity was 5%. The maximum load used was 250 kg./cm.^{2}

Nickel. Two specimens were used from two different sources. Both were what is known commercially as pure nickel, and were somewhat over 99% pure nickel. Both were in the form of sheet 0.012 cm. thick, and were annealed.

It is well known that the pure longitudinal effect is abnormal both with respect to sign, and departure from linearity, and hysteresis. The results obtained with any single specimen are a strong function of the past history. The disentanglement of all the complicated effects in nickel would constitute an elaborate study, and was much beyond the scope of this work. I contented myself here with merely determining whether the sign of the transverse effect would also be found to be abnormal.

The results found with the two specimens agreed in character, although the numerical agreement is not close. It is not worth while to try to reproduce here the complicated results found, but I will merely indicate the general nature.

Both purely longitudinal and geometrical mean effects show hysteresis, saturation (or more probably a maximum), and complicated dependence on the past history. The following numerical values were found for one of the samples. Under a load of 378 kg./cm.^{2} the measured purely longitudinal effect was a decrease of resistance of 0.00540. Correcting for distortion, this becomes a proportional decrease of specific resistance of 0.00570 for this load. Under the same load the geometrical mean of longitudinal and transverse effects was found to be -0.00189. Correcting for distortion this becomes -0.00200. This is less than half the pure longitudinal coefficient, so that it is evident that the transverse specific resistance increases under tension. Detailed calculation gives a proportional increase of transverse specific resistance by 0.00170 for the load of 378 kg./cm.^{2}

The chief conclusion to be drawn from these measurements on nickel is that the longitudinal and transverse coefficients are of opposite sign. Because of the abnormal character of nickel, I shall not attempt to discuss further the significance of the results.

NUMERICAL RESULTS.

In Table I are reproduced the numerical results found for all the metals except nickel. In column 1 are given the names of the metals, in column 2 the measured longitudinal effect, expressed as fractional change of resistance for a tension of 1 kg./cm.^{2}, in column 3 the meas-

TABLE I.

Collected Numerical Results.

Metal		Longitudinal Fractional Change of Resistance, for 1 kg./cm.^{2}		Geometrical Mean of Longitudinal and Transverse Fractional Change of Resistance for 1 kg./cm.^{2}		Calculated Fractional Change of Transverse Resistance for 1 kg./cm.^{2}
		Measured	Ccrrected for Distortion	Measured	Corrected for Distortion	
Al		$+5.8 \times 10^{-6}$ 6.9	$+4.0 \times 10^{-6}$			
Au		$+6.25 \times 10^{-6}$ 5.94 6.33	$+3.87 \times 10^{-6}$	Not greater than $+0.40 \times 10^{-6}$	$-.17 \times 10^{-6}$ (upper limit)	-4.5×10^{-6} (very uncertain)
Cu		$+3.01 \times 10^{-6}$ 3.15	$+1.75 \times 10^{-6}$	$-.1$ to $.0 \times 10^{-6}$	$-\ .3 \times 10^{-6}$	-2.4×10^{-6}(?)
Ag		$+5.06 \times 10^{-6}$ 5.03	$+2.86 \times 10^{-6}$	$+1.89 \times 10^{-6}$	$+1.41 \times 10^{-6}$	$-.04 \times 10^{-6}$
Pt		$+2.82 \times 10^{-6}$	$+1.78 \times 10^{-6}$	$+1.29 \times 10^{-6}$	$+1.06 \times 10^{-6}$	$+.34 \times 10^{-6}$
Pd		$+2.90 \times 10^{-6}$	$+1.37 \times 10^{-6}$	$+1.29 \times 10^{-6}$	$+\ .94 \times 10^{-6}$	$+.51 \times 10^{-6}$
Fe	(1)	$+2.13 \times 10^{-6}$	$+1.42 \times 10^{-6}$	$+\ .99 \times 10^{-6}$	$+\ .86 \times 10^{-6}$	$+.30 \times 10^{-6}$
	(2)	$+2.13 \times 10^{-6}$		$+1.33 \times 10^{-6}$	$+1.20 \times 10^{-6}$	$+.78 \times 10^{-6}$

(1) Tension parallel to direction of rolling.
(2) Tension perpendicular to direction of rolling.

ured results of column 2 are corrected for the distortion by tension so as to give the change of specific resistance for a tension of 1 kg./cm.^{2} The correction by which column 3 is obtained from column 2 is $(1 + 2\sigma)/E$. Column 4 contains the measured change of resistance as given by the four point apparatus with the points in the transverse position. In column 5 the results of column 4 are corrected for the distortion produced by tension by subtracting the term σ/E, giving the geometrical mean of the effect of 1 kg./cm.^{2} on longitudinal and transverse resistance. Finally, in column 6 is given the effect of tension on specific resistance transverse to the direction of the tension. Column 6 is obtained by subtracting from column 5 half of column 3 and multiplying by 2. The essentially new results, to obtain which this investigation was made, are those contained in column 6; the results of column 3 are also necessary for any theoretical discussion.

The transverse coefficient may have either sign. The cases of negative sign are much less certain than the positive cases. The negative value for gold is very uncertain, that for silver is so small that within experimental error it might well be positive, and the value for copper is also very uncertain, as may be seen by examining the measured geometrical mean effects. It is perhaps significant that the negative coefficients are shown by the softer metals. The positive values for palladium, platinum and iron, are, however, much more certain experimentally; the effects were larger, larger stresses could be applied, and the corrections for distortions were smaller and more certain. The positive sign is the reverse of that found by Tomlinson in the two cases examined by him.

Theoretical Discussion.

In the first place, we are now in a position to answer a question of somewhat formal character raised in connection with the effect of tension on the resistance of the abnormal metals, namely whether it is possible to connect the changes of resistance directly with the changes of dimensions, changes of dimensions at right angles to the direction of current flow affecting the resistance differently from changes parallel to the flow. The coefficient of proportionality may be written k_l for parallel (longitudinal) changes, and k_t for perpendicular (transverse) changes. This involves in general writing for the change of resistance;

$$\frac{\Delta R}{R} = k_l \frac{\Delta \delta_l}{\delta_l} + k_t \left[\left(\frac{\Delta \delta_t}{\delta_t} \right)_1 + \left(\frac{\Delta \delta_t}{\delta_t} \right)_2 \right],$$

where $\Delta\delta_l/\delta_l$ denotes the strain in the direction of flow, and $\Delta\delta_t/\delta_t$ that at right angles to it, the latter appearing twice because there are two directions perpendicular to the flow. The equation as assumed involves two coefficients. The assumption may be checked by comparing the values of $\Delta R/R$ given by three independent kinds of experiment, which the data of this paper now for the first time place at our disposal, namely the changes of resistance under hydrostatic pressure (given by previous work), longitudinal changes under tension, and transverse changes under tension. For these three kinds of stress we have the following deformation:

Hydrostatic pressure

$$\frac{\Delta\delta_l}{\delta_l}=\frac{\Delta\delta_t}{\delta_t}=\frac{1}{3}\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t p.$$

Tension, longitudinal effect

$$\frac{\Delta\delta_l}{\delta_l}=\frac{T}{E},\qquad \left(\frac{\Delta\delta_t}{\delta_t}\right)_1=\left(\frac{\Delta\delta_t}{\delta_t}\right)_2=-\frac{\sigma T}{E}.\qquad (T\text{ is tension.})$$

Tension, transverse effect

$$\frac{\Delta\delta_l}{\delta_l}=-\frac{\sigma T}{E},\qquad \left(\frac{\Delta\delta_t}{\delta_t}\right)_1=\frac{T}{E},\qquad \left(\frac{\Delta\delta_t}{\delta_t}\right)_2=-\frac{\sigma T}{E}.$$

Hence we have the following three equations for the two coefficients k_l and k_t.

Hydrostatic pressure

$$\frac{1}{p\frac{1}{3}\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t}\cdot\frac{\Delta R}{R}=k_l+2k_t.$$

Tension, longitudinal

$$\frac{E}{T}\cdot\frac{\Delta R}{R}=k_l-2\sigma k_t.$$

Tension, transverse

$$\frac{E}{T}\cdot\frac{\Delta R}{R}=-\sigma k_l+k_t(1-\sigma).$$

Putting p and T equal unity gives to $\Delta R/R$ the values of pressure or tension coefficient. The values used in checking the equations are given in Table II.

TABLE II.

Metal	Pressure Coefficient of Specific Resistance	Longitudinal Tension Coefficient of Specific Resistance	Transverse Tension Coefficient of Specific Resistance	Linear Compressibility $-\frac{1}{3}\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t$	$\frac{1}{E}$	σ	k_l	k_t	$-\sigma k_l+(1-\sigma)k_t$ Obs.	$-\sigma k_l+(1-\sigma)k_t$ Calc.
Au	-3.33×10^{-6}	$+3.87 \times 10^{-6}$	-4.9×10^{-6}	$.193 \times 10^{-6}$	1.25×10^{-6}	.42	7.3	5.0	−3.92	−0.17
Ag	−3.83	+2.86	− .04	.329	1.26	.38	4.84	3.38	− .03	+ .26
Cu	−2.45	+1.75	−2.4	.240	.81	.34	4.20	3.00	−2.96	+ .55
Pd	−2.16	+1.37	+ .51	.173	.89	.39	4.62	3.94	+ .57	+ .60
Pt	−2.07	+1.78	+ .34	.120	.59	.39	7.0	5.1	+ .58	+ .38
Fe	−2.60	+1.42	+ .54	.196	.48	.28	5.2	4.05	+1.13	+1.45
Al	−4.87	+4.0		.448	1.42	.34				

The check is applied by comparing the observed value of $-\sigma k_l + (1 - \sigma)k_t$ with the calculated value. The check is probably as good as could be expected for Pt, Pd, and Fe, the metals for which the transverse coefficient was determined with the greatest accuracy, is not bad for Ag, considering that the calculated value is the difference of two numbers nearly equal, while for Au and Cu there is no agreement, but the experimental values were also exceedingly uncertain. I believe that these results give rather high probability to the legitimateness of the assumption made as to the connection between distortion and change of resistance, and that for the soft metals, for which the transverse effect is exceedingly difficult to measure, the transverse effect may probably be calculated from the hydrostatic pressure and longitudinal tension effects with greater accuracy than it can be measured.

The numerical values of k_l and k_t are of interest. In all cases they are positive (that is, increasing the distance between atoms increases resistance), and k_t is numerically less than k_l. It is perhaps at first unexpected that k_t is not zero. A reason is at once suggested by any free path theory of conduction, for the motion of those electrons which are in paths inclined to the e.m.f. is affected by transverse strain because the transverse strain has a component along the path.

This formal expression for the change of resistance in terms of strain allows considerable latitude in the underlying physical picture. For instance, if the free flight is from atom to atom, through the substance of the atom, as I have supposed in my theory for normal metals, the coefficient k_l would be expected to be the most important and positive, whereas if the path lies between the atoms, k_t would be most important and negative. The values found above correspond to the expectation for a normal metal.

It is one of the tasks of any complete theory to reproduce the experimental connection between these three effects, or to reproduce k_l and k_t. This I believe no theory is at present in a position to attempt, but we may get some indications of the relative magnitudes of the transverse and longitudinal effects. I have already given some discussion of this question in connection with the longitudinal tension coefficient. The conclusion there drawn was that the classical free electron theory, in which the electron is free to drift in the direction of an applied e.m.f. irrespective of the direction of the free path relative to the e.m.f., must be modified by picturing the electron as moving in something like a fixed groove, so that the velocity of drift imparted by the applied e.m.f. is directed along the original path, and is effectually produced only by the component of e.m.f. along the path.

This conclusion is much strengthened by the new evidence of this paper. There was, however, an error in the mathematical analysis of the former paper[5] which must be corrected. The problem is to integrate the effect of strain over all directions. The physical assumption made is that the change in length of any electronic free path is proportional to the geometrical elongation in that direction. In terms of my previous theory, this means a normal metal, in which there are no free paths between the atoms. It is also assumed that when a body is strained there is no redistribution of the relative number of electronic paths in different directions, nor any redistribution of velocity. This amounts to maintaining one of the fundamental assumptions of the classical theory, namely that after the free paths of the electrons have been terminated by collision, the new distribution of velocities is entirely at random. This means that the entire effect on resistance produced by strain is due to an alteration of length of free path.

Consider now the strain due to tension. In the direction of the tension there is an elongation

$$\frac{\delta l}{l} = \frac{T}{E},$$

and at right angles to the tension a contraction

$$\frac{\delta l}{l} = -\sigma \frac{T}{E}.$$

For any intermediate direction, making an angle θ with the tension, the elongation is

$$\frac{\delta l}{l} = \frac{T}{E}(\cos^2 \theta - \sigma \sin^2 \theta).$$

In the preceeding paper an incorrect expression, $\frac{T}{E}[1 - (1 + \sigma) \sin \theta]$ was used for this. We now assume that the free path, L, of an electron in any direction in the strained metal is related to the free path in the unstrained metal, L_0, by the equation

$$L = L_0\left(1 + \alpha \frac{\delta l}{l}\right) = L_0\left[1 + \alpha \frac{T}{E}(\cos^2 \theta - \sigma \sin^2 \theta)\right],$$

where α is an empirical constant, and may most easily be evaluated in terms of the pressure coefficient of resistance.

Consider now a metal to which an e.m.f. $\mathcal{E}$ is applied, and an electron moving in a free path making an angle φ with $\mathcal{E}$. The conventional free electron analysis gives for the velocity of drift in the direction of $\mathcal{E}$ the expression $\frac{1}{2}\frac{\mathcal{E}e}{m}\cdot\frac{L}{v}$, provided the force $\mathcal{E}$ produces full drift in its own direction. Here v is the normal electron velocity, supposed large compared with the velocity of drift. The result is obtained as follows. The time of free flight is L/v, the force acting is $\mathcal{E}e$, the acceleration is $\mathcal{E}e/m$, the velocity of drift at the end of free flight is $\frac{L}{v}\cdot\frac{\mathcal{E}e}{m}$, and hence the average velocity of drift is $\frac{1}{2}\frac{\mathcal{E}e}{m}\frac{L}{v}$. The total current is obtained by integrating, or

$$I = \int_0^{2\pi} d\psi \int_0^{\pi} n\frac{\mathcal{E}e}{m}\cdot\frac{L_0}{v}\sin\theta\left[1 + \alpha\frac{T}{E}(\cos^2\theta - \sigma\sin^2\theta)\right]d\theta,$$

where θ and ψ are ordinary polar coordinates, and n is the number of free electrons per unit volume per unit solid angle. The conductivity is obtained from this equation by dividing both sides by $\mathcal{E}$, or by setting $\mathcal{E}$ equal to unity.

But if the electron is not free to drift completely in the direction $\mathcal{E}$, but is constrained to maintain its original direction, then the component drift in the direction of $\mathcal{E}$ is $\frac{1}{2}\frac{\mathcal{E}e}{m}\cdot\frac{L}{v}\cdot\cos^2\varphi$, $\cos\varphi$ entering once because $\mathcal{E}$ must be resolved along the path, and again because the drift so produced must be resolved along $\mathcal{E}$. The conductivity is now

$$X = \int_0^{2\pi} d\psi \int_0^{\pi} n\frac{eL_0}{mv}\sin\theta\cos^2\varphi\left[1 + \alpha\frac{T}{E}(\cos^2\theta - \sigma\sin^2\theta)\right]d\theta.$$

Our problem is now to evaluate these expressions corresponding to different special cases. First consider the expression given by the hypothesis of unrestrained drift. It is at once evident that the conductivity is independent of the direction of $\mathcal{E}$, that is, the transverse change of resistance is the same as the longitudinal change. The reason of course, is that under the assumptions every electron makes a contribution to the total conductivity determined only by its time of free flight, and independent of the direction of $\mathcal{E}$. *This conclusion is directly contrary to the experimental evidence,* and demands that we abandon this conception of free flight. The same assumptions would also lead us to the conclusion that in a crystal the resistance must be the same in all directions, which again is directly contradicted by experiment.

For convenience of reference we record the result of carrying out the integration on the hypothesis of unrestricted drift. The result is

$$X_l = X_t \backsim 1 + \frac{1}{3}\alpha\frac{T}{E}(1 - 2\sigma), \quad \text{unguided path.}$$

Turn now to the assumption of guided paths. We have to consider two cases, $\mathcal{E}$ along the tension, or at right angles. In the first case $\cos\varphi = \cos\theta$, and in the second $\cos\varphi = \sin\theta\cos\psi$. Hence we have respectively for the longitudinal and transverse conductivities

$$\left.\begin{aligned} X_l &\backsim 1 + \frac{3}{5}\alpha\frac{T}{E}\left(1 - \frac{2\sigma}{3}\right) \\ X_t &\backsim 1 + \frac{1}{5}\alpha\frac{T}{E}(1 - 4\sigma) \end{aligned}\right\} \text{guided path.}$$

For pure hydrostatic pressure, $L = L_0(1 + \alpha kp)$, where k is the linear compressibility. The two hypotheses give

$$Xp \backsim \int_0^{2\pi} d\psi \int_0^{\pi} \sin\theta(1 + \alpha kp)d\theta \backsim 1 - \alpha kp, \quad \text{unguided path,}$$

$$Xp \backsim \int_0^{2\pi} d\psi \int_0^{\pi} \sin\theta\cos^2\theta(1 + \alpha kp)d\theta \backsim 1 + \alpha kp, \quad \text{guided path.}$$

In applying this analysis, we have assumed n unaffected by the strain. But now it has been one of the assumptions of our theory that the number of free electrons *per atom* is constant. This means that in making experimental comparisons we must use the various coefficients "corrected for change of volume," as was done in the 1923 paper.

We may first apply the formulas given above to correct the previous results.[5] The ratio of pressure coefficient to longitudinal tension coefficient for the unguided path should be

$$\frac{3kE}{1 - 2\sigma}.$$

The sign of this does not agree with experiment, as already indicated.

On the guided path theory, the ratio is

$$\frac{5kE}{3\left(1-\frac{2\sigma}{3}\right)}.$$

Assuming for σ the mean value 1/3, this is $(15/7)kE$. The value previously found was $4.7kE$.

Using the new data, we now compare this expression with experiment. The results are given in Table III, columns 4 and 5. The agreement is not good. It is to be noticed, however, that the new value for the longitudinal tension coefficient of aluminum removes aluminum from a markedly exceptional position. In the last column of the table, is given the corrected transverse coefficient, X_t, for purposes of reference.

Returning now to the expressions for X_l and X_t on the guided path theory, we see that there is provision for a variation of sign. X_l is always positive for those normal metals whose resistance decreases under hydrostatic pressure. The reason is that $1 - 2\sigma/3$ is always positive, the maximum value of σ consistent with stability being 0.5. X_t on the other hand, is negative if σ is greater than 0.25. Now σ is greater than 0.25 for all the metals listed here, but nevertheless X_t is positive for the more certain ones. It is evident, therefore, that the picture of the free path given above does not correspond exactly to the facts, but it must be credited with making a step in the right direction in allowing either sign.

It is evident that a closer approach to experiment may be made by taking some sort of a mean between the guided and the unguided path formulas. The physical significance of this may be perhaps that both effects are present.

We turn now to another question connected with the tension coefficient, namely connection with crystal symmetry. The metals examined in this paper all belong to the cubic system crystallographically, and may therefore be expected to show a certain simplicity as compared with other metals. It should nevertheless be recognized that even with the cubic metals crystal structure has an effect, and that the quantities X_l and X_t may be expected to be different for samples cut in different directions from a single crystal. The reason of course is that the strain produced by tension is a function of the orientation. The values found above experimentally are mean

TABLE III.

Comparison of Experimental and Calculated Relation between Pressure and Longitudinal Tension Coefficients of Resistance.

Metal	Pressure Coefficient of Specific Resistance Corrected by $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t$	Longitudinal Tension Coefficient of Specific Resistance corrected by $\frac{1-2\sigma}{E}, X_t$	Ratio, $\frac{\text{Corrected Press. Coeff.}}{\text{Corrected L. Ten. Coeff.}}$	$\frac{5}{3}\frac{kE}{1-\frac{2\sigma}{3}}$	Transverse Tension Coefficient of Specific Resistance, corrected by $\frac{1-2\sigma}{E}, X_t$
Al	-3.97×10^{-6}	$+3.55 \times 10^{-6}$	1.12	.68	—
Au	-2.75	$+3.65$	.75	.37	—
Ag	-2.83	$+2.56$	1.10	.58	$-.34 \times 10^{-6}$
Cu	-1.73	$+2.19$	.79	.64	$-.27$
Pd	-1.64	$+1.96$	.84	.44	$+.31$
Pt	-1.71	$+1.94$	.88	.46	$+.21$
Fe	-2.01	$+2.39$	.84	.84	$+.33$

values. The pressure coefficient of resistance, on the other hand, is independent of the orientation of the specimen. Experimentally we may have a manifestation of the effect in the different results obtained for iron parallel and perpendicular to the direction of rolling.

The general problem of determining the number of constants required to completely determine the resistance in all directions for any system of stress in a single crystal belonging to any crystallographic system does not seem to have been discussed.

SUMMARY

A new experimental method for measuring the change of resistance in a metal under tension when the direction of flow is at right angles to the tension (transverse coefficient) has been developed. The method gives immediately the geometrical mean of the transverse and longitudinal coefficients. An independent determination of the longitudinal coefficient permits a calculation of the transverse coefficient. Measurements are presented for 8 metals. Nickel is abnormal, as it also is with respect to the longitudinal coefficient. It is established that the signs of the two coefficients are opposite for Ni. The values found for the softer of the other metals, Al, Au, Ag, and Cu, are uncertain because of the smallness of the effect and the necessity for remaining below the elastic limit. It is probable that the transverse coefficients of some of these metals are negative. The results for Pd, Pt, and Fe are much more certain. The transverse coefficients of these are positive; this sign is the opposite of that found by the only previous observer, Tomlinson.

In the discussion it is shown that probably all changes of resistance due to deformation at constant temperature may be described in terms of two coefficients, one connecting the change of resistance with changes of dimensions at right angles to the current flow, and the other connecting with changes of dimension parallel to the flow. It is the task of theory to reproduce these two coefficients. It is pointed out that if some form of free path theory of conductivity is maintained it is not possible to suppose the electrons free to drift unrestrainedly in the direction of the applied force, for under such conditions the transverse and longitudinal coefficients are equal. It must be, therefore, that the electrons are constrained to a certain extent to move along guided paths in the metal. No theory is at present able to account satisfactorily for all the deformational effects. With regard to my own particular form of free path theory, a cor-

rection in a previous calculation of the longitudinal tension coefficient from the pressure coefficient is made by which the agreement between calculation and experiment becomes less good. It must be recognized, however, that this form of theory is so far in accord with the facts as it allows a transverse coefficient of either the same or opposite sign from that of the longitudinal coefficient.

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The Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.

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ON THE SYMBIOSIS OF CERTAIN BERMUDA COELENTERATES AND ZOOXANTHELLAE.

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ON THE SYMBIOSIS OF CERTAIN BERMUDA COELENTERATES AND ZOOXANTHELLAE.

BY H. BOSCHMA.

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. NO. 150.

By H. Boschma.

ON THE SYMBIOSIS OF CERTAIN BERMUDA COELENTERATES AND ZOOXANTHELLAE.

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The association of coelenterates with unicellular algae (zooxanthellae or zoochlorellae) is usually defined as a kind of symbiosis. There are, however, only very few data on the advantages which each of the two organisms obtains from this association. In a previous paper (Boschma, 1924) I have published the results of a study of the contents of the gastric cavity in a large number of East Indian coral-polyps. In these contents I often found partly digested foreign organisms, or parts of them, and besides these invariably zooxanthellae in different stages of disintegration. Thus I have confirmed the results of Gardiner (1903), who found that most of the reef-corals derive a considerable part of their food from their symbiotic algae, and even that the polyps of many species feed exclusively on their zooxanthellae.[1]

To give definite proof that the zooxanthellae are digested by the polyps of reef-corals and by other coelenterates it was important that the process of digestion should be studied in living polyps more accurately than has been done hitherto. I was therefore glad of the opportunity to work for some time at the Bermuda Biological Station for Research during the latter half of September, 1924. I am greatly indebted to the Director of this Station, Dr. E. L. Mark, who allowed me to use the laboratory after it was already closed, and who kindly revised the manuscript of this paper. I also wish to thank Dr. Roy W. Miner, who loaned me a microscope of American manufacture, which could be brought back to the United States without paying importation duties.

As I could spend only a short time (ten days) in the Bermudas, and as the weather during this period was too rough to permit the collecting of corals at the outer reefs, I confined my experiments to coelenterates which were easily to be collected in the neighborhood of Agar's Island. The four which I studied were: *Isophyllia dipsacea* (Dana), *Siderastraea radians* (Pallas), *Zoanthus sociatus* (Ellis), and *Condylactis passiflora* Duch. et Mich.

The polyps were kept alive in large glass jars containing sea-water that was renewed every day. This, however, did not seem to be quite sufficient to keep the animals in a perfectly normal state. After some days they were less sensitive to mechanical and chemical stimuli than during the first day of captivity.

The greater part of the experiments was carried out with Isophyllia,[2] which proved to be a very useful object because its polyps are large. In the literature the statement is often found that remnants of food are hardly ever to be detected in the gastric cavity of coral-polyps (cf. Duerden, 1902; Pratt, 1906; Carpenter, 1910). These statements, however, in most cases are based upon the study of preserved material. Owing to the strong contraction produced by the preserving fluids, such polyps have discharged the contents of the gastric cavity.

In the polyps of freshly collected colonies one may often find a small amount of mucous matter protruding from the stomodaeum. In Isophyllia I found that these small masses of protruding material consisted for the greater part of matter of indeterminable nature (detritus), but also contained diatoms (both living and dead) and remnants of appendages of small crustaceans. Usually nematocysts also are to be found in this extruded matter. Besides other material, zooxanthellae were invariably found in these mucous masses. Some of these yellow cells were in various stages of disintegration, but others were quite normal.

The entoderm of the oral disk, of the tentacles, and of the edge-zone of Isophyllia is crowded with zooxanthellae, as already stated by Duerden (1902).[3] These zooxanthellae are spheroidal yellow algae with a diameter of 8½ to 11½ μ. They have a light yellow color, which seems to be distributed uniformly through the contents of the cell, but probably is located only in the small chromatophores which are found in great numbers in contact with the cell wall of the alga. In the central part of the alga one or two—sometimes more—small, strongly refringent bodies are to be found, which consist of an amyloid substance (as shown by the iodine reaction). In the living state the nucleus is not to be seen.

These zooxanthellae are always found in the digestive organs of the polyps of Isophyllia which have been freshly collected. It is well known that in actinians digestion takes place only in the mesenterial filaments (Metschnikoff, 1880; Mesnil, 1901; Jordan, 1907, to quote only a few authors), and it is also here that we have to trace the digestion of the food in the Madreporaria. The mesenterial filaments in Isophyllia consist of the somewhat thickened free edges of the septa, which are bordered by a thick margin. The latter part is almost circular in cross-section and consists chiefly of nematocysts (cf. also Duerden, 1902). We find that the food is taken up and digested in the region next to that of the nematocysts. In mesenterial filaments from freshly collected polyps this region is crowded with zooxanthellae. When examined under a low magnification, therefore, a dark band is found adjacent to the marginal part containing the nematocysts. In the remaining part of the septum only sparsely distributed zooxanthellae are present in the entoderm cells. Some of the yellow cells in the mesenterial filaments still have the same form and contents as those found in the tissues of the oral part of the polyp. The condition of the contents of the greater number of the algae, however, differs from that of normal ones. A large proportion of them are irregularly discolored, and the most of these also show small corpuscles of dark brown color in contact with the wall of the cell. In some cases the zooxanthellae have lost the spheroidal form, being more or less irregularly distorted. There is very little doubt that these abnormal algae present different stages in the process of disintegration caused by the digestive action of the coral-polyps. This view is also supported by the fact that such zooxanthellae are always present in the remnants of food which are found protruding from the stomodaeum of freshly collected specimens.

Feeding experiments were carried out with a large number of polyps belonging to different colonies of *Isophyllia dipsacea.* The food, consisting of the meat of mussels, was readily taken, especially by specimens which had not yet been many days in the laboratory. Usually I mixed litmus with the food, so as to be able to trace the food in the internal organs after it had been swallowed. This mixture of mussel-meat and litmus was as readily ingested as pure mussel-meat.

The experiments were made in diffuse daylight in the laboratory. I found that Isophyllia could ingest food in three different ways. (1) Often, especially in fresh specimens, the food produced a sinking of the oral disk and a contraction of the sphincter muscle, causing the edge-zone to form a kind of roof over the oral disk and the tentacles in the manner described by Carpenter (1910). (2) In other specimens, also in colonies which had been recently collected, another method was seen: the food caused the mouths of the polyps to open, and then by ciliary action the food was slowly passed down through the stomodaeum into the gastric cavity, whereupon the mouth was again closed. In this case the oral disk and the edge-zone underwent almost no change, although the tentacles increased slightly in size. In general the colonies, the polyps of which had fed in this way, filled their gastric cavity with more water, the tissues thereby projecting a little more above the skeleton. (3) The third manner of feeding observed in Isophyllia was the one which was usual for polyps of colonies which had been four or more days in the laboratory, though it also occurred sometimes in freshly collected specimens. When mussel-meat was placed on the oral disk of the polyps, mesenterial filaments protruded through the mouth and through different parts of the oral disk, and especially through the tissues of the edge-zone. When the mesenterial filaments came into contact with a piece of meat, the meat became more or less enveloped by these organs and partly ingested. This variability in the method of feeding observed in different polyps of Isophyllia was, perhaps, partly caused by the unnatural state of the animals due to the infrequency with which the sea-water was changed.

In a great many of the mesenterial filaments of polyps which had been fed with the mixture of mussel-meat and litmus red spots were found after some hours in the part next to the marginal zone of nematocysts, which proved that it was at this point that the meat had been ingested (the litmus accompanied the meat only as an indicator), and, furthermore, that the reaction in the food-vacuoles was acid. The same phenomenon has been found in actinians (Mesnil, 1901). In some of the mesenterial filaments no red vacuoles were present. In these large polyps of Isophyllia, therefore, only a part of the mesenterial filaments had ingested the food.

During several days the litmus remained in the zone of ingestion of the mesenterial filaments. After about three days the greater part of the litmus-colored vacuoles had assumed a blue color, showing that the reaction had changed from acid to alkaline. The final digestion and resorption of the food probably took place at this stage.

As I already have pointed out, in freshly collected, not artificially fed, polyps the mesenterial filaments are crowded, in the part next to the free margin, with zooxanthellae in various stages of disintegration. In the mesenterial filaments that had ingested a fairly large quantity of mussel-meat (as shown by the large number of litmus-colored vacuoles) only a very small number of zooxanthellae were to be seen a few days after the feeding. The few zooxanthellae still found in these organs never looked like those in the entoderm of the oral parts of the polyp; they were always in an advanced stage of disintegration showing brown corpuscles and white spots. From these facts we may conclude that after abundant feeding on mussel-meat no more zooxanthellae had been ingested by these mesenterial filaments, and that the algae present in them before the feeding of the meat were, with a few exceptions, completely digested. This conclusion is further supported by the fact that in artificially fed polyps some of the mesenterial filaments do not ingest the colored mussel-meat. In these filaments, however, there is a zone full of zooxanthellae next to the marginal part.

Summarizing the above facts, we may describe the feeding of Isophyllia in the following way. Under normal circumstances the food of the polyp consists chiefly of the algae which are living in its entoderm cells. Besides yellow cells, small planktonic organisms are captured and digested, but food from this source seems under normal conditions to be too scanty to serve as exclusive nutriment for the polyps. When, however, the digestive organs (the mesenterial filaments) can obtain a sufficient quantity of food from other sources, they do not ingest any more zooxanthellae. The algae which were already present in the mesenterial filaments are gradually and completely digested, so that after abundant feeding on meat the digestive organs may become wholly devoid of zooxanthellae. Consequently the feeding of the polyps on zooxanthellae appears to be a condition due to the scarcity of food of extraneous origin. The symbiosis of coral-polyps and yellow cells represents, therefore, at least partly, a parasitic relationship between the two organisms, the polyps parasitizing on the zooxanthellae. Probably, on the other hand, the algae derive some profit from living in the entoderm, especially in that of the oral part of the polyps, the advantage consisting in their being exposed to the light and, perhaps, in their obtaining waste nitrogenous products from the polyp. At least, they thrive here quite well and increase rapidly in number by fission. The final result of this symbiosis is that the surplus of yellow cells is constantly eaten by the polyp.

These experiments have led to a result quite similar to that obtained by Van Trigt (1919) in fresh-water sponges. In the amoebocytes of these sponges Van Trigt found symbiotic algae (zoochlorellae) in various stages of being digested. In nature, however, the sponges digest only few symbiotic algae, their food supply of other origin being quite sufficient. When they were kept in aquaria the number of zoochlorellae which were digested by the amoebocytes considerably increased, owing to the lack of food from other sources. In

Isophyllia, on the contrary, under normal circumstances the extraneous food supply usually seems to be not very abundant, and therefore the zooxanthellae are largely eaten by the polyps. The algae are left unmolested only when enough food of other kinds is available.

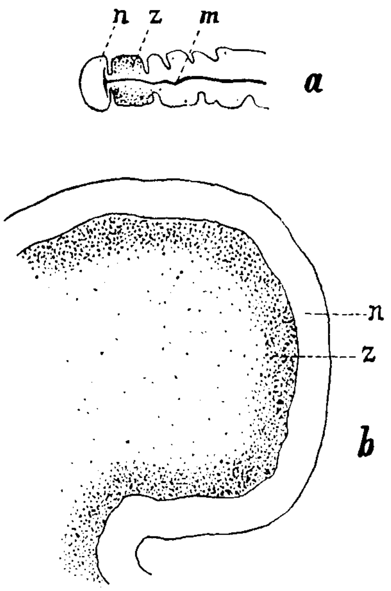
I also made some feeding experiments on *Siderastraea radians.* A thick juice was obtained by pounding up the internal organs of mussels. This juice after being thoroughly mixed with litmus was poured over the surface of the colonies with a pipette. This colored food substance was readily ingested by the polyps. After a few hours a marked contrast between the artificially fed colonies and those which had not obtained mussel-juice was to be seen. In the former the polyps expanded so as to rise a little above the skeleton, with slightly extended tentacles, whereas in the unfed colonies the polyps remained in a contracted state.

It was more difficult to experiment with this animal than with Isophyllia. Owing to the smallness of the polyps, the mesenterial filaments were far less easily obtained. Therefore the experiments on Siderastraea were much less complete than those on Isophyllia. On the whole, however, the results were quite the same. In freshly collected polyps of Siderastraea the mesenterial filaments showed a zone next to the marginal part crowded with partly digested zooxanthellae. After feeding mussel-juice and litmus abundantly, the litmus became visible as red spots in this zone, while the zooxanthellae gradually disappeared in the course of a few days. The symbiotic algae in the entoderm cells of the oral part of the polyps of Siderastraea have the same structure as those of Isophyllia.

Numerous zooxanthellae are also found in *Zoanthus sociatus.*[4] They occur in the entoderm of the oral zone, of the tentacles, and of the column. In size, color, and structure these algae agree completely with those of the two species of Bermudian Madreporaria in which I have studied the yellow cells, as recorded above. They contain usually one, often two, and only rarely more corpuscles consisting of amyloid substances. The mesenterial filaments of Zoanthus have a dark brown color, for they are completely filled with zooxanthellae. Many of these still have the same form and contents as those in the tissues of the oral part of the polyp. The yellow cells in the mesenterial filaments, however, are for the greater part more or less digested, their contents having assumed a different shape. In the interior of these zooxanthellae are found dark brown corpuscles and discolored spots, just as in the yellow cells in the mesenterial filaments of Isophyllia. The zone of the mesenterial filaments of Zoanthus in which the digestion takes place is very broad; it lies next to the uncolored marginal part, which contains a large quantity of nematocysts.[5]

The feeding experiments on this species gave entirely negative results. The polyps refused to ingest mussel-meat or mussel-juice, and reacted only with strong contraction when these substances came in contact with either oral disk or tentacles. Meat of Chiton and of Fissurella was also invariably refused.

In the large sea-anemone *Condylactis passiflora* the entoderm of the oral disk and the tentacles is richly supplied with zooxanthellae. The brown color of the tentacles is entirely due to the presence of these algae. The yellow cells are quite similar to those of the other species dealt with in this paper. Usually they have the same size (average diameter 10 μ), though some smaller ones also occur among the others. The cells contain, one, two, or more amyloid corpuscles.[6]



Parts of mesenterial filaments of *Condylactis passiflora*, showing the distribution of the zooxanthellae in these organs. **a**, transverse section of a preserved, strongly contracted filament; **b**, part of a living mesenterial filament, in side view.

m, mesogloea; *n*, free edge with nematocysts; *z*, digestive part with zooxanthellae (indicated as black dots). × ca. 50.

The mesenterial filaments of Condylactis contain a large number of zooxanthellae in the region next to the colorless marginal part where nematocysts are found in great numbers (cf. the accompanying figure). In the remaining part of the mesentery only sparsely distributed zooxanthellae are present. The zone where great nunbers of zooxanthellae are found is doubtless the region where digestion chiefly takes place. Some of the yellow cells of this region, as well as those of the tentacles, are quite normal in appearance, but the larger part of them show various stages of disintegration, so that there results a discoloration of a portion of the cells and the formation in them of dark brown corpuscles. Consequently in Condylactis also the zooxanthellae seem to constitute an important part of the normal diet of the polyps.

As I collected these actinians on one of the last days of my stay in the Bermudas, I had no time to undertake feeding experiments with colored food. Had I been able to remain longer, the experiments with the large polyps of this species would doubtless have yielded good results, for the animals readily eat mussel-meat.

SUMMARY.

The zooxanthellae of *Isophyllia dipsacea*, *Siderastraea radians*, *Zoanthus sociatus*, and *Condylactis passiflora* have approximately the same size and structure, and probably belong to one species of algae.

In the digestive organs (the mesenterial filaments) of freshly collected polyps of the four species mentioned large numbers of zooxanthellae are always found. They are in different stages of disintegration owing to their being in process of digestion by the polyps.

When the polyps of Isophyllia and Siderastraea are abundantly fed with mussel-meat, this is ingested by the mesenterial filaments. Under this condition the mesenterial filaments no longer ingest zooxanthellae. As a result the algae gradually disappear from the mesenterial filaments after the polyps are fed with meat. This suggests that the symbiotic algae are ingested by the polyps only when other food is scarce. In other Bermudian Anthozoa which live in symbiosis with zooxanthellae these algae also form a large part of the food supply of the polyps.

Consequently we may safely conclude that in general the symbiosis of Anthozoa with the algae known as "yellow cells" consists, in large part at least, of a parasitism of the polyps on the zooxanthellae.

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[1] Also Hickson (1906) states that reef-corals perhaps derive in some cases a considerable part of their food from their symbiotic algae. In my former paper (Boschma, 1924) I have wrongly interpreted Hickson's views, inasmuch as I stated that this author denies the possibility that reef-corals feed exclusively on zooxanthellae. Hickson has only stated that he thinks it *improbable* that polyps with a complete set of organs for catching, swallowing, and digesting animal food never use them.

[2] Throughout this paper I have used the generic name Isophyllia for this coral, since it is by this name that it is generally known. Now, however, this genus is incorporated in the larger genus Mussa (cf. Verrill, 1901b). The specimens used in my experiments all belonged to the species *I. dipsacea*, which is to be distinguished from *I. fragilis* by its thicker septa and shallower calicles (Verrill, 1901a).

[3] The occurrence of a holophytic organism in certain species of Isophyllia is also recorded by Fulton (1921). According to this author the holophyte is probably allied to zooxanthella. A comparison of the symbiotic algae of Isophyllia with those of Zoanthus and Condylactis showed, however, that in all three genera these organisms are fairly alike in form, size, and structure.

[4] The form, color, and structure of my specimens agree completely with the description of *Zoanthus sociatus* by McMurrich (1889a). This species, however, is not mentioned by the same author (McMurrich, 1889b) in his report on the Bermudian actinians. Verrill (1900) records *Zoanthus sociatus* from the Bermudas. The number of tentacles in my specimens was 55-60, which is one of the specific features of *Z. sociatus* (cf. Verrill, 1900).

[5] Already in 1889 McMurrich (1889b) ascribed to this part of the mesenterial filaments a digestive function. The green corpuscles which he found in the mesenterial filaments were in all probability partly digested zooxanthellae. His studies were made on material preserved in alcohol. The yellow color of the zooxanthellae had been readily extracted by the alcohol, but the green color, which is less easily soluble therein, had remained.

[6] The zooxanthellae found in the species dealt with in this paper have the same size and color as those of the Madreporarian polyps of the East Indian region (cf. Boschma, 1924). The latter, however, generally contain one corpuscle of amyloid substance, and only very rarely two. In the Bermudian zooxanthellae often two or more of these corpuscles are present.

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Vol. 60. No. 10.—October, 1925.

COMPUTATION OF BEHAVIOR OF ELECTRIC FILTERS UNDER LOAD.

By A. E. Kennelly and Arthur Slepian.

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(Continued from page 3 of cover.)

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By A. E. Kennelly and Arthur Sleplan.

Presented January 14, 1925. Received January 15, 1925.

Object of the Paper.

In the quantitative discussion of the behavior of an alternating-current electric filter under variable impressed frequency, it has commonly been assumed[1] that the load at the receiving end has an impedance equal to the characteristic or surge impedance of the filter; in other words that it behaves as though the load at the receiving end was an indefinite prolongation of the same filter. It is proposed to consider a case of a simple uniform, low-pass, five-section filter, with a constant load at the receiving end, and to indicate that in the case of such a relatively high degree of complexity, the behavior of the filter can be readily computed on the principle of simple alternating-current artificial lines.

The results have been obtained by calculation only, as the assumed filter was not actually tested or constructed. A sufficient number of observations have, however, been collected upon actual filters of the same type, to justify reporting the computations. As is customary in such calculations, the filter has been assumed as operating without internal losses or dissipation of power. If, however, definite small uniform resistance and dielectric losses had been assumed, corresponding to those which occur in filters actually employed, the computations would not thereby have been made much more difficult. While they would have taken a little more time to work out, the difference in such a case as this would not be great. The method here indicated is recommended as of general and convenient application.

Particulars Concerning the Filter Selected for Computation.

Fig. 1 represents the T section selected. It has an inductance of $\mathcal{L} = 0.040$ henry in the series element AOB, and a capacitance of $C = 10^{-6}$ farad, or 1 microfarad, in shunt. The resistance of the inductor is taken as negligible, and likewise the leakance of the condenser.

Fig. 2 represents the series connection of 5 such identical T filter sections. The filter is thus assumed to contain a total series inductance of 0.2 henry, and a total shunt capacitance of 5 microfarads. The load σ between the receiving-end terminal B and the ground is a constant resistance of 183.3 ohms, either alone, or in series with a constant inductance of 0.0265 henry, according to the position of the switch S.

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[1] One case has however already been briefly reported by F. S. Dellenbaugh, using the method described in this paper. See Bibliography 10, p. 21, Fig. 11.

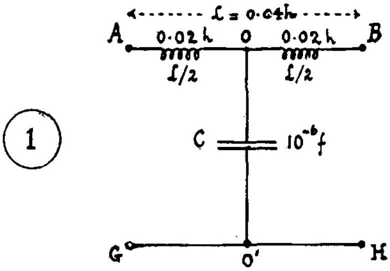
$\mathcal{L}$ = 0.04h
A
0.02 h
0
0.02 h
B
$\mathcal{L}/2$
$\mathcal{L}/2$
1
C
10^{-6}f
G
0'
H

FIG. 1. Section of the Low-Pass Filter Selected.

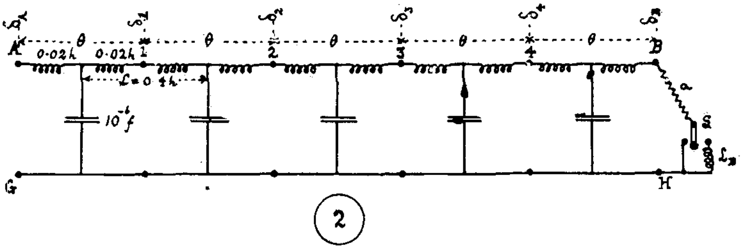
σ_A
σ_1
σ_2
σ_3
σ_4
σ_B
A
θ
1
2
3
4
B
0.02h
0.02h
$\mathcal{L}$ = 0.04h
10^{-6}f
a
S
$\mathcal{L}_s$
G
H
2

FIG. 2. Five-Section Low-Pass Filter corresponding to the Constants of Fig. 1 with a Load σ at the Receiving end B.

It is well known that, at a given impressed frequency, the behavior of any symmetrical filter depends on two and only two quantities connected therewith; namely, (1) the hyperbolic angle θ of the filter and (2) its surge impedance z_0 ohms $\angle$. Both θ and z_0 vary with the frequency. Moreover, in the case of a filter like that of Fig. 2, containing n identical symmetrical sections, each section will have the same angle θ, and surge impedance z_0 so that

$$\Theta = n\theta \qquad \begin{matrix}\text{hyperbolic} \\ \text{radians or hyps } \angle\end{matrix} \qquad (1)$$

According to the rule[2] for determining the angular velocity of cut off in a filter like that of Figs. 1 and 2, if $\mathcal{L}$ is the series inductance in henrys per section, and C is the shunt capacitance in farads per section,

---

[2] Bibliography, 2, 6.

$$\omega_0 = \frac{2}{\sqrt{\mathcal{L}C}} = 2\pi f_0 \qquad \frac{\text{radians}}{\text{sec.}} \qquad (2)$$

f_0 being the cut-off frequency.

If the angular velocity impressed on the filter is ω, we may call the ratio of this to the cut-off angular velocity, the frequency ratio u, or

$$u = \frac{\omega}{\omega_0} = \frac{\omega\sqrt{\mathcal{L}C}}{2} = \pi f\sqrt{\mathcal{L}C} = \frac{f}{f_0} \qquad \text{numeric} \qquad (3)$$

This type of filter allows alternating currents to pass for values of the frequency ratio is less than unity, and arrests them more or less completely when this ratio exceeds unity.

At any impressed frequency ratio ω, the section angle of this type of filter is defined by the relation

$$\sinh\frac{\theta}{2} = ju \qquad \text{numeric } \angle \qquad (4)$$

or

$$\theta = 2\sinh^{-1}(ju) \qquad \text{hyps } \angle \qquad (5)$$

Fig. 3 shows a portion of a chart[3] for obtaining antihyperbolic sines, or $\sinh^{-1}x$. At A is the origin of coördinates. Entering the chart on the rectangular coördinate network, we start from A and move up along the j line ABC, until we reach the value chosen for u. Holding this point with a blunt needle, we read off its corresponding value on the curvilinear coördinates, which are of the type $x + jq$. The real component x is in hyperbolic radians. The j component q is in circular quadrants of arc, each quadrant being 90°.

On page 465 is a brief Table of $\sinh^{-1}(ju)$, as obtainable from the chart Fig. 3, but to a lower degree of precision.

It will be seen that as u increases from 0 to 1.0, θ increases from $0 + j0$ to $0 + j2$ in quadrant measure, or to $0 + j\pi$ in circular measure. This means that θ is entirely imaginary, or has no real component, as far as $u = 1.0$. Immediately on passing $u = 1$, however, the value of θ undergoes no further change in the imaginary part $j\theta_2$; but rapidly develops a real part θ_1. An artificial line section of angle θ, forming part of an indefinitely long line, subjects any voltage or current which traverses it to an attenuation $\varepsilon^{-\theta} = \varepsilon^{-(\theta_1 + j\theta_2)} = \varepsilon^{-\theta_1} \diagdown \theta_2$, where $\varepsilon = 2.718\ldots$ the Napierian base.

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[3] Bibliography 4.

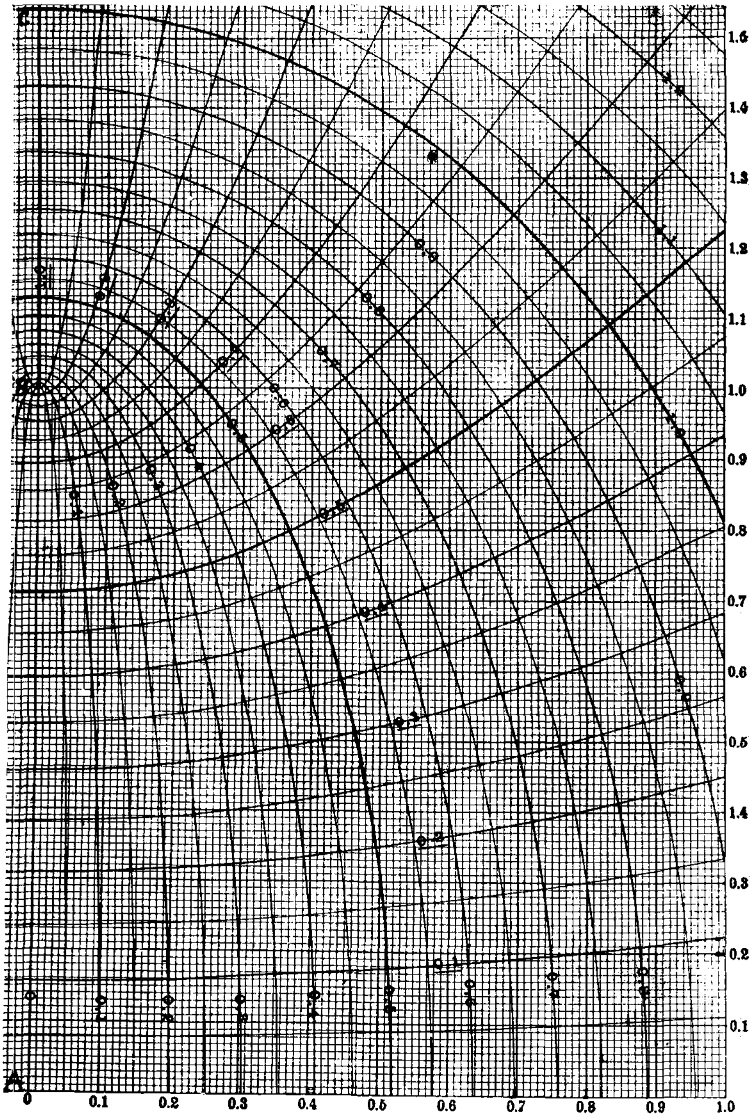

Fig. 3. Portion of Chart for evaluating $\sinh^{-1} jy = \frac{\theta_1 + j\theta_2}{2}$.

TABLE I

Section Angle θ for a T-Filter of the Type shown in Fig. 1 as a Function of the Frequency Ratio u.

ω	$ju = \frac{j\omega}{\omega_0}$	$\frac{\theta}{2} = \sinh^{-1}(ju)$		$\theta = \theta_1 + j\theta_2$	
		quadrants	cir. radians	quadrants	cir. radians
		$\frac{\theta_1}{2} + j\frac{q}{2}$	$\frac{\theta_1}{2} + j\frac{\theta_2}{2}$	$\theta_1 + jq$	$\theta_1 + j\theta_2$
0	0	$0 + j\,0$	$0 + j\,0$	$0 + j\,0$	$0 + j\,0$
1000	$j\,0.1$	$0 + j\,\underline{0.0638}$	$0 + j\,0.1002$	$0 + j\,\underline{0.1276}$	$0 + j\,0.2004$
2000	$j\,0.2$	$0 + j\,\underline{0.1283}$	$0 + j\,0.2014$	$0 + j\,\underline{0.2566}$	$0 + j\,0.4028$
3000	$j\,0.3$	$0 + j\,\underline{0.194}$	$0 + j\,0.3047$	$0 + j\,\underline{0.388}$	$0 + j\,0.6094$
4000	$j\,0.4$	$0 + j\,\underline{0.262}$	$0 + j\,0.4115$	$0 + j\,\underline{0.524}$	$0 + j\,0.8230$
5000	$j\,0.5$	$0 + j\,\underline{0.333}$	$0 + j\,0.5236$	$0 + j\,\underline{0.666}$	$0 + j\,1.0472$
6000	$j\,0.6$	$0 + j\,\underline{0.4095}$	$0 + j\,0.6435$	$0 + j\,\underline{0.819}$	$0 + j\,1.2870$
7000	$j\,0.7$	$0 + j\,\underline{0.493}$	$0 + j\,0.7753$	$0 + j\,\underline{0.986}$	$0 + j\,1.5506$
8000	$j\,0.8$	$0 + j\,\underline{0.590}$	$0 + j\,0.9273$	$0 + j\,\underline{1.180}$	$0 + j\,1.8546$
8100	$j\,0.81$	$0 + j\,\underline{0.601}$	$0 + j\,0.9442$	$0 + j\,\underline{1.202}$	$0 + j\,1.8884$
9000	$j\,0.9$	$0 + j\,\underline{0.712}$	$0 + j\,1.1197$	$0 + j\,\underline{1.424}$	$0 + j\,2.2394$
10000	$j\,1.0$	$0 + j\,\underline{1.0}$	$0 + j\,1.5708$	$0 + j\,\underline{2.0}$	$0 + j\,3.1416$
11000	$j\,1.1$	$0.4436 + j\,\underline{1.0}$	$0.4436 + j\,1.5708$	$0.8872 + j\,\underline{2.0}$	$0.8872 + j\,3.1416$
12000	$j\,1.2$	$0.6223 + j\,\underline{1.0}$	$0.6223 + j\,1.5708$	$1.2446 + j\,\underline{2.0}$	$1.2446 + j\,3.1416$
13000	$j\,1.3$	$0.7565 + j\,\underline{1.0}$	$0.7565 + j\,1.5708$	$1.513 + j\,\underline{2.0}$	$1.513 + j\,3.1416$
14000	$j\,1.4$	$0.8670 + j\,\underline{1.0}$	$0.8670 + j\,1.5708$	$1.734 + j\,\underline{2.0}$	$1.734 + j\,3.1416$
15000	$j\,1.5$	$0.9624 + j\,\underline{1.0}$	$0.9624 + j\,1.5708$	$1.9248 + j\,\underline{2.0}$	$1.9248 + j\,3.1416$

Consequently if θ_1 vanishes, $\varepsilon^{-\theta} = \varepsilon^{-j\theta_2} = 1 \diagdown \theta_2$ a mere versor, or phase-shifting operator, causing the voltage or current at its receiving-end terminals to lag in phase θ_2 circular radians with respect to the corresponding quantity at sending-end terminals; without altering the magnitude or size of the traversing quantity. When, however, there is a real component θ_1, not only is there slope attenuation (in phase) but there is also size attenuation (in magnitude). At $u = 1.5$, for example, $\theta = 1.9248 + j\underline{2}$ and $\varepsilon^{-\theta} = \varepsilon^{-1.9248-j\pi} = 0.146 \diagdown 180°$. The voltage or the current will fall in this case to 14.6 per cent in size, and by 180° in slope, when passing through the filter section, provided that there are a large number of similar sections beyond.

In the case of the particular values of $\mathcal{L}$ and C shown in the T section of Fig. 1, the cut-off frequency $\omega_0 = 10{,}000$ radians per second, ($f_0 = 1591$ ∾) by (2). The values of impressed angular velocity are then given in the last column of Table I.

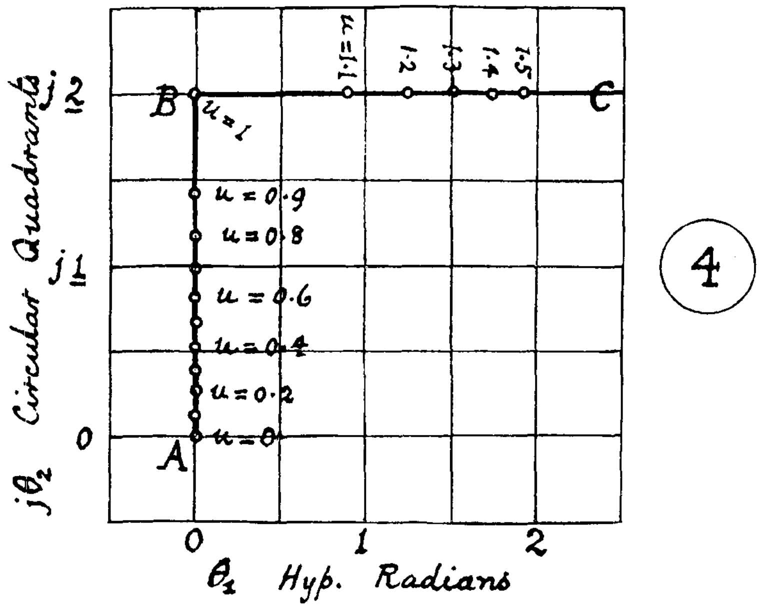

FIG. 4. Vector Locus of Section Angle θ as the impressed frequency ratio u is varied from 0 to 1.5.

Fig. 4 is a vector diagram of the section angle θ for the filter Fig. 1, as a function of impressed frequency ratio u. The vector locus is a pair of perpendicular straight lines, AB and BC. The turning point at $u = 1$ corresponds to the cut off angular velocity ω_0. As u is increased from 1.5 to infinity, the horizontal locus BC advances continuously toward the right hand.

The rate of increase of section angle θ with respect to a change in u, is expressed by

$$d\theta = j\frac{2du}{\sqrt{1 - u^2}} = j\frac{2du}{\cosh\left(\frac{\theta}{2}\right)} = j\frac{2du}{\cos\left(\frac{\theta_2}{2}\right)} \qquad \text{hyps} \angle \qquad (6)$$

below the cut-off value $u = 1$ and

$$d\theta = \frac{2du}{\sqrt{u^2 - 1}} = \frac{2du}{\cosh\left(\frac{\theta}{2}\right)} = \frac{2du}{\sinh\left(\frac{\theta_1}{2}\right)} \qquad \text{hyps} \qquad (7)$$

beyond the cut-off. Thus, at $u = 0.8$, or $\omega = 8000$ in the case considered, $d\theta = j2\,(du/0.6) = j(du/0.3)$. That is for a small change in du amounting say to 0.01, or 100 in ω, $d\theta = j\,0.033$ hyps; i.e. 0.033 circular radians or 0.0212 quadrant. This is in substantial agreement with the entry in Table I for $u = 0.81$.

At $u_0 = 1$, or the cut-off frequency, $d\theta/du$ becomes infinite, which would mean that an indefinitely small change in u would produce an indefinitely large change in θ at this point; but this refers only to a perfect filter devoid of all losses. In the presence of any appreciable loss, the rate of change $d\theta/du$ becomes finite, but may be large. The larger it is, the better the cut-off behavior of the filter.

ANGLE OF MULTIPLE-SECTION FILTER.

Fig. 5 gives the vector locus diagram ABC of the angle Θ subtended by the entire filter of Fig. 2. By comparison with Fig. 4, it will be observed that the effect of the five sections is to make the rate of cut-off five times as great as with a single section. This is known to be a characteristic property of multiple-section filters. That is in changing u from 1.0 to 1.1, the increase in θ is 0.89 hyp. in Fig. 4, and 4.45 in Fig. 5.

SURGE IMPEDANCE OF ONE SECTION OR OF THE FIVE-SECTION FILTER.

By the regular rule for determining the surge impedance of a T section as in Fig. 1,

$$z_0 = \sqrt{\frac{\mathfrak{L}}{C}}\cosh\frac{\theta}{2} = z_{00}\cosh\frac{\theta}{2} \qquad \text{ohms} \angle \qquad (8)$$

or in the case considered, 200 cosh $\theta/2$, where θ is the section angle. The surge impedance thus varies with the impressed frequency.

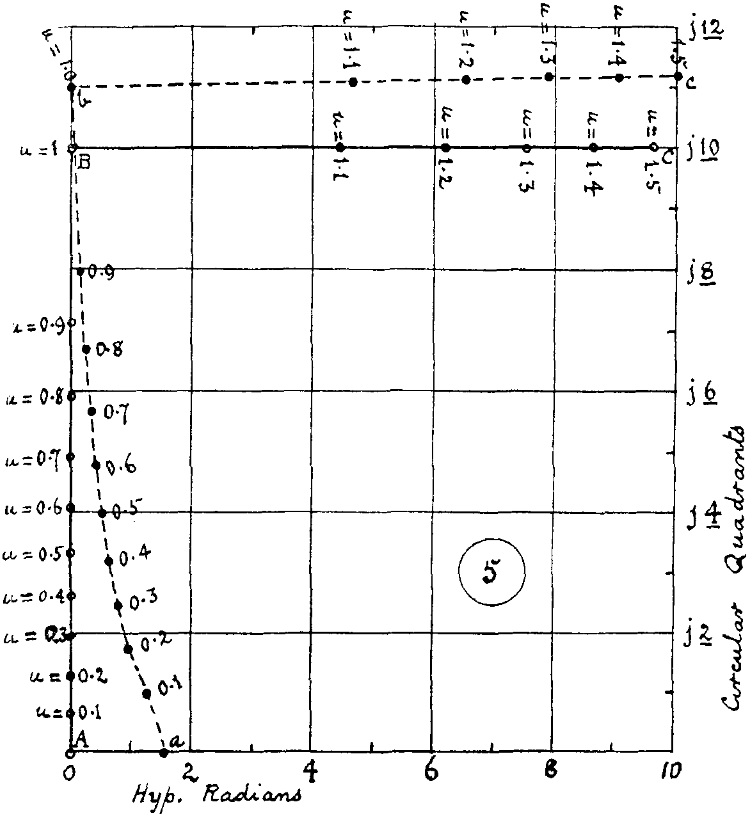

FIG. 5. Vector Locus *ABC* of the total filter angle Θ as u varies from 0 to 1.5; also the vector locus *abc* of the position angle δ_A when the load σ = 183.3 ohms + 0.0265 h.

Fig. 6 shows the vector locus. As ω increases from 0 to $\omega_0 = 10{,}000$, $(u = 1)$, z_0 moves along the graph *AB* following the resistance axis from 200 ohms to 0. At the cut-off point, z_0 vanishes, for the case assumed of a lossless filter. From and beyond ω_0, z_0 becomes a pure inductive reactance or $+j$ quantity, along the branch *BC*, increasing rapidly at first, and afterwards more slowly.

POSITION ANGLE δ_B AT THE RECEIVING END OF THE FILTER UNDER LOAD.

The position angle δ_B (Fig. 2) of the receiving end B of the filter line Fig. 2, under a given load σ ohms ∠ and under an impressed angular velocity ω, is equal to the angle θ' of that load, and is found from the well known relation

$$\delta_B = \theta' = \theta_1' + j\theta_2' = \tanh^{-1}\frac{\sigma}{z_0} \qquad \text{hyps} \angle \qquad (9)$$

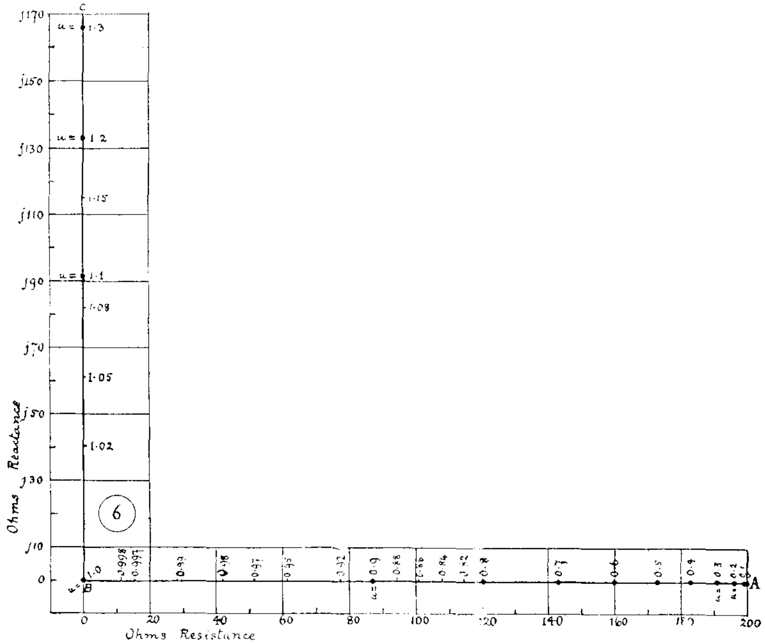

FIG. 6. Vector Locus of the surge impedance z_0 of the filter as the frequency ratio u is varied from 0 to 1.3 or ω from 0 to 13000.

But the impedances σ and z_0 will, in general, vary with the angular velocity ω; so that the angle θ' also varies with ω. The antitangent of the complex ratio σ/z_0 is found from a chart of tanh and $\tanh^{-1}$. Fig. 7 shows four vector loci of δ_B at varying angular velocities, for different loads σ connected to the receiving-end B of the filter in Fig. 2. The abscissas of Fig. 7 are in real hyperbolic radians θ_1'. The ordinates are in circular quadrants of arc $(j\theta_2')$. Thus $j\,\underline{1.0}$ represents one quadrant, ninety degrees, or $\pi/2$ radians. The four loads σ whose angles θ' are plotted in Fig. 7 as a function of u are:

(1) A pure resistance load $\sigma = 183$ ohms, graph $A\ b'\ d'\ e'\ D\ f$.
(2) A resistance of 183.3 ohms in series with a condenser of 2.65 microfarads, graph $D\ F\ G\ H\ D\ J\ K$.
(3) A resistance of 183.3 ohms in series with an inductance of 26.5 millihenries, graph $A\ B\ C\ D\ E$.
(4) A resistance of 156 ohms in series with an inductance of 22.5 millihenries, graph $a\ b\ c\ D\ e$.

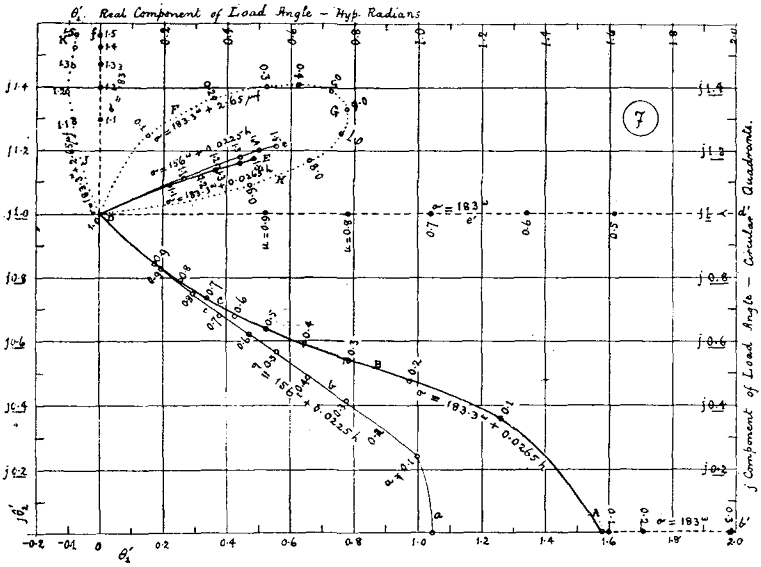

FIG. 7. Vector Loci of δ_B or of angles subtended by four different loads σ connected successively to the receiving end B of the filter shown in Fig. 2.

Different values of u are marked along each graph. Thus $u = 0.7$, corresponding to $\omega = 7000$ for the filter here discussed, is found near the point e' on graph 1, near G on graph 2, and near C on graphs 3 and 4. Graph 1 for the pure resistance goes off to infinity along the base line as ω approaches 4000, or as u approaches 0.4. It then returns along the parallel line $d'\ e'\ D$ as u approaches 1.0, then turns sharply through a positive quadrant and follows the straight line $D\ f$.

The load (2) containing the series condenser executes the closed oop $D\ F\ G\ H\ D$ and then pursues the curve $D\ J\ K$.

Loads (3) and (4) pursue somewhat similar curves which change suddenly at D. This point D at $0 + j\,\underline{1}$ is common to all four graphs at $u = 1$, or $\omega = \omega_0 = 10{,}000$. At the cut-off frequency, therefore, all four loads subtend one circular quadrant when connected to this particular filter in one or more T sections.

POSITION ANGLE δ_A AT THE SENDING END OF THE FILTER LINE.

In conformity with the regular rule affecting uniform artificial or real lines, operating at a single impressed frequency in the steady state, the position angle at the sending end A, as shown in Fig. 2, is

$$\delta_A = \Theta + \delta_B = n\theta + \theta' \qquad \text{hyps } \angle \qquad (10)$$

δ_A clearly varies with the frequency ω, as do both the section angle θ and the load angle θ', according to (5) and (9). In Fig. 5, the broken line graph $a\ b\ c$ traces the vector position angle at A as u increases from 0 to 1.5.

INPUT AND OUTPUT CURRENTS.

If I_A is the input current at A (Fig. 2) in rms. amperes, and I_B the output current given to the load σ at B, in rms. amperes, we have[4] the well known relation

$$\frac{I_B}{I_A} = \frac{\cosh \delta_B}{\cosh \delta_A} \qquad \text{numeric } \angle \qquad (11)$$

So that if the input current I_A is given, the output current I_B is obtained directly therefrom, with the cosine ratio of the two position angles δ_B and δ_A. The current strength at any section junction is likewise obtainable from the same rule.

INPUT AND OUTPUT IMPEDANCES.

The impedance at and beyond the receiving end B in Fig. 2, or the output impedance is σ ohms $\angle$, a function of the frequency, unless the load σ is a pure resistance. The impedance at the input terminal A, or the input impedance is well known to be

$$Z_A = z_0 \tanh \delta_A \qquad \text{ohms } \angle \qquad (12)$$

Both z_0 and δ_A vary with the impressed frequency f. If either the current I_A, or the emf. E_A is known at A, the other can be immediately found from Z_A.

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INPUT AND OUTPUT VOLTAGES.

The voltages at A and B being defined as E_A and E_B rms. volts $\angle$, the input and output voltages respectively, we have the well known relation

$$\frac{E_B}{E_A} = \frac{\sinh \delta_B}{\sinh \delta_A} \qquad \text{numeric } \angle \qquad (13)$$

or the output voltage at B is the input voltage E_A times the sine ratio of corresponding position angles. The voltage at any intermediate junction can likewise be derived, for any given frequency, by the same rule.

INPUT AND OUTPUT POWER.

The vector output power P_B watts $\angle$ is obtainable directly from the vector output voltage E_B volts $\angle$, and the vector output current I_B amperes $\angle$. This vector power will be expressed either to voltage standard phase, or to current standard phase. In the former case, the voltage E_B is taken as of zero slope, and the current I_B is then taken with its slope in local reference thereto. Thus, taking the impressed input voltage E_A as of initial standard phase say $10 \angle 0°$ volts, the output voltage E_B might be say $8 \overline{\diagdown} 600°$ volts, or 8 volts 600° in phase behind E_A; while the output current I_B might be say $0.1 \overline{\diagdown} 540°$ amperes, or one-tenth ampere lagging 540° behind E_A. To find the output power at B, taking E_B as of local standard phase, we write

$$E_B = 8 \angle 0° \qquad \text{volts } \angle$$

and

$$I_B = 0.1 \angle 60° \qquad \text{amperes } \angle$$

because I_B leads E_B in phase by $600 - 540 = 60$ degrees. We then have

$$P_B = E_B \times I_B \qquad \text{watts } \angle \qquad (14)$$

or in this case $0.8 \angle 60$ watts $= 0.4 + j\,0.693$ watts; i.e. 0.4 watt active, or delivered power, and 0.693 watt reactive, or circulating power, of the $+j$ type, which will represent condensive activity. If, on the other hand, we take the local current at B as the standard of phase, $E_B = 8 \overline{\diagdown} 60°$ volts, and $I_B = 0.1 \angle 0°$ ampere, whence by (14), $P_B = 0.8 \overline{\diagdown} 60°$ watt $= 0.4 - j\,0.693$. The active power is the same as before, but the reactive power is $-j$, or condensive.

At the sending-end, with E_A as phase standard,

$$P_A = E_A \times I_A \qquad \text{watts } \angle \qquad (15)$$

BEHAVIOR OF FILTER UNDER SURGE-IMPEDANCE LOAD.

If in Fig. 2, the load σ be kept equal to z_0 as the frequency is changed, the load will behave like an indefinite prolongation of the same filter, or the line becomes infinitely long. The entering current at A, is then

$$I_A = \frac{E_A}{z_0} = E_A\, y_0 \qquad \text{amperes} \angle \qquad (16)$$

or if the impressed voltage is kept at 1 volt, at standard phase, over the entire range of frequency, the entering current will be vectorially equal to the surge admittance y_0. The voltage and current at B will then be

$$E_B = E_A \varepsilon^{-\Theta} \qquad \text{volts} \angle \qquad (17)$$

and

$$I_B = I_A \varepsilon^{-\Theta} \qquad \text{amperes} \angle \qquad (18)$$

From an inspection of Figs. (5) and (6), it will be seen that the entering current I_A will continuously increase as ω is varied from 0 to ω_0. At the critical frequency $f_0 = \omega_0/2\pi$, the entering current will be indefinitely great, or the filter acts like a short circuit, (assuming no internal losses in the filter); while above f_0, the current falls off rapidly. This is shown in Fig. 8 by the curve I_A, commencing at 0.005 ampere with $\omega = 0$, and running off the sheet near $\omega = 9900$. At $\omega = 12000$, I_A has returned to 0.075 and is falling. The received current I_B is numerically equal to I_A up to the critical frequency. It then falls much faster than I_A and is very nearly 0 at $\omega = 11000$

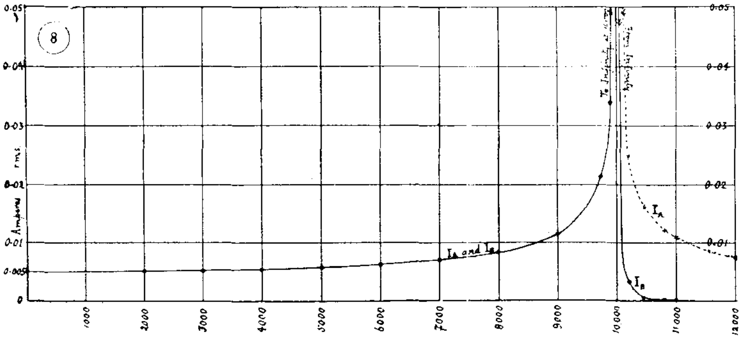

FIG. 8. Current strengths at terminals A and B of 5-Section filter Fig. 2 under surge-impedance load $\sigma = z_0$, as impressed frequency is varied from $\omega = 0$ to $\omega = 12000$.

Fig. 9 shows the corresponding vector diagram of I_B under varying impressed frequency ratio. At zero frequency, $I_B = PA = 5 \angle 0°$

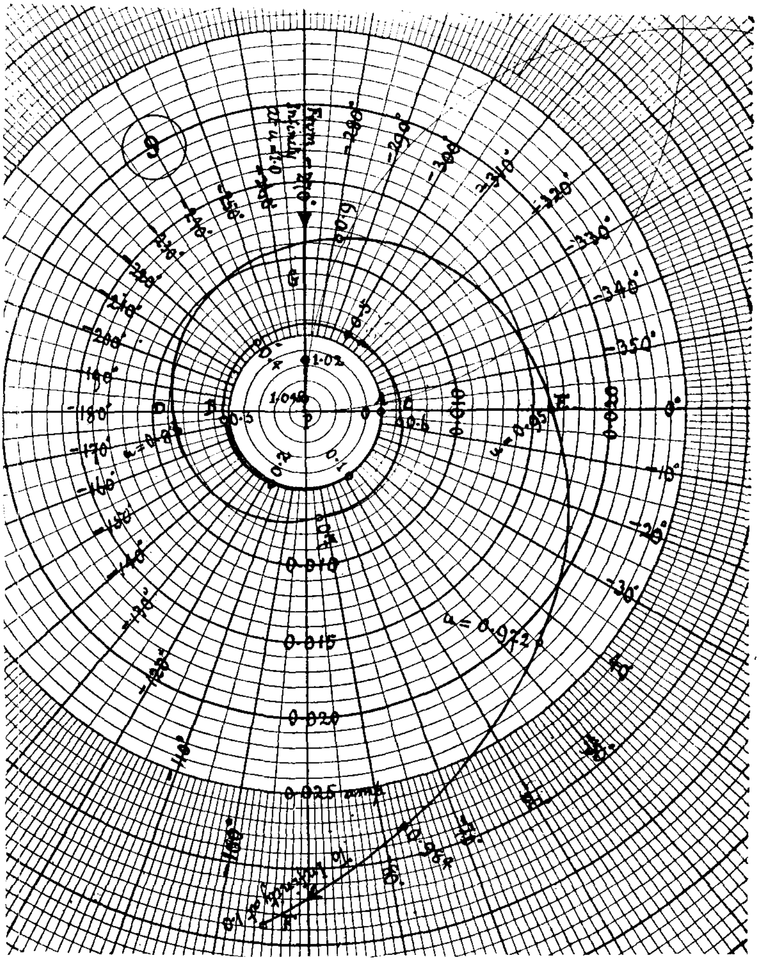

FIG. 9. Vector graph of current I_B under condition of Surge-Impedance Load.

milliamperes. At $u = 0.6$ or $\omega = 6000$, $I_B = OC$ or 6.25 ⟍ 368° milliamperes. The received current I_B is then more than one complete cycle behind the impressed emf. E_A. At $u = 0.9511$, or $\omega = 9511$, $I_B = OE = 16 \diagdown 720°$ milliamperes. At OF, near $u = 0.984$, the received current is $32 \diagdown 810°$, and rapidly increasing. At $u = 1.0$ it becomes infinite, and above this critical value it returns rapidly along the line $G\ P$. At $u = 1.02$, it is $3.375 \diagdown 990°$ or 11 quadrants behind E_A in phase. The entering current I_A does not pursue this spiral path. As far as $u = 1$, it is numerically equal to I_B, but is in phase with E_A, or extends along the initial line $P\ A\ E$.

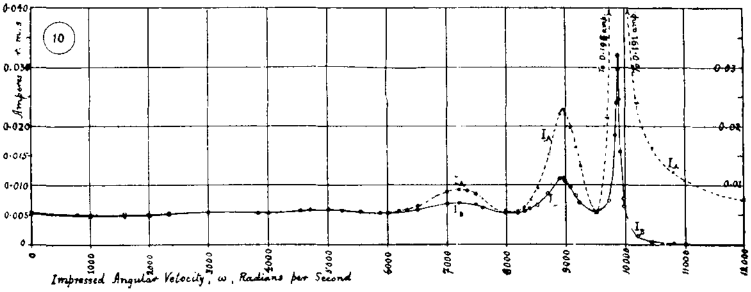

FIG. 10. Current strengths at terminals A and B of Five-Section Filter Fig. 2, under Constant-Resistance Load $\sigma = 183.3$ ohms, as the impressed angular velocity is varied from $\omega = 0$ to $\omega = 12000$.

Above $u = 1$, I_A returns from infinity along the line $F\ P$, opposite to $G\ P$. Before reaching the critical frequency, therefore, the phase difference between I_A and I_B increases from 0 to 10 quadrants or 900°. Beyond $u = 1$, it remains constant at 10 quadrants. I_B falls much more rapidly.

The expanding spiral executed by I_B in Fig. 9 is, up to $u = 1$,

$$I_B = \frac{y_{00}}{\sqrt{1 - u^2}} \diagdown (2n \sin^{-1} u) = y_{00} \sec \left(\frac{\beta}{2n} \right) \diagdown \beta$$

$$= y_{00} \sec \beta' \diagdown 2n\beta' \qquad \text{amperes } \angle \qquad (19)$$

where $y_{00} = \sqrt{C/\pounds}$, the value of the surge admittance at zero frequency, in this case 0.005, n is the number of sections in the filter between A and B, in this case 5, and $u = \sin \beta$ and $\beta' = n\beta$. This type of expanding spiral may be called the *secant of the 2nth arc* spiral. If we consider a filter made up of a very large number of sections like Fig. 1, the vector current at the junctions 1, 2, 3, . . . in Fig. 2, will be $2n$th secant spirals, where $n = 1, 2, 3$ . . . successively. Such secant spirals characterize the vector currents at successive junctions along an infinitely long low-pass filter, of the types shown in Figs. 1 and 2.

BEHAVIOR OF THE FILTER UNDER THE CONSTANT PURE RESISTANCE LOAD σ = 183.3 OHMS.

Fig. 10 shows the corresponding current strengths I_A and I_B when the load σ at the receiving end is a constant resistance of 183.3 ohms, as ω is increased up to 12000. Instead of having one and only one resonant frequency as in Fig. 8, there are now five resonant frequencies; namely one at ω_0, no longer a short-circuit resonance, and lesser resonances at or near $\omega = 9000$, 7000, 5000 and 3000. There is very little difference between the magnitudes of I_A and I_B until the resonance near $\omega = 7000$ is reached. At this and subsequent resonances, I_A exceeds I_B. After reaching the frequency ratio $u = 0.9877$, when I_A goes to 0.191 ampere, and I_B to 0.032 ampere, both the currents diminish; but I_B falls much more rapidly. At the cut-off ratio $u = 1$, I_B has fallen to about 0.006 ampere.

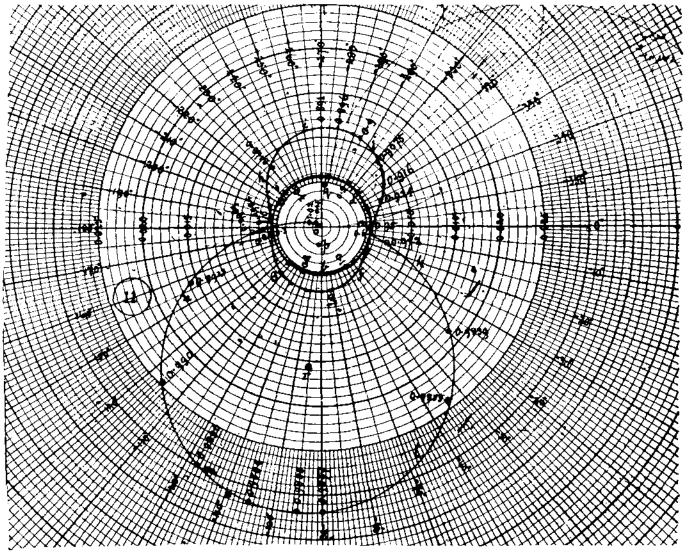

FIG. 11. Vector Graph of Current I_B under condition of Constant Resistance Load σ = 183.3 ohms, as the frequency ratio is increased from $u = 0$ to $u = 1.2$.

The corresponding vector graph of the current I_B is given in Fig. 11. At $u = 0$, this current is 5.4 milliamperes at standard phase, or along the initial line *oa*. At $u = 0.6$, the current has completed a nearly circular loop *a b c d*, and has reached the value 5.47 ⦪ 366°.9, or more than 4 quadrants behind the phase of E_A. It now executes the loop $f\,g\,h\,j$, and at $u = 0.95$, it has become $5.46 \diagdown 720°$ or 8 quadrants in lag. It now executes the nearly circular loop $k\,l\,m\,n$, with its diametral maximum near $u = 0.988$. At $u = 1.0$, it is about 6 milliamperes, 10 quadrants behind E_A (2 quadrants per section of filter).

The effect of substituting this constant resistance load for the surge-impedance load has been to change the vector graph of I_B from a secant $(\beta'/2n)$ spiral, such as that of Fig. 9, to a graph of a succession of approximately circular arcs, as in Fig. 11. The center of the arc $k\,l\,m\,n$ is at r, that of $h\,i\,j$ at q, and that of $f\,g$ at p. These centers lie nearly on one straight line, and fall successively on opposite sides of the origin O.

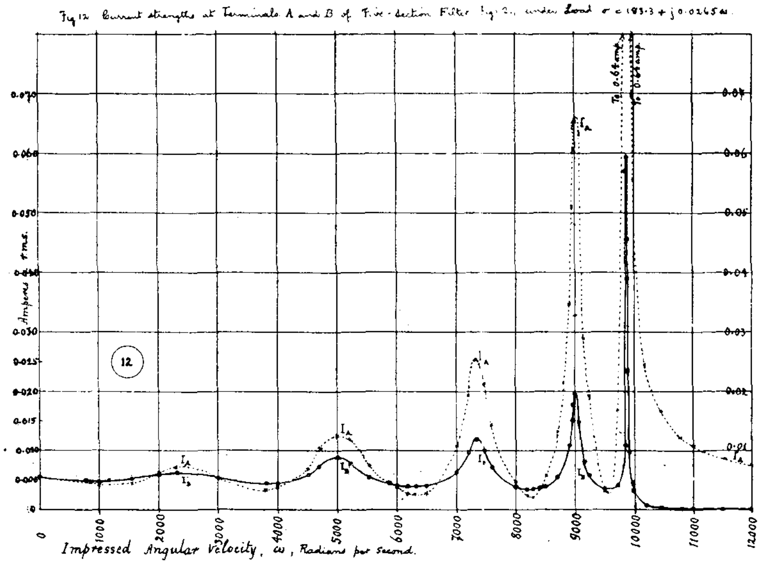

FIG. 12. Current strengths at Terminals A and B of Five-Section Filter Fig. 2, under Load $\sigma = 183.3 + j\,0.0265\,\omega$.

BEHAVIOR OF THE FILTER UNDER A CONSTANT REACTIVE LOAD $= 183.3 + j\,0.0265$ ohms $\angle$.

The effect of inserting a pure inductance of 26.5 millihenrys in circuit with 183.3 ohms resistance load, is shown in Fig. 12. The resonances produce in both I_A and I_B are now much more marked, owing to the influence of the inductance in the load. There are five such resonances, one for each section of the filter. The principal resonance is at $\omega = 9886$, when I_A reaches 0.64 ampere, and I_B 0.059 ampere. The other resonances are near $\omega = 9000$, 7350, 5000 and 2400. At each resonance I_A exceeds I_B; but at intermediate frequencies I_A falls below I_B.

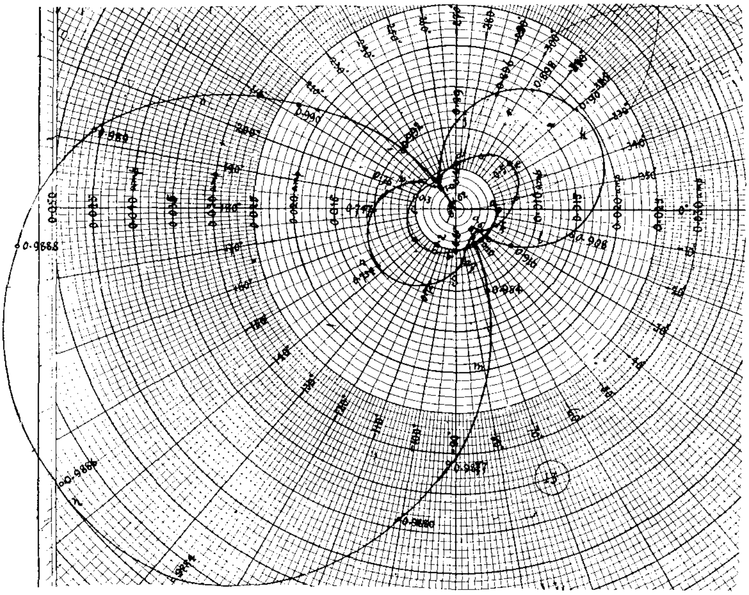

FIG. 13. Vector Graph of Current I_B under condition of a Reactive Load $\sigma = 183.3 + j\,0.0265\,\omega$, as the frequency ratio is increased from $u = 0$ to $u = 1.2$.

The corresponding vector graph of I_B appears in Fig. 13. *ba* is the initial current at $u = 0$. The first half loop *a b c*, goes as far as $u = 0.4$. The next *d e f* reaches $u = 0.6$. The third *g h i* goes to $u = 0.8$. The fourth *j k l* reaches $u = 0.95$, and the last *m n o*, to $u = 1.02$ and beyond. These loops are only roughly circular, and are not nearly such good approximations to circles as those of Fig. 11. The centers of the loops in Fig. 11 lie nearly on a straight line inclined 85° with the reference axis; but in Fig. 13, the approximate centers lie more nearly on a line *n p k*, making an angle of 30° with the reference axis *p a*.

It may be noticed that between $u = 0.984$ and $u = 0.992$, the received current is very sensitive to change in impressed frequency, both in regard to magnitude and to phase, or the filter is a sensitive frequency indicator.

Table II indicates, for a few entries only, the method of computation employed, for the case of $= 183.3 + j\,0$ ohms, and Table III a few similar examples for the case of $= 183.3 + j\,0.0265\omega$.

SUMMARY

(1) Any alternating-current filter, consisting of sections in series having the same surge impedance, forms an artificial electric line, and may advantageously be dealt with, in computation, by means of hyperbolic functions.

(2) The behavior of any such filter under surge-impedance load (virtually infinite line), is apt to be very different from its behavior under a constant load at the receiving end.

(3) In the case of a low-pass inductance-capacitance T-section filter with negligible losses, having a large number of sections, the vector current graphs at n successive junction points, as the impressed frequency is increased up to cut-off, are expanding secant spirals of the type $\rho = \rho_0 \sec (\beta/2n)$.

(4) Under the same conditions last named, but with a constant pure resistance load, the vector received current graph is a close approximation to an aggregation of resonant circles, one for each filter section.

(5) When the load in the last named case includes a constant inductance, the received current graph is a succession of resonant roughly circular loops, one for each filter section.

List of Symbols Employed.

β, β'	Circular angles of radius vector in a spiral (radians, quadrants or degrees).
C	Capacitance of a condenser in a filter (farads).
δ_A, δ_B, δ_P	Position angles of terminals A and B, or of a junction point P (hyps $\angle$).
E_A, E_B, E_P	Voltages at terminals A and B, or of a junction P (volts $\angle$).
f	Impressed frequency at the sending end (cyps).
f_0	Cut-off frequency (cyps).
$\theta = \theta_1 + j\theta_2$	Angle subtended by a filter section (hyps $\angle$).

TABLE II

EXAMPLES OF TABULAR COMPUTATION OF 5-SECTION FILTER IN FIG. 2, UNDER DIFFERENT SUCCESSIVE FREQUENCIES, WITH 1.0∠0° VOLT IMPRESSED AT A, AND A LOAD OF 183.3 OHMS AT B.

u	$\frac{\theta}{2}$ hyps	θ hyps	Θ_q hyps	$\cos\frac{\theta}{2}$	z_0 ohms	$\frac{\sigma}{z_0}$	δ_B hyps	δ_A hyps	tanh δ_A	$Z_A = z_0 \tanh \delta_A$ ohms
0.2	$j\,0.2014$	$j\,0.4028$	$j\,\underline{1.282}$	0.9797	195.9	0.935	1.70	$1.700 + j\,\underline{1.282}$	1.043 ⦣ 3°.	204.4 ⦣ 3°.0
0.4	$j\,0.4115$	$j\,0.8230$	$j\,\underline{2.620}$	0.9165	183.3	1	∞	∞	1	183.3 ∠ 0°
0.6	$j\,0.6435$	$j\,1.287$	$j\,\underline{4.093}$	0.800	160	1.145	$1.347 + j\,\underline{1}$	$1.347 + j\,\underline{5.093}$	1.138 ⦣ 2°.35	182.2 ⦣ 2°.35
0.8	$j\,0.9273$	$j\,1.8546$	$j\,\underline{5.905}$	0.600	120	1.528	$0.783 + j\,\underline{1}$	$0.783 + j\,\underline{6.905}$	1.490 ∠ 7°.5	178.8 ∠ 7°.5
0.99	$j\,1.429$	$j\,2.858$	$j\,\underline{9.10}$	0.141	28.2	6.50	$0.155 + j\,\underline{1}$	$0.155 + j\,\underline{10.1}$	0.220 ∠ 44°	6.20 ∠ 44°

$I_A = Y_A = 1/Z_A$ amp $\times 10^{-3}$	cosh δ_A	cosh δ_B	I_B amp $\times 10^{-3}$	sinh δ_A	sinh δ_B	E_B volts	$\sigma = E_B/I_B$ ohms.
4.892 ∠ 3°.0	2.675 ∠ 117°	2.828 ∠ 0°	5.172 ⦣ 114°	2.800 ∠ 113°.8	2.646 ∠ 0°	0.945 ⦣ 113°.8	182.7 ∠ 0°.2
5.455 ∠ 0°	∞	∞	5.455 ⦣ 235°.8	∞	∞	1 ⦣ 235°.8	183.3 ∠ 0°
5.488 ∠ 2°.35	1.80 ∠ 99°.3	1.793 ∠ 90°	5.467 ⦣ 6°.9	2.055 ∠ 97°	2.053 ∠ 90°	0.999 ⦣ 7°	182.7 ⦣ 0°.1
5.593 ⦣ 7°.5	0.88 ∠ 257°	0.8655 ∠ 90°	5.501 ⦣ 174°.5	1.315 ⦣ 264°.2	1.323 ∠ 90°	1.006 ⦣ 174°.2	182.8 ∠ 0°.3
161.2 ⦣ 44°	1.020 ∠ 182°	1.0120 ∠ 90°	24.6 ⦣ 136°	0.225 ∠ 226°	0.1556 ∠ 90°	4.498 ⦣ 136°	182.8 ∠ 0°

TABLE III.

EXAMPLES OF TABULAR VOLTAGES AND CURRENTS AT SUCCESSIVE TERMINALS OF FIVE-SECTION FILTER IN FIG. 2 WITH LOAD AT B OF $183.3 + j\,0.0265\,\omega$ OHMS, UNDER DIFFERENT IMPRESSED FREQUENCIES. $E_A = 1.0 \angle 0°$ VOLT.

u	ω	Z_A	I_A	E_1	I_1	E_2	I_2
		ohms	milliamp.	volts	milliamp.	volts	milliamp.
0.2	2000	160.0 ∠ 348°.2	6.25 $\overline{\diagdown}$ 348°.2	1.12 $\overline{\diagdown}$ 24°.9	5.69 $\overline{\diagdown}$ 8°.3	1.23 $\overline{\diagdown}$ 44°.9	5.10 $\overline{\diagdown}$ 34°
0.4	4000	281 ∠ 339°.2	3.56 $\overline{\diagdown}$ 339°.2	0.958 $\overline{\diagdown}$ 26°	3.90 $\overline{\diagdown}$ 54°.5	0.624 $\overline{\diagdown}$ 76°	5.53 $\overline{\diagdown}$ 90°.2
0.6	6000	278 ∠ 34°.3	3.6 $\overline{\diagdown}$ 34°.3	0.429 $\overline{\diagdown}$ 91°.3	6.78 $\overline{\diagdown}$ 83°.7	1.04 $\overline{\diagdown}$ 166°.8	3.04 $\overline{\diagdown}$ 143°.7
0.8	8000	214 ∠ 56°.4	4.66 $\overline{\diagdown}$ 56°.4	0.786 $\overline{\diagdown}$ 157°.7	6.97 $\overline{\diagdown}$ 96°.4	0.61 $\overline{\diagdown}$ 195°.9	8.19 $\overline{\diagdown}$ 254°.7
0.99	9900	3.71 ∠ 67°.8	268 $\overline{\diagdown}$ 67°.8	2.88 $\overline{\diagdown}$ 165°.3	250 $\overline{\diagdown}$ 247°.4	4.73 $\overline{\diagdown}$ 345°.2	210 $\overline{\diagdown}$ 424°.8

E_3	I_3	E_4	I_4	E_B	I_B	σ
volts	milliamp.	volts	milliamp.	volts	milliamp.	ohms
1.26 $\overline{\diagdown}$ 62°.4	4.85 $\overline{\diagdown}$ 63°.5	1.22 $\overline{\diagdown}$ 80°.4	5.46 $\overline{\diagdown}$ 92°.8	1.10 $\overline{\diagdown}$ 100°.9	5.85 $\overline{\diagdown}$ 117°	189.3 ∠ 16°.1
0.750 $\overline{\diagdown}$ 160°.2	5.05 $\overline{\diagdown}$ 120°.8	1.03 $\overline{\diagdown}$ 195°	3.30 $\overline{\diagdown}$ 173°.2	0.904 $\overline{\diagdown}$ 214°.2	4.28 $\overline{\diagdown}$ 244°.3	211.8 ∠ 30°.1
0.626 $\overline{\diagdown}$ 208°.3	6.25 $\overline{\diagdown}$ 249°.2	0.802 $\overline{\diagdown}$ 330°	5.17 $\overline{\diagdown}$ 283°.7	0.946 $\overline{\diagdown}$ 364°.3	3.89 $\overline{\diagdown}$ 405°.2	243 ∠ 40°.9
1.08 $\overline{\diagdown}$ 349°.1	3.20 $\overline{\diagdown}$ 317°.6	0.398 $\overline{\diagdown}$ 450°.9	9.32 $\overline{\diagdown}$ 443°.4	1.05 $\overline{\diagdown}$ 537°.9	3.77 $\overline{\diagdown}$ 587°	280 ∠ 49°.1
6 $\overline{\diagdown}$ 520°	157 $\overline{\diagdown}$ 603°.3	7 $\overline{\diagdown}$ 699°	93.2 $\overline{\diagdown}$ 783°.3	7 $\overline{\diagdown}$ 877°.8	23.1 $\overline{\diagdown}$ 932°.9	320.5 ∠ 55°.1

θ'	Angle subtended by a load σ at receiving end (hyps $\angle$).
Θ	Total angle subtended by a multiple-section filter (hyps $\angle$).
I_A, I_B, I_P	Current strengths at terminals, or at a junction (amperes $\angle$).
$j=\sqrt{-1}$.	
$\mathcal{L}$	Inductance in the series branches of a T-filter section (henrys).
n	Number of sections in a multiple-section filter.
$\pi=3.14159\ldots$	
P_A, P_B	Power at terminals (watts $\angle$).
q	Imaginary component of a complex angle expressed in circular quadrants.
ρ	Radius vector of an expanding spiral.
ρ_0	Initial radius vector of an expanding spiral.
σ	Impedance of a load at receiving end (ohms $\angle$).
u	Frequency ratio f/f_0 (numeric).
$y_0 = 1/z_0$	Surge admittance of a T-filter at a given impressed frequency (ohms $\angle$).
$y_{00} = \sqrt{C/\mathcal{L}}$	Surge admittance of T-filter at zero frequency (ohms).
z_0	Surge impedance of T-filter at a given impressed frequency (ohms $\angle$).
$z_{00} = \sqrt{\mathcal{L}/C}$	Surge impedance of T-filter at zero frequency (ohms.)
$\omega = 2\pi f$	Impressed angular velocity (radians per second).
$\omega_0 = 2\pi f_0$	Cut-off angular velocity of filter (radians per second).

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INDETERMINISM IN THE PHYSICAL WORLD.

By Wm. S. Franklin.

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INDETERMINISM IN THE PHYSICAL WORLD.

By Wm. S. Franklin.

INDETERMINISM IN THE PHYSICAL WORLD.

By Wm. S. Franklin.

Presented[1] October 10, 1923. Received January 15, 1924.

(*a*) Is everything in this world of ours completely determined by antecedent conditions, and is there no such thing in nature as an essentially unpredictable occurrence?

(*b*) Is every choice, every decision, of our lives determined by a definitely unbalanced condition among all of the influences which weigh for and against the choice or decision, and is there no such thing as essential freedom of the will and therefore no such thing as moral responsibility among men?

Of the two questions *a* and *b*, it seems that *a* is the more important because it is simpler, and because, as Huxley says "Nobody doubts that, at any rate within certain limits, you can do as you choose," and no one but a narrow fatalist or blind dialectician can doubt the reality of our moral responsibilities. The strongest evidence of the essential freedom of the will is our nâive sense of freedom, and the thing which nowadays tends more than anything else to discredit this nâive sense is the almost universal sophistication of latter-day civilized men in matters relating to the systematic sciences. If we except a small amount of traditional fatalism among sectarians in religion as of no consequence it would seem that the only strong presumption against the essential freedom of the will comes from an almost universal bias regarding question *a*.

This essay is devoted chiefly to question *a* in its application to lifeless nature. As the main purpose of the essay is to set forth a point of view which will force us to admit that something essentially unpredictable may occur tomorrow it is highly important at the start to recognize and to some extent to understand the universal philosophical bias which gives to everyone an instant answer to question *a*; "Yes, everything in lifeless nature is completely determined by antecedent conditions, and there is no such thing in lifeless nature as an essentially unpredictable occurrence." I have yet to find a single person who has returned other than this answer to question *a;* even those who persistently withhold an affirmative answer to question *b* have become so completely sophisticated in matters relating to the formal aspects of the physical sciences that they instantly accept an affirmative answer to question *a*.

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[1] The title of this paper as read before the Academy was Determinism in Biology and Indeterminism in Physics. This title was intended to convey the idea that the only determinisms in the world of actual happenings are the determinisms of genetics and physiology. The "equilibrium" world of the classicist in physics is almost as nothing in contrast with the biological world, and the perfect determinisms of this "equilibrium" world are of a lower order than the real determinisms in the biological world.

The paper has been revised since its delivery, and the author is greatly indebted to the outgoing chairman of the Publication Committee for the many alterations and additions made since January 1924.

What is here referred to as a philosophical bias may seem to be much more than that, for many scientists have come to believe in the existence of an objective principle or condition of universal validity, a principle which Helmholtz called "die Gesetzmäsigkeit der Natur." Of course Gesetzmäsigkeit is a fact, otherwise there could be no such thing as science, but it may not be of universal validity; a complete philosophy of nature may have to contemplate something beyond Gesetzmäsigkeit, something essentially unpredictable, lawless. At any rate it is the purpose of this essay to make this general point of view thinkable.

A Philosophical Bias.

What then is this philosophical bias and what is its source? To frame a tentative answer to this question let us note that the world in which we live is extremely complex and let us consider what characteristic of human psychology and what mode of human thinking would seem to be demanded for purely practical reasons in an extremely complex world.

Psychological. It is difficult to imagine an effective human psychology other than that with which we are familiar in which every sense perception is completed, as it were, by all but overwhelming subjective contributions so as to drown out extraneous and conflicting material, and lead, even in the face of infinite complexity, to a clearly defined behavior reaction. Otherwise there could be no behavior, or at best, like Brer Rabbit, we could only rush around! The subjective strengthening of one particular aspect or phase of a sense-complex until this aspect or phase stands out alone and becomes completely dominant, this seems to be the necessary characteristic of human psychology.

This characteristic of human psychology seems to be closely connected with what has been called the "all or none condition" in physiological psychology. A particular mode or phase of nerve activity may be likened to the explosion of a complex layout of powder or dynamite. The layout explodes completely or not at all. There is no such a thing as a partial explosion and there seems to be no such thing as degree or intensity of nerve action. This is the "all or none condition" expressed in simple concrete terms. In a strong vigorous man the "complex layout of powder or dynamite" is quickly and completely repaired after an "explosion" and this quick and complete repair makes for sustained nerve activity, but gradations of nerve activity as respondent to gradations of external stimuli do not exist. Sensations which we describe as more or less *intensive* must be merely more or less sustained in time or more or less widespread in our nervous system, or, in other words, merely more or less *extensive;* and the above described strengthening and control of a sense-complex by subjective contributions must be dependent on *inhibitory action* on the one hand and on *inciting action* on the other hand, inhibitory action being the avoidance of certain aspects of a nerve discharge by back firing as in the control of a prairie or forest fire, and inciting action being the mere leaving open of certain bridgeheads or perhaps the laying of certain bridgeheads with powder trains. Of these two phases of control inhibitory action is certainly the more thinkable and perhaps real inciting action does not exist.

This mechanistic conception of psychological processes is of course very crude and probably all wrong, but it is a conception and therefore usable in thinking of this complicated business; and a very old idea receives a strong emphasis from this mechanistic conception, namely, that however completely the behavior of a human being may be correlated with the world in which he lives, his psychology must consist of elements so purely symbolic of world conditions themselves as to be hopelessly inadequate as a basis for any really adequate philosophy of nature. This old idea gets badly worn by neglect, and it fails to limit any man's philosophy; we ourselves possess a mysterious kind of curiosity which breeds a philosophic optimism that laughs at limitations! We do, however, wish to use this old limiting idea somewhat drastically, but we confess that we do not ourselves really submit to it.

It would seem that the subconscious contributions to behavioristic trains in psychology, however omnipotent, must be extremely rudimentary or animal-like and of little significance in our philosophy. Let us therefore, neglect these subconscious contributions, and we have left an almost complete rational control of behavior in response to sense impressions. This rational control is embodied in ideas, and postulates, and proverbs, and articles of belief. Recall if you can an idea or postulate or proverb or article of belief which does not bring to mind a definite thing or condition as an antecedent and another definite thing as a consequence, or which does not tie two or more definite things together. We think only of things as tied together, we think only in terms of one-to-one correspondences. This is the philosophic bias to which we have referred, and it is putting the case mildly to call it a philosophic bias when it is the essential substance of our philosophic method, and indeed it has hitherto been nearly the whole of our method in science.

Mode of thinking. A good example of human thinking is furnished by the gardner who plants in the dark of the moon. There is a vast amount of human thinking of this gardener's kind, and it is essentially as reasonable as other modes of thinking for which we have less impatient contempt, for the gardener does plant and his success is such that he does not need to rectify his formula; therefore, let no one begrudge him his planting cue, *for cue he must have.* But a parlor discussion of the dark-of-the-moon idea is another thing! Once the writer was dreadfully oppressed by the apparent lack of qualifying humor in a long-drawn out and highly specific discussion of this idea, as it were a fact, but where no experimental use of the idea ever had or ever would be made; when, bursting out with Faust in the Witches' Kitchen:

> "Mich dünkt ich hor ein ganzes Chor
> Von hundert-tausend Narren sprechen"

the spell was broken because one other of the assembled company understood and appreciated Goethe's German! But the essence of human thinking is doubtful or wholly unjustifiable postulate and quick conclusion, and to conclude is to come to an end!—unless the conclusion is put to the test of experiment. Without the test of experiment our modern research institute would be a witches' kitchen ten thousand times confounded. This digression is intended to suggest the weakness of human reasoning when not continually rectified by critical appeal to experience, a weakness that is by no means so widely appreciated as it should be.

Natural Philosophy. Only two phases of natural philosophy have ordinarily been stressed in recent times, namely, (*a*) the philosophy of continua as exemplified by nearly the whole of modern mathematics, and (*b*) the philosophy of natural law in the mode of Newton and Maxwell as exemplified by nearly the whole of modern physics. These two aspects of natural philosophy may be called the classical natural philosophy for convenience.

a. Experimental. A very remarkable fact concerning the physical sciences is that every laboratory study of a physical condition or thing involves the bringing of that condition or thing into an unchanging state by laboratory control. Classical physics is the study of permanent or quasi-permanent states.

A simple example of a piece of physical research is the work of Boyle and Gay Lussac, and I will set forth this example as if the whole of the experimental study of gases had been completed, as we now know it, by Boyle and Gay Lussac. They took a batch of gas and bottled it up and protected it as completely as possible from outside disturbing influences, "killing it dead" as it were, then subjecting it to very slow changes of volume and temperature, they measured volumes, temperatures and pressures with the utmost care. Thus they found definite one-to-one correspondences between the measured values of volume, temperature and pressure.

The equation $pv = RT$ expresses approximately[2] the laws of Boyle and Gay Lussac which are one-to-one correspondences between p, v and T, where p is the pressure, v is the volume, and T is the absolute temperature of a batch of gas, and R is a constant. This equation is applicable to the gas only where the gas is in thermal equilibrium. When the gas is not in thermal equilibrium it has no definite pressure or temperature, and its volume is, strictly speaking, a measurable property of the containing vessel which is to be thought of as in thermal equilibrium. If a slow reversible change of state of the gas is represented graphically in a pv, or pT, or Tv diagram, we get a continuous curve, but if at any stage the process becomes irreversible the gas becomes turbulent and there is a gap in the process curve. The important thing is to recognize that there are definite one-to-one correspondences of the Boyle and Gay Lussac kind which can be established with a degree of precision which is dependent only on the completeness of the control, on the precision[3] of the measurements, and on the use of a sufficiently large amount of gas.

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[2] It is a common mistake to think of the classical laws of physics as *inexact* when they are merely *not simple* from the arithmetic or algebraic point of view. The simple algebraic expression of the gas laws, namely, $pv = RT$, is known to be only approximate, but there are very definite one-to-one relationships between p, v, and T when a considerable body of gas is kept in a quiet condition.

[3] Every physical measurement depends on a congruence operation, the fitting together of the thing to be measured and of the measuring structure; and errors of measurement (erratic errors of measurement) are of two distinct kinds, namely, *intrinsic* errors which are due to the inherent variability of the thing which is being measured, and *extrinsic* errors which are due to variability of the measuring structure. The so-called probable error of a measured result should therefore be separated into two parts, namely the *true probable*

All of the classical laws of physics are one-to-one correspondences between the various measured aspects of a system, they relate to substances or systems in permanent states, they are intrinsically approximate only when the permanence is incomplete or when the substance or system under test is so small that atomic or molecular irregularities become perceptible, and the essential narrowness of the classical method in physics has never been more strikingly pointed out than by Goethe, who, being a poet, could not be expected to recognize the necessity or appreciate the tremendous importance of a narrow prosaic point of view.

> "Wer will was Lebendiges erkennen und beschreiben
> Sucht erst den Geist heraus zu treiben."

Instead of bottling a batch of gas and wrapping the bottle in cotton wool, let it be turned loose and allowed to run wild! We can still follow it to some extent idealistically in terms of fleeting quasi-permanencies and roughly approximate one-to-one correspondences, and this is what your classicist in physics takes to be the whole story, but the point I shall wish to make is that a batch of gas turned loose and allowed to function in the forward movement of nature does perhaps run wild!

Quantitative chemistry, again, is not concerned with chemical actions themselves[4] but with their results. Consider, for example, a steam boiler plant, and fancy an engineer looking into the furnace and attempting to observe and record the height and breadth and momentary temperature of each flicker of flame! It is not done. The important thing is the amount of steam that is produced by each pound of coal, and this depends only on (1) the condition of the feed water from which the steam is made, (2) the quality of the coal before it is put into the furnace, (3) the initial condition of the air which enters the furnace, (4) the condition of the flue gases as they enter the chimney, and (5) the pressure and temperature of the steam which is

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error which is due to variability of the measuring system and to the inaccuracies of congruence; and the *probable departure*, this departure being the probable departure of the measured system from the carefully considered estimate at any instant.

[4] In some cases chemical action shows itself, approximately as a permanently varying state, and the various aspects of a chemical system in a permanently varying state can be measured and correlated. Thus the slow saponification of an oil is approximately a permanently varying state and the study of slow saponification gives rise to the notion of reaction velocity. Permanently varying states of chemical systems constitute what the writer has called *steady sweeps*. See pages 128–148 of Franklin and MacNutt's *Heat*, Lancaster, Pa., 1923.

produced. That is to say, it is necessary to consider only the states of things *before* and *after* the combustion takes place, and the only measurements that need be made, the only measurements that can be made, are measurements of substances in thermal equilibrium. The laboratory method for the study of thermal and chemical changes is what may be, in mild derision, called the "before and after" method!

b. Methods. There are four distinct methods in physics, namely, the method of mechanics, the method of thermodynamics, the method of atomics, and the method of statistics; and all four methods come into play in the broad study of any particular condition or thing.

The method of mechanics is exemplified by what is narrowly called mechanics; under the name of generalized mechanics it includes the whole of the classical treatment of electricity and magnetism, of light and of sound; and it stops short where thermal and chemical changes come into consideration.

The method of thermodynamics is essentially an extension of the method of mechanics so as to include the study of thermal and chemical changes and this extension is based on two generalizations, namely, on the first and second laws of thermodynamics.

The methods of mechanics and thermodynamics are not to be apprehended by definition, but by actual use. *Both methods establish correlations directly in terms of measured data without any consideration whatever of the essential nature of physical conditions and things;* both methods are highly abstract and they neglect much that is by no means always negligible; both methods establish one-to-one correspondences between the various measured aspects of systems in permanent or quasi-permanent states; and what I have called the "before and after" scheme gives to the method of thermodynamics a truly remarkable range in the correlation of the consequences of irreversible processes or forward movements.

The atomic method is the building of elaborate conceptual pictures of the essential nature of physical conditions and things on a postulate foundation. The molecule, the atom, the ion, the proton, and the electron *as used in theory* are mere postulates in view of the fact that as physical things our knowledge of them is so incomplete.

Everyone is familiar with the use of the atomic method in elementary chemistry, and with the somewhat vague but nevertheless important use of the atomic theory in Tyndall's "Heat, a Mode of Motion." The so-called kinetic theory of gases is also quite well known, but the use of the atomic method in crystallography by

Sohncke and others before the days of Laue and the Braggs, and the elaborate attempt which was made a hundred years ago to use the atomic method in the theory of elasticity are not generally known.

The atomic method has been extensively used in the elucidation of previously known one-to-one correspondences. The kinetic theory of gases, for example, gives a remarkable rational insight into the laws of Boyle and of Gay Lussac. The atomic method has also been of use in the discovery of classical one-to-one correspondences, and the most remarkable example of this sort of use is, perhaps, Maxwell's discovery of the invariance of gas viscosity with respect to pressure when the temperature is constant. Furthermore an important use of the atomic method is in the rationalization of simple types[5] of forward movements such as reaction velocities, radio-activity, thermoionic emission, electrical conduction, etc.

The statistical method[6] has been used but little in physics, except in meteorology which I will consider later. The best known statistical studies in the laboratory are Perrin's study of the Brownian motion, Rutherford's study of the scattering of alpha and beta particles, and Millikan's study of the movement of charged droplets of oil through ionized air. All of these studies are intimately connected with the atomic theory in that they deal with atoms or something like atoms as the actual objects of study. Perhaps the greatest achievements of the atomic method in the future will be statistical—only then it will not be the atomic method as above defined for atoms themselves will be the physically real objects of study.

Helmholtz has used the word "systematik" to designate the methods and results of classical physics, which includes the methods of mechanics and thermodynamics and also the method of atomics when used as it has been hitherto in the discovery and elucidation of one-to-one correspondences. However, we often extend the classical systematic physics to systems which are very far removed from permanent states, but the ideas which are used, the measurements (approximate measurements) which are made, and the approximate one-to-one correspondences which are established are all borrowed from and interpreted in terms of ideal permanencies.

c. *Idealization.* It is necessary in what follows to refer repeatedly to actual happenings in the physical world. Such happenings are so-called irreversible or sweeping processes, they always involve an increase of entropy,[7] and it is helpful to refer to them as *forward movements*.

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[5] These are what the writer has called steady sweeps or permanently varying states which are amenable to measurement.

[6] What is here referred to is the use of the theory of probabilities in the arrangement and interpretation of observational material. The kinetic theory of gases and statistical mechanics (Gibbs) are postulate structures, they are not what is here called statistical physics.

Let us look very briefly into the most familiar branch of physics, namely, Mechanics. In the study of the performance of any kind of a mechanism a great variety of change is involved, and yet the science of mechanics is no exception to the general statement that physical science is the study of permanent or quasi-permanent states. To justify broadly this statement in its application to mechanics it would be necessary to show that every phase of laboratory control in any detailed mechanical study is for the sake of permanency; it would be necessary to explain at length the peculiarly idealistic[8] character of mechanical studies, in general, and it would be found that the superficial and idealistic character of the science of mechanics is bound up with and grows out of the fact that the irreversible aspects of physical happenings are completely ignored[9] in all mechanical studies. To undertake to do all this would lead us away from the main theme of this paper and it must suffice therefore to refer to the science of hydraulics as a partial illustration of the method of mechanics. A brief discussion of hydraulics is indeed worth while because the idealistic character of this branch of mechanics is easily explained and because everyone can understand the utter inadequacy of the idealized science of hydraulics on the basis of what can be easily seen in any actual case of fluid motion.

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[7] No generalization in physics is more important to the biologist than the "law of entropy" or the second law of thermodynamics. The elementary discussion of this law which is given in Franklin and MacNutt's *Heat* is recommended to the biologist because this discussion dwells at first on the physical facts, because the postulate character of the generalization itself is then set forth, because the meaning of the generalization is then exemplified in simple concrete terms, and because the mathematical arguments which lead to the experimentally verifiable consequences of the generalization are made as simple as it is possible to make them. Of all the generalizations of physics the second law of thermodynamics is the most deeply rooted in intuition.

[8] One of the most far reaching discussions along this line is an appendix in Sir Joseph Larmor's "Aether and Matter" on "The Scope of Mechanical Explanation." This appendix of Sir Joseph's is difficult to read, but the reader will be helped in getting a clear idea of what is here referred to by reading the simple discussion of the mechanical nature of force which is given on pages 174–178 of Franklin and MacNutt's "Electricity and Magnetism," Lancaster, Pa., 1924. This discussion is an exposition of the simpler ideas which are set forth in the above mentioned appendix to Sir Joseph's "Aether and Matter."

[9] Friction is indeed taken into account in many mechanical studies, but in a manner so utterly incomplete that this statement may be allowed to stand.

Hydraulics, as the term is here used, is the study of fluids in motion, and the phenomena of fluid motion are excessively complicated. The apparently steady flow of a river in a smooth channel is seen on close inspection to be an endlessly intricate combination of boiling and eddying motion. The jet of spray from a fire nozzle; the burst of steam from the safety valve of a locomotive; the fitful motion of the wind as indicated by fluttering leaves and quivering blades of grass, and as actually visible in driven clouds of dust and smoke; and the visible sweep of the flames in a conflagration—these are actual examples of fluid motion and they are indescribably, infinitely complicated! Everyone concedes the idea of infinity which is based on abstract numbers, one, two, three, four, and so on *ad infinitum*, and the idea of infinity which comes from the contemplation of a straight line; but most men are so narrowly concerned with the humanly significant and more or less persistent phases of the material world that their perception does not penetrate into the substratum of erratic action which underlies every physical happening, and they balk at the suggestion that the phenomena of fluid motion, for example, are infinitely complicated. Surely the abstract idea of infinity is nothing in comparison with the awful intimation of infinity which comes from things that are seen and felt.

The science of hydraulics is based on ideas which relate to average aspects of fluid motion. We can determine the time required to draw a pail of water from a hydrant or the drop of pressure along a pipe line from a pump to a fire nozzle, or the force exerted by a jet on the buckets of a water wheel; but such things are never perfectly steady even when the ruling conditions are completely controlled, they are always subject to perceptible variations of an erratic character, and to think of any of these effects as definitely quantitative is, of course, to think of their average character under the given conditions.

The most important conceptual element in the science of hydraulics is the idea of *simple flow*, and if we limit ourselves to what is called permanent or steady flow this idea of concept is that the velocity of the fluid at each point remains constant in value and unchanging in direction so that the motion of a fluid can be visualized in terms of what are called stream lines. Simple flow is however, an idea, not a fact; and stream lines never exist in actual fluid motion. Nevertheless, the whole of the science of hydraulics is based on the idea of simple flow and one or two additional sharply defined ideas just as the science of geometry is based on sharply defined ideas of which the most familiar is the idea of a straight line.

Now it is not the purpose of this paper to discuss the very great importance of precise ideas in the physical sciences nor to justify the useful and even necessary habit among physicists of accepting and talking about such ideas as facts, a habit which is similar to that of the ordinary man who thinks of a straight line as a physical fact rather than as a mere idea. Any other point of view involves too many qualifications in speech and thought, and qualifications always weaken speech and thought. It is a great help, says Helmholtz, to form the most concrete possible pictures of abstractions even when such pictures involve assumptions that are in all strictness unnecessary. It is not my purpose, I say, to point out the importance of precise ideas, but it is quite necessary for me to call your attention to the fact that the many precise ideas which underlie the physical sciences operate not only as a stimulus to, but also as a constraint on, the imagination.[10]

What would you say of a fire chief who might become absorbingly interested in the infinite complex of minute detail of the stream of spray from a fire nozzle? Of a farmer who would employ an innumerable squad of disembodied spirits who could be stationed in his fields without interfering with the drops of rain and set observing and recording the size and shape and placing of each individual drop of rain? And yet it is on minute details of this kind that I ask you to fix your attention and exercise your imagination, because it is the aim of this paper to show that any infinitesimal detail or element in the infinitive aggregate of such that exists in any forward movement of nature may conceivably dominate the movement as a whole and determine finite consequences.

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[10] Reference may be made to an essay of C. S. Peirce, *The Monist*, April, 1892, reprinted in his Chance, Love and Logic (edited by W. R. Cohen), 1923. A quotation: "I propose here to examine the common belief that every single fact in the universe is precisely determined by law. It must not be supposed that this is a doctrine accepted everywhere and at all times by all rational men. Its first advocate appears to have been Democritus, the atomist, who was led to it, as we are informed, by reflecting upon the 'impenetrability, translation, and impact of matter.' That is to say, having restricted his attention to a field where no influence other than mechanical constraint could possibly come before his notice, he straightway jumped to the conclusion that throughout the universe that was the sole principle of action,—a style of reasoning so used in our day with men not unreflecting as to be more than excusable in the infancy of thought. But Epicurus, in revising the atomic doctrine and repairing its defences, found himself obliged to suppose that atoms swerve from their courses by spontaneous chance; and thereby he conferred upon the theory life and entelechy."

The Real State of Affairs.

What I have to say to you about indeterminism in physics is simple enough and it will take only a short time to tell; it has already been adumbrated in the discussion of idealization, but I must count on your continuing to take a realistic view of physical things, and by realistic I mean a freely imaginative sensuous view as opposed to a constrained aperceptive view which sees only what is in conformity with accepted ideas. For we must certainly break away from the idealization of classical natural philosophy if we are to give adequate attention to the discontinuous and to that which in particular is lawless and becomes lawful only in general.

The attempt is made in this paper to show that, conceivably, there are gaps in nature wherein there are no physical or chemical laws, no one-to-one correspondences expressible in terms of differential or integral equations, and that the finite consequences of such a gap or break are indeterminate.

Turbulence. The more an experimental system departs from an ideal permanent or permanently varying state, the more clearly we recognize an underlying erratic action which is indescribably complex. This erratic action exists everywhere and in everything that goes forward in nature, we call it *turbulence,* and I think it may be assumed to be infinitely complicated and never twice alike in detail even when the large-scale conditions are exactly the same. We have a macro-physics, a large scale physics, which is the physics of the classicist, and we have a new micro-physics, a small scale physics. The terms macro-physics and micro-physics are however misleading because turbulence often shows itself in large-scale phenomena such as storm movements in our atmosphere and storm movements in the sun, to say nothing of such frequent large-scale turbulences as automobile and train wrecks and collapsing structures! It is these large-scale turbulences which I wish especially to consider, not because they are essentially different from omnipresent small-scale turbulences, but because they are humanly significant and therefore real to everybody, and the point I wish to make is that *a large-scale turbulence can conceivably be attributed to infinitesimal causes.*

Turbulence is characteristic of those physical and chemical changes which are called irreversible or sweeping processes. Such processes involve increase of entropy and they are normally impetuous. The most familiar example of such a process is ordinarily fire, and, as everyone knows, a fire is not dependent on external driving cause but when once started it goes forward of itself and with a rush.

Impetuous processes, such as explosions and wrecks and the storm movements of our atmosphere, are intimately connected with conditions of instability. Indeed an impetuous process seems always to be the collapse of an unstable state. Let us therefore consider two ideal examples in which the condition of instability is supposed to be completely established at the beginning.

(*a*) Imagine a warm layer of air near the ground overlaid with cold air. Such a condition of the atmosphere is unstable and any disturbance however minute, may conceivably start a general collapse. Thus a grasshopper in Idaho might conceivably initiate a storm movement which would sweep across the continent and destroy New York City or a fly in Arizona might initiate a storm movement which would sweep out harmlessly into the Gulf of Mexico. These results are different surely, and the grasshopper and the fly may be of entirely unheard of varieties, more minute and insignificant than anything assignable. Infinitesimal differences in the earlier stages of an impetuous process may, therefore, lead to finite differences in the final trend of the process.

(*b*) Consider a smooth spherical ball traveling through still air. There certainly is no more reason to expect the ball to jump to the right than to the left. Therefore, we may conclude that it will not jump either way. Similarly, a sharp-pointed stick stands in a perfectly vertical position in a perfectly quiet room, and there is no more reason to expect the stick to fall one way than another: therefore the stick will not fall at all. Everyone appreciates the fallacy of this argument as applied to the stick, and the moving ball does, in fact, jump sidewise.

To understand the behavior of the ball, let us think of the ball as standing still and of the air as blowing past it in a steady stream. The air streams past the ball and slides over a body of still air behind the ball,[11] the surface which separates the moving air and still air is called a vortex sheet, and a vortex sheet is unstable. Any cause however minute, is sufficient to start an eddy or whirl, and once started such an eddy or whirl develops more and more. Such an eddy or a whirl means that the air streaming past one side of the ball is thrown inward or outward and the reaction on the ball pushes the ball sidewise. This effect can be shown by dropping a marble in a deep jar of water. Instead of moving straight downwards the marble follows an erratic zigzag path. This effect is familiar to everyone in the sidewise quivering of a stick or fishline in a stream of water; and the hissing of a jet of steam is due to the rapid fluttering of the boundary between steam jet and air because of the formation of innumerable eddies.

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[11] The motion of the air is somewhat idealized in this statement.

The above examples refer to the breaking of an unstable state by a minute disturbance. In the case of an ideal or complete instability the breaking disturbance can be smaller than anything assignable, that is to say, the breaking disturbance can be an infinitesimal disturbance in the strict mathematical meaning of this term, many of the finite characteristics of the resulting forward movement will be determined by the time and place of the infinitesimal disturbance, and we must admit that the idea of cause and effect is here inapplicable because a finite effect cannot be attributed to an infinitesimal cause.

A postulate. In that branch of mathematical physics which is called statistical mechanics and which includes the atomic theory, we speak of the complection of a system when we wish to refer to the positions and velocities of all the elements or particles of the system. Let us use this word in the statement of the POSTULATE OF INDETERMINISM. *The complection of the world tomorrow is not determinate*—that is to say, it does not grow out of the complection of the world today as a single-valued determinate thing. This is a postulate which, as it seems, must be accepted as a working hypothesis in the "extra-equilibrium" world, the world of actual happenings, where things never do stand still, but go forward by fits and starts impetuously and beyond all control.

Whenever the postulate of erratic action is set forth, and the probable departure of a natural phenomenon from the most carefully considered prediction is urged as in the nature of things inevitable, we meet objections from two classes of men, namely, the average man who thinks frankly in terms of human values (the gardener who is not concerned with individual drops of rain) and the classicist in science who idealizes nature in one-to-one correspondences. Surely the classicist says, "if we knew all the data we could make an unqualified prediction in any case." But, ignoring the hopelessly unscientific attitude of mind of one who can postulate infinite knowledge, let it be understood that to speak of data in physics is to speak of a very narrow and limited kind of thing, for data are conceivable only where measurements can be made or where we have, contrary to Bacon's exhortation, accepted a dream of fancy for a model of the world.

Meteorology has for many years been the only branch of physics dealing with actual uncontrolled happenings, and the statistical methods which have heretofore been used in meteorology have been too much of the kind that is applicable to a homogeneous aggregate of erratic elements. Statistical meteorological studies have in most cases involved the leveling process of simple averaging, whereas the conditions would seem to require classification ten thousand times more exhaustive than any hitherto made. Rythmic variations of day and night, the succession of the seasons, bodies of land and water, mountains and valleys, and the influence of underlying regularities of atmospheric movement which show themselves in what we call prevailing winds, all these would seem to have much to do with the characteristics of incipient atmospheric instabilities and they must certainly operate as "grooves" as it were, in their influence on storm movements and determine a host of families and genera and species. Any adequate statistical study of a "grooved" system must use exhaustive classification studies. To be content with averages and probable departures derived from meteorological data uninfluenced by exhaustive classification studies is as hopeless as a study of plants and animals which ignores the existence of species.[12]

The conception of indeterminism, as dependent on the collapse of unstable states is decidedly incomplete in a realistic sense because every unstable state is an ideal. Static and dynamic instabilities of systems of rigid bodies are very highly idealized, and so also are static and dynamic instabilities of fluids; and all such instabilities of mechanical system are, perhaps, in their limits, incompatible with the atomic or molecular constitution of matter. However, the main purpose of this paper is to make indeterminism thinkable! Critical instabilities are highly idealized, as stated, but it does seem that an interpretation of nature which is based on the postulate that *critical instabilities do occur* is more realistic (on the whole less highly idealized) than the usual mechanistic interpretation which is based on the even higher degree of idealization which rules out the critical instability.

The postulate of indeterminism and its basis on instabilities is not new. It is set forth in a "practical" form in a remarkable paper by J. Boussinesq, entitled "Conciliation du Véritable Determinism Méchanique avec l'Existence de la Vie et de la Liberté Morale," Paris, 1878; reprinted in the third volume of his Theorie de la Chaleur, etc., Paris, 1922. That Boussinesq should incorporate this essay in his great treatise on heat and light shows that he still holds to his earlier views, and it may indicate that he considers these views to be worthy of attention although they have been almost completely neglected hitherto. "Scientists are in agreement" says Boussinesq "that physical and chemical laws are reducible in the last analysis to differential equations" and indeterminism is connected with the behavior of differential equations at or near their singular points. The equations have several distinct solutions which differ only infinitesimally at or near a singular point; so that in passing through a singular point an infinitesimal influence is sufficient to determine the outcome of one or the other of the distinct solutions and thus lead to widely different consequences.

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[12] For a further discussion of meteorology see my paper in the Monthly Weather Review, September 1918, in which paper is given a preliminary sketch of my point of view relative to indeterminism.

About the same time as Boussinesq, Maxwell was making a similar suggestion.[13] Says he: Every existence above a certain rank has its singular points, the higher the rank the more of them. At these points influences too small to be taken into account by a finite being may produce results of the greatest importance. All great results produced by human endeavor depend on taking advantage of these singular states when they occur. But singular points are by their very nature isolated and form no appreciable fraction of the continuous course of our existence. This view of Maxwell's makes indeterminism the exceptional, determinism the ordinary course of events, but the exceptional the more significant. An anology from biology might be mutation and from astronomy the cataclysmic change which leads to the birth of a double star.[14]

A Generalization. Although through the ideas of singular states and the collapse of unstable states, it may be possible to go some distance towards giving a rational theory and a convincing illustration of indeterminism in physics, we do not believe that these suggestions are adequate. So pervasive are the cases of observable indeterminism, so obvious does it appear that the whole mechanistic system and philosophy of systematic physics is an idealization directly away from intimate happenings for the purpose of simplifying microscopic explanation, that we prefer to regard the facts of nature as indicating indeterminism as not exceptional but general. A batch of gas turned loose and allowed to function in the forward movement of nature does perhaps run wild! We have, of course, chemical instabilities, and the recent developments of atomic theory, especially those which include the quantum theory, seem to indicate the existence in nature of kinds of instability and of chance happenings previously undreamed.[15]

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[13] See "The Life of Clark Maxwell" by L. Campbell and W. Garnett, 1882, pp. 434 ff.

[14] See for example, J. H. Jeans' Cosmogony and Stellar Dynamics, Chapter VI.

The point of view relative to indeterminism which we have tried to subsume in a postulate and to illustrate by pointing out what does seem to happen in the world of physics is cognate not so much to that of Boussinesq and of Maxwell as to that of C. S. Peirce. In an imaginary dialogue with an imagined opponent, after the style of Plato, Peirce says[16] "Very well, my obliging opponent, we have now reached an issue, you think all the arbitrary specifications of the universe were introduced in one dose, in the beginning, if there was a beginning, and that the variety and complication of nature has always been just as much as it is now. But I, for my part, think that the diversification, the specification, has been continually taking place." And again,[17] "An elememt of pure chance survives and will remain until the world becomes an absolutely perfect, rational, and symmetrical system in which mind is at last crystallized in the infinitely distant future."

Whether the last phrase is merely rhetorical for emphasis or whether it represents Peirce's notion of the inevitable line of evolution in nature's infinite variety is not clear, but with respect to evolution and the idealized laws of nature he does say this:[18] "Uniformities are precisely the sort of facts that need to be accounted for. Now the only possible way of accounting for these laws of nature and for uniformity in general is to suppose them results of evolution. This supposes them not to be absolute, not to be obeyed precisely. It makes an element of indeterminacy, spontaneity, or absolutely chance in nature." These are our *gaps;* they are inherent and not necessarily mere singular points in differential equations which cannot, as a matter of fact, be set up except on a basis of idealizations and uniformities.

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[15] It is interesting to note that in the work of Einstein and his followers we find both determinism and indeterminism. On the one hand his generalized relativity with its developments to include electrical phenomena is the apothesis of the Democritus-Newton-Laplace-Maxwell school of classical natural philosophy and has already found its interpreters among philosophers, notably A. N. Whitehead. On the other hand his quantum theory and theory of radiation with their experimental and theoretical consequences make a return to the Epicurean doctrine of the swerving of atoms (electrons and quanta are the prime movers now) from their courses by spontaneous chance; if such phenomena and notions become in due time thoroughly basic and permanent in physics, they, too, will find their interpreters among philosophers.

[16] Chance, Love and Logic, p. 195.

[17] *Ibid.*, p. 177.

[18] *Ibid.*, p. 162.

It is not necessary to follow Peirce in all his ideas, but it is advantageous to recognize with him and a long line of predecessors that a purely mechanistis philosophy does not cover all the phenomena of nature. It is not necessary to slacken our endeavors to explain, as duly caused, all phenomena which may be so explained, and we are entirely sympathetic with the mechanistic developments in genetics, physiology, and biology in general; but it is important to realize that there will always remain points at which such explanations break down, and recourse must be had to a classificatory procedure such as has been urged relative to meteorology. It is possible that the present difficulties in reconciling the new quantum phenomena with the old classical physics are in part due to the fact that physicists have long since abandoned the classificatory method and are too loath to return to it.

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THE EFFECT OF HYDROSTATIC PRESSURE ON THE MAGNETIC PERMEABILITY OF IRON, COBALT AND NICKEL.

By Chi-Sun Yeh.

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THE EFFECT OF HYDROSTATIC PRESSURE ON THE MAGNETIC PERMEABILITY OF IRON, COBALT, AND NICKEL.*

BY CHI-SUN YEH.

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INTRODUCTION.

THE various phenomena of magnetostriction together with their inverse effects, namely, the effects of stress on magnetization, have been known for a long time. In particular, Nagaoka and Honda (1) studied the effect of hydrostatic pressure on the magnetization of iron and nickel, and later on, Miss Frisbie (2) repeated the work for iron. The highest pressure used by Nagaoka and Honda was about 300 atmospheres; that by Miss Frisbie about 1000 atmospheres. For iron, while Nagaoka and Honda obtained only a decrease of magnetization by pressure, Miss Frisbie obtained an increase in low fields and a decrease in high fields, the range of field covered by Miss Frisbie being within that covered by Nagaoka and Honda. Besides this disagreement in results, the data presented by these authors are not sufficient for giving a comprehensive view of the pressure effect on magnetization.

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* The material of this paper constitutes essentially the thesis submitted by Dr. Yeh for the doctor's degree in June 1923. The manuscript of this paper was handed to me by Dr. Yeh immediately before he left this country in the late summer of 1923. The delay in publication has been caused by my difficulty in getting into communication with Dr. Yeh to obtain his consent to various changes in the text and diagrams affecting only the method of presentation.—P. W. BRIDGMAN.

The present research was undertaken because we thought that the subject could now be attacked more comprehensively and also to better advantage. There are three reasons for this: (1) the work of Professor Bridgman has enabled us to extend the pressure to a much wider range than covered by earlier workers; (2) the ferromagnetic metals can now be obtained in a much purer condition than those used by the earlier experimenters; (3) the results may stimulate interest in developing magnetic theories along modern lines.

Experimental Method in General.

In order to avoid end effects, toroidal specimens were used. The method of balancing the magnetic deflection against that due to another specimen, made as nearly similar as possible to the pressure sample, was rejected at the very beginning of the work, since it cannot give reliable results on account of the great difficulty of getting exactly similar specimens. The *increase* of magnetization of iron under pressure for H about 5 obtained by Miss Frisbie might be easily explained by this error. (This wrong effect might also be due to imperfect demagnetization as we will see later.) Instead of the method of Miss Frisbie, we balanced the deflection due to the specimen under pressure against that due to a mutual air inductance.

Two kinds of runs were made, one at a constant magnetizing field, and the other at a constant pressure. They will be described separately.

Constant Field Runs.—For these runs, the ballistic deflection due to the magnetic specimen was nearly completely balanced against that due to a mutual inductance, leaving however a residual reversal deflection greater than the total change of reversal deflection at 12000 kg./cm^2. This deflection usually amounted to several centimeters. It was not possible to balance the magnetic deflection exactly when there was no pressure for the reason that the two parts of the magnetic reversal deflection, one obtained on breaking the circuit and the other on making the circuit in the opposite direction, were unequal because of hysteresis. The result was that during the initial stages of the reversal there was a slight deflection in a direction opposite from the final deflection. The effect probably could be eliminated by increasing the period of the galvanometer or by increasing the rapidity of reversal. Too rapid reversals are, however, not desirable on account of the difficulty of insuring uniform operation. The final scheme adopted was to minimize the initial reverse deflection as much as possible by using two mutual inductances of different values, such that one of them is in circuit when the magnetizing current goes one way, while the other is in circuit when the magnetizing current goes the other way. This arrangement was made possible by using a reversing switch of eight poles. The mutual inductances were made of solenoids with sliding secondary coils inside the primary. They were specially made to give the large variation demanded in this work.

From the direct observations, the differences of residual reversal deflections were calculated. When these are divided by the total reversal deflection of the specimen alone (without the compensating mutual inductance) under no pressure, we obtain the percentage change of permeability or induction. This is practically the same as the percentage change of susceptibility or magnetization for the ferromagnetic substances in the range of fields we are using. These percentage changes have to be corrected for the change of dimensions under pressure, because in the calculation of B and H from experimental data, the cross section and circumferential length of the ring come into consideration. When we consider the decrease of area under pressure, twice the absolute value of linear compressibility must be added to the uncorrected pressure coefficient. When we consider further the circumferential linear contraction and the consequent higher value of H we must subtract the absolute value of the linear compressibility, making a total additive correction of the absolute value of the linear compressibility. The magnitude of this correction is .02% for each 1000 kg./cm^2. It should be deducted from the numerical value of the observed percentage change, when the pressure coefficient of magnetization is negative; it should be added when positive.

When the corrected percentage changes were plotted against pressure, the first few runs showed a flat region in the curve plotting $\Delta B/B_0$ against pressure or else showed a curve of continually increasing slope. Further, the results were often irregular. The cause of the trouble was later traced to imperfect demagnetization. The final scheme adopted was to demagnetize the specimen completely before every change of pressure. When this was done, the percentage change became linear with pressure within the limits of experimental error, and the results also became reproducible.

A word should be said about the meaning of the reversal deflection as measured here. As is well known, the first reversal deflection is greater than the later ones. What was measured here was the constant reversal deflection obtained after several reversals, which can be determined with greater accuracy than the initial reversal deflection. The question of accuracy is of particular importance here because we are dealing with small differences. The values of permeability and also of differential permeability therefore come out considerably lower than when measured by the step-by-step method.

In using the balanced reversal method, we have to be particularly careful about the effect of imperfect demagnetization. For suppose we start with a specimen not completely demagnetized, then the reversal deflection under no pressure and low fields will be smaller than it ought to be. If, now, the residual magnetization is a function of the pressure, the error in the residual deflection will vary with pressure so that the apparent percentage change produced by pressure may be complicated in character and entirely erroneous; it may even be of the wrong sign. Under high fields an erroneous result is less likely.

In general, the effects of imperfect magnetization in iron and nickel are opposite, since their pressure coefficients are of opposite signs.

Constant pressure runs.—This type of run was to determine the percentage change of permeability as a function of the field. It is less conducive to accurate measurement than the constant field run; but it gives a more comprehensive idea about the course of phenomena. In this type of run, we simply obtain a normal magnetization curve under no pressure, and then one under high pressure. This method gives reliable results if the pressure is sufficiently high. The method of balancing the magnetic deflection against a mutual inductance deflection is here not used on account of the inconvenience of changing the mutual inductance adjustments before every increase of the magnetizing current. Even if the balancing method were used, it would not give as accurate results as with the constant field runs.

EXPERIMENTAL DETAILS.

The pressure apparatus designed by Professor Bridgman has been thoroughly described in his papers (3). It suffices to say that the apparatus can hold pressure with practically no leak for the range covered in this paper, namely, 0–12000 kg./cm^2. The measurement of pressure by the change of resistance of manganin wire under pressure has also been described in Professor Bridgman's papers (4). By comparing with an absolute gauge he showed that the change of resistance of manganin is linear with pressure for the range covered here.

The pressure values reported below were obtained as follows: the total resistance of the gauge and the change of resistance under pressure were directly measured on the bridge. The pressure was then calculated with the coefficient 2.325×10^{-6} per kg/cm^2. The maximum error that may be introduced in the absolute value of pressure on account of the slight variation of the pressure coefficient for different specimens from same spool is about 2%. The error in the relative values of pressure is less than 1%.

The diameter of the hole of the cylinder containing the specimen was about 16 mm., and its external diameter about 8 times the diameter of the hole. These dimensions of the high pressure chamber necessitated the use of toroidal specimens having an outer diameter of about 16 mm. and an arm width of about 3.2 mm. The toroids were made into shape with a semi-circular cutter.

The specimen was mounted horizontally on a collar screwed to a three-terminal plug. The three terminals served for the secondary leads and one of the primary leads. The other primary lead was attached to the body of the plug. Through each hole passed a steel stem surrounded by packing which served for keeping the pressure as well as for insulating the stem from the main body of the plug. The perfection of the insulation was tested both on the bridge for measuring pressure and with the ballistic galvanometer, the criterion with the latter being that its zero should be exactly the same no matter whether the primary current was going one way or the other. No particular care was taken to compensate or minimize the earth's field, because the effective earth's field inside the pressure cylinder must be exceedingly small.

The primary winding was such that one ampere gave about 50 gausses. The total number of secondary turns ranged from 200 to 500. For the primary winding, enameled copper wire was used; the diameter of the bare wire was about .2 mm. The change of resistance of the primary by pressure does not affect our results, because the constancy of current was watched with a potentiometer. For the secondary winding, silk insulated copper wire was used, the diameter of the bare wire being about 0.76 mm. The total resistance of the secondary winding varied from 10 to 20 ohms. Since the total change of resistance of copper under 12000 kg./cm^2. is about 2% of the resistance under no pressure, the maximum change of resistance of the secondary that might affect our results is about .4 ohm. Since there are more than 9000 ohms in the secondary circuit, the correction to be applied is at most .005%. We need not consider this correction since we give results only to a hundredth of a per cent.

Before putting on the primary winding, the anchor ring was insulated with a thin layer of enamel, baked on in an oven at a temperature around 205° C. A single layer of the primary was wound all over the circumference of the ring, care being taken to keep the winding radial. The secondary was then wound outside of the primary, no care being taken to keep the winding radial, because it was not necessary. What enters the measurement of the induction is the projection in the direction of the field of the area through which the lines of force run. This projection is always the cross section of the arm of the ring. Winding was done by hand and the number of turns was counted at the same time.

The magnetizing current was kept constant by watching a potentiometer of very simple scheme, (see fig. 1) a standard cell being used directly in closed circuit in series with a 50,000 ohm resistance during balancing. The e.m.f. of the standard cell was compared with that of a better standard from time to time. It kept its value within one thousandth of a per cent.

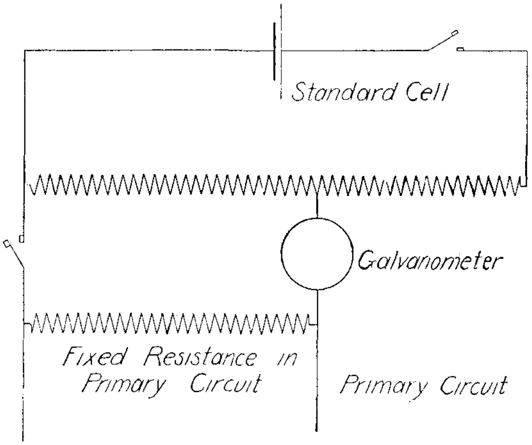

FIGURE 1. Potentiometer.

The field, H, is given by the relation $H = (4/10)NI$ for a thin toroid, where I is the current in amperes and N the number of primary turns per cm. I was measured on the potentiometer. N was obtained by dividing the total number of primary turns by the mean circumference of the toroid. The largest uncertainty in the measurement of field lies in the fact that our toroids were not thin, the ratio of arm width to mean diameter varying from 0.26 to 0.31. The error that may be introduced on this account is about 3% according to a calculation by Lloyd (5). Errors in measuring N and I are certainly much less than this.

The magnetic measurements were carried out with a ballistic galvanometer. It was used in the nearly critically damped condition (slightly over damped) with 9680 ohms in the circuit. The galvanometer (Cat. No. 2285–D Serial No. 73779) was obtained from Leeds and Northrup. Its free period is 28 sec., ballistic period, 6 sec. It was used at a distance of about 3 meters and gave a sensitivity of .0092 microcoulomb per millimeter.

The equation for calculating the induction, B, is

$$B = \frac{KRd}{nA}$$

where $d = \frac{1}{2}$ reversal deflection, n = total number of secondary turns, A = cross section area of the arm of the anchor ring, R = total resistance in the secondary circuit, K = galvanometer constant. Since R was kept constant, the calibration of the ballistic galvanometer amounts to finding the combination of constants $K \times R$. This was done by means of a mutual inductance standard, of the value 0.964 millihenries. The relation for calculating KR is the following:

$$K \times R = \frac{MI}{d_s}$$

where M = mutual inductance, d_s = deflection observed when we make or break a current of value I. The absolute value of $K \times R$ is only needed for the $B - H$ curves, not for the pressure coefficients of magnetization. Throughout the series of runs, the galvanometer constant was found to keep its value within .08%.

When the deflections became too large, a shunt of ratio 2.55 was used in parallel with the ballistic galvanometer. The shunt ratio was directly determined by comparing the deflections obtained with and without the shunt.

The electrical connections, except for minor conveniences, are sketched in fig. 1–3. With regard to the primary circuit, the reader will note that when the switch is closed one way, the current follows the path $AC(MI_1)$ D—toroid—EB; when reversed the path is $AC'(MI_2)$ $D'E$—toroid—$DE'B$. The exact equality of the two paths of the primary circuit as far as resistance is concerned was tested with the potentiometer. Outside of the ferromagnetic specimen, the circuits were free from iron, except the steel stems which serve as electric leads through the three-terminal plug. These steel stems can intro-

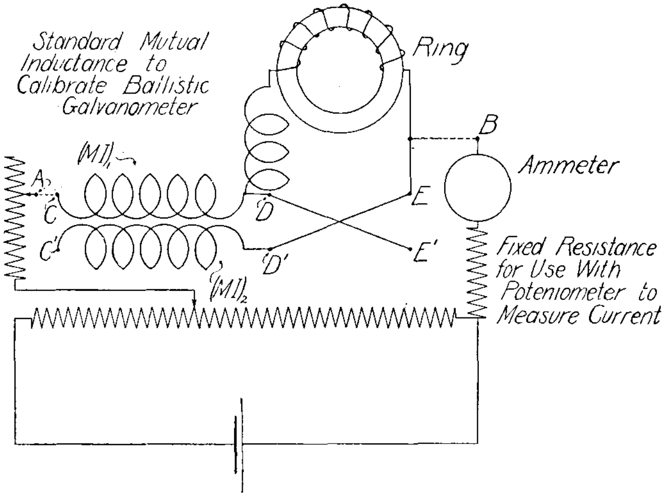
Standard Mutual Inductance to Calibrate Ballistic Galvanometer
Ring
B
Ammeter
(MI)₁
A
C
D
E
C'
D'
E'
(MI)₂
Fixed Resistance for Use With Potentiometer to Measure Current

Fig 2 Primary Circuit

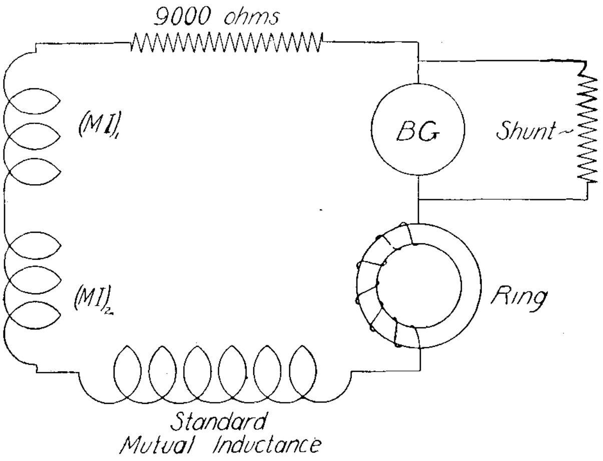
9000 ohms
(MI)₁
BG
Shunt
(MI)₂
Ring
Standard Mutual Inductance

Fig. 3 Secondary Circuit

duce no error in the magnetic measurement as long as there is no secondary turn linking with them.

The specimens were demagnetized by using a gradually decreasing alternating current (60 cycles), starting from a current higher than the maximum magnetizing current. The current was varied by varying the potential with a sliding resistance of tubular type.

The pressure cylinder containing the specimen was surrounded with a tank of water with a stirrer. The temperature of the bath was kept around 20° C. Since the temperature coefficient of magnetization around room temperature is small, the pressure coefficient could be measured without taking elaborate means to keep the temperature absolutely constant; the temperature of the bath was constant within a degree.

The heating effect of the magnetizing current is a factor that must be considered. With one ampere current, the temperature of the specimen is probably 1° higher than that of its surroundings. This is one of the reasons for not pushing the magnetizing current beyond two amperes.

THE MATERIALS

The specimens examined are described as follows:

Pure Iron.—The specimen was of French preparation, obtained from Professor Sauveur. It is known to contain 99.98% iron. It is indeed very pure as shown by its microphotograph (fig. 4). The material had been previously annealed by imbedding in lime. The toroid, after being shaped, was annealed again by heating to 1000° C. in an electric furnace, followed by cooling in the furnace. The specimen, while annealing, was imbedded in a large quantity of iron dust

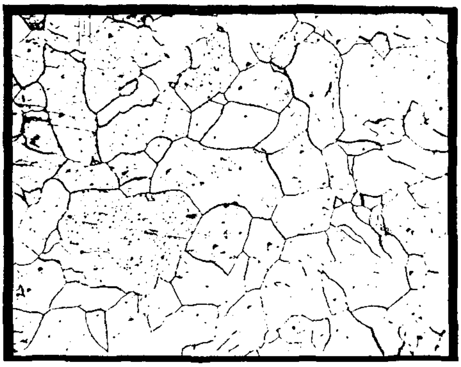

FIGURE 4. Microphotograph of Pure Iron.

to prevent oxidation. Microphotographic examination of the toroid after annealing shows that the material remained very pure.

Slightly Carbonized Iron.—This specimen started its career as a very pure specimen of iron, obtained from the Bureau of Standards, having the following analysis: C .005%, Si .007%, S .011%. The toroid made from it was annealed in the same way as described above. The toroid, after annealing, was not examined at once microphotographically, and it was thought that it had remained very pure. After the series of readings was taken with this specimen, the pure iron described in the last paragraph was examined to see whether the results would agree. The results, however, came out widely different. Microphotographic examination of the specimen from Bureau of Standards then showed that it had been slightly carbonized while annealing. The results obtained with this specimen are therefore to be taken as results for slightly carbonized iron. The difference between the specimen of pure iron and that of slightly carbonized iron is clearly seen in figures 4 to 5. The microphotographs were taken with the two anchor rings themselves after their pressure coefficients had been determined. An alcoholic solution of 5% nitric acid was used as the etching agent, with a magnification of 125 diameters.

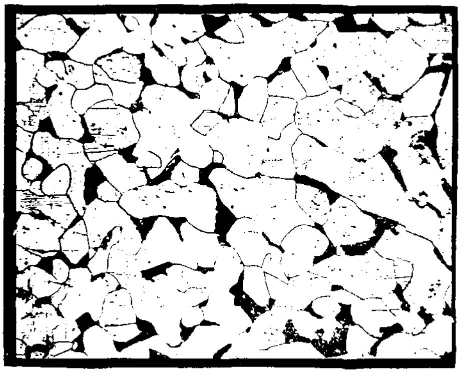

FIGURE 5. Microphotograph of Slightly Carbonized Iron.

After much work was done on the specimen from the Bureau of Standards, it was certainly a great disappointment to know that the specimen had been slightly carbonized. The impressive lesson is that for any magnetic measurement of pure iron, a microphotograph should be taken before and after any heat treatment.

Steels.—Two kinds of steel, of carbon content .10% and .30% were examined. The method of annealing was the same as for iron.

Nickel.—The specimen of nickel examined had the following analysis: Approximately 99.1% Ni, 0.5% Co, together with small amounts of Fe, Si, C, and Cu. It was annealed by heating to 900° C., followed by slow cooling.

Cobalt.—The specimen was obtained from the Bureau of Standards which again obtained it from Kahlbaum. It came in the form of little cubes. A casting was made in a vacuum furnace. The toroid made from the casting was first annealed by heating to 1000° C. After a series of readings was taken, it was annealed again, using the method of Kalmus (9), i.e. keeping the specimen between 500° and 600° C. for several hours. The second series of readings, however, gave no different results.

In table I are given the dimensions of the toroids.

TABLE I.

Specimen	Mean diameter	Arm width	Arm width/mean diameter
pure iron	1.110 cm.	.294 cm.	.265
slightly carbonized iron	1.153	.300	.260
.10% C. steel	1.018	.317	.312
.30% C. steel	1.054	.305	.289
nickel	1.092	.292	.267
cobalt	1.067	.336	.312

These dimensions will be needed in applying a certain correction to our experimental results, to be explained later, as well as in calculating H and B.

EXPERIMENTAL RESULTS.

$\Delta B/B_0$ *at Room Temperature.*—Sample curves of constant field runs are given in figures 6 to 8. They show that, for iron, the decrease of magnetization is proportional to pressure within the limits of experimental error. The individual data for $H = 0.52$ and $H = 15.6$ are not as regular as for $H = 1.30$ and 4.69. The reason is that for $H = 0.52$ the total deflection is small, and for $H = 15.6$, the percentage change becomes small.

In the case of nickel, the increase of magnetization is also approximately linear with pressure. That the curve has a slightly increasing slope is explained by the fact that the specimen, though demagnetized before taking the series of readings for the curve, was not further

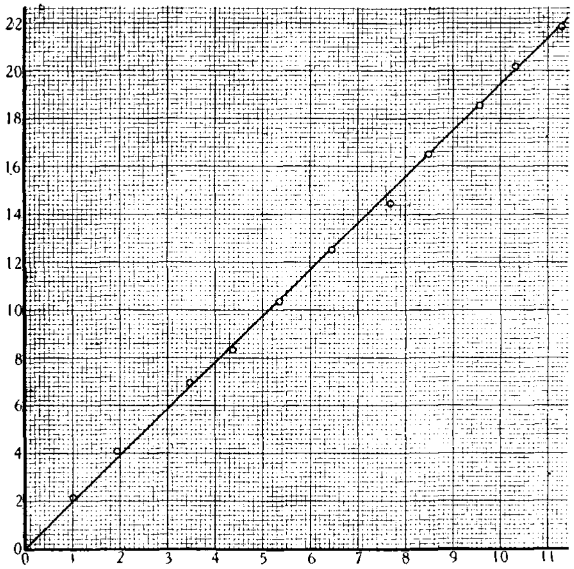

FIGURE 6. Effect of pressure, at 20°, on the induction of slightly carbonized iron, under a constant field of 1.30 Gauss. Ordinates are $-(\Delta B/B_0)$ in per cent, abscissae pressure in thousands of kg./cm^2.

demagnetized in the course of increasing pressure. (See the paragraph on the effect of imperfect demagnetization).

Sample data sheets for constant field runs are given in tables II,

TABLE II.

CONSTANT FIELD RUN FOR SLIGHTLY CARBONIZED IRON AT $H = 1.30$

Pressure	Residual Rev. Defl.	$\Delta B/B_0$ (per cent) Uncorrected	Corrected
0 kg./cm.^{2}	5.09 cm.	0	0
1010	4.72	− 2.17	− 2.15
1930	4.39	− 4.10	− 4.06
3470	3.89	− 7.04	− 6.97
4370	3.65	− 8.44	− 8.35
5360	3.30	−10.49	−10.38
6460	2.93	−12.67	−12.54
7590	2.60	−14.60	−14.45
8490	2.24	−16.70	−16.52
9570	1.89	−18.76	−18.56
10320	1.61	−20.40	−20.19
11290	1.32	−22.08	−21.85

data for fig. 6, and III, part of data for fig. 7. A sample data sheet for a constant pressure run is given in table IV, data for fig. 9.

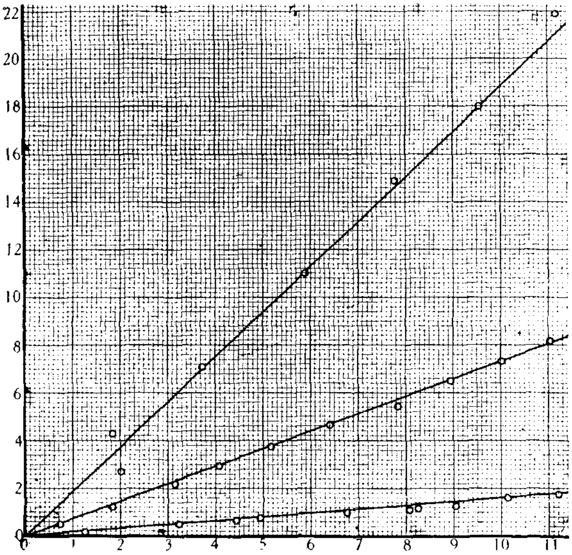

FIGURE 7. Effect of pressure, at 20°, on the induction of slightly carbonized iron under constant fields of respectively 0.52, 4.69 and 15.6 Gauss, reading from the top down. Ordinates are $-(\Delta B/B_0)$ in per cent, abscissae pressure in thousands of kg./cm^2.

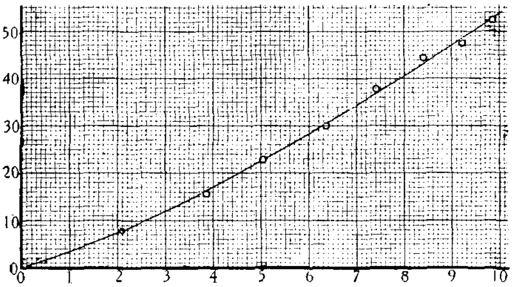

FIGURE 8. Effect of pressure, at 20°, on the induction of nickel under a constant field of approximately 2 Gauss. Ordinates are $+(\Delta B/B_0)$ in per cent, abscissae pressure in thousands of kg./cm^2.

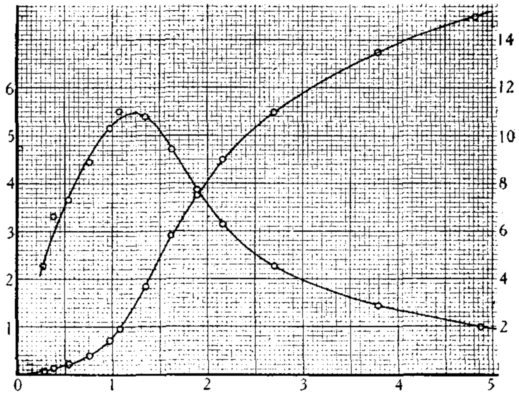

FIGURE 9. Results for pure iron at 20°. Abscissae are H, in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, B in thousands of Gauss.

TABLE III.

CONSTANT FIELD RUN FOR SLIGHTLY CARBONIZED IRON AT H = 0.52

Pressure	Residual Rev. Defl.	$\Delta B/B_0$ (per cent) Uncorrected	$\Delta B/B_0$ (per cent) Corrected
0	2.53		
1850	2.42	− 4.4	− 4.3
2020	2.46	− 2.8	− 2.7
3740	2.35	− 7.1	− 7.1
5880	2.25	−11.1	−11.0
7770	2.15	−15.0	−14.9
9540	2.07	−18.2	−18.0
11180	1.97	−22.2	−21.9

NOTE: The straight line passing through the origin and fitting well the points corresponding to high pressures passes between the points corresponding to 1850 and 2020 kg./cm.^{2} The irregularity at low pressures is evidently due to difficulty of accurate measurement.

TABLE IV.

CONSTANT PRESSURE RUN FOR PURE IRON AT P = 7510 KG./CM.^{2}

H	Total Rev. Defl. at $p = 0$	Total Rev. Defl. at $P = 7510$	$\Delta B/B_0$ (per cent) Uncorrected	$\Delta B/B_0$ (per cent) Corrected	$\Delta B/B_0$ (per cent) for 1000 kg./cm.^{2} Corrected
0.27	.48	.40	17	17	−2.27
0.38	.80	.60	25.0	24.8	−3.31
0.54	1.41	1.02	27.7	27.5	−3.67

TABLE IV.—*Continued.*

H	Total Rev. Defl. at $p = 0$	Total Rev. Defl. at $P = 7510$	$\Delta B/B_0$ (per cent) Uncorrected	$\Delta B/B_0$ (per cent) Corrected	$\Delta B/B_0$ (per cent) for 1000 kg./cm.^{2} Corrected
0.76	2.62	1.74	33.6	33.4	−4.45
0.97	4.72	2.89	38.8	38.6	−5.15
1.08	6.38	3.73	41.5	41.3	−5.51
1.35	12.31	7.33	40.5	40.3	−5.38
1.62	19.41	12.50	35.6	35.4	−4.72
1.89	25.10	17.81	29.1	28.9	−3.86
2.16	30.00	22.93	23.7	23.5	−3.14
2.70	14.37*	30.29	17.2	17.0	−2.27
3.79	17.63*	15.71*	10.9	10.7	−1.43
4.87	19.58*	18.10*	7.57	7.42	−0.99

The remaining data are given in Tables V to IX, and in figures 10

TABLE V.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR SLIGHTLY CARBONIZED IRON.

H	B	Pressure Coefficient
0.26	244	−1.30
0.52	1048	−1.88
0.62	1102	−2.14
0.73	1566	−2.36
0.78	1915	−2.66
0.83	2170	−2.43
0.93	2800	−2.38
1.04	3640	−2.19
1.30	5480	−1.89
1.56	6020	−1.71
1.82	7050	−1.46
2.08	7620	−1.27
4.69	11200	−0.73
15.6	16580	−0.16
52.0	18760	−0.01

TABLE VI.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR .10% C STEEL.

H	B	Observed Pressure Coefficient
0.57	300	−0.57
1.13	836	−0.95
1.42	1292	−1.28

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* Constants of circuits changed.

TABLE VI.—*Continued.*

H	*B*	Observed Pressure Coefficient
1.70	1928	–1.59
1.98	2695	–1.69
2.26	3570	–1.57
2.83	4870	–1.52
3.40	6030	–1.33
3.96	6890	–1.15
4.53	7640	–1.06
5.15	8390	–0.97
5.66	8910	–0.93
11.32	12210	–0.58
16.98	13960	–0.38
22.64	15000	–0.25
28.30	15660	–0.18
33.96	16170	–0.10

TABLE VII.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR .30% C STEEL.

H	*B*	Observed Pressure Coefficient
0.69	263	–1.66
1.24	614	–1.80
1.48	851	–1.88
1.73	1163	–1.99
1.98	1588	–2.05
2.47	2565	–2.30
2.96	3610	–2.36
3.46	4560	–2.16
3.96	5510	–2.02
4.50	6440	–1.84
4.94	7090	–1.70

TABLE VIII.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR NICKEL.

H	*B*	Observed Pressure Coefficient
.223	33	+ 2.5
.446	76	+ 3.2
.624	118	+ 4.2
.892	210	+ 5.4
1.115	303	+ 5.56
1.338	402	+ 5.62
1.561	503	+ 5.34

TABLE VIII.—*Continued.*

H	B	Observed Pressure Coefficient
1.784	603	+ 5.22
2.01	692	+ 4.97
2.23	775	+ 4.76
4.46	1478	+ 4.57
6.69	2000	+ 3.87
8.92	2480	+ 2.82
11.15	2910	+ 2.32
13.38	3300	+ 1.82
15.61	3640	+ 1.37
17.84	3930	+ 1.10
20.07	4170	+ 0.78
22.30	4360	+ 0.71

TABLE IX.

OBSERVED PRESSURE COEFFICIENTS OF MAGNETIZATION FOR COBALT.

H	B	Observed Pressure Coefficient
10.9	380	− 0.2
16.3	668	− 0.2
21.8	981	− 0.2
27.3	1278	− 0.2
32.8	1562	+ 0.090
41.0	1942	+ 0.39
45.2	2130	+ 0.27
48.0	2250	+ 0.26
56.1	2580	+ 0.25
64.3	2885	+ 0.30
72.4	3165	+ 0.38
79.5	3420	+ 0.41

to 14; the pressure coefficients are given as before as percentage changes per 1000 kg./cm^2. The numbers are obtained from the slope of the best fitting straight line in the case of the constant field runs. When pressure coefficients are deduced from constant pressure runs, use is made of the linear relation between percentage change and pressure established by the constant field runs. On account of the fact that our toroids were not thin, the pressure coefficient thus reported is a sort of average pressure coefficient corresponding to some average field. To reduce these experimental values to true coefficients corresponding to definite values of field involves the solution of an integral equation. The writer had not been able to solve the integral equation, but believes that the data presented in this paper, namely, the *B-H* curves, the observed pressure coefficients, and the dimensions of the specimens, are sufficient for reducing the experimental values to true coefficients when the method of solving the integral equation is discovered.

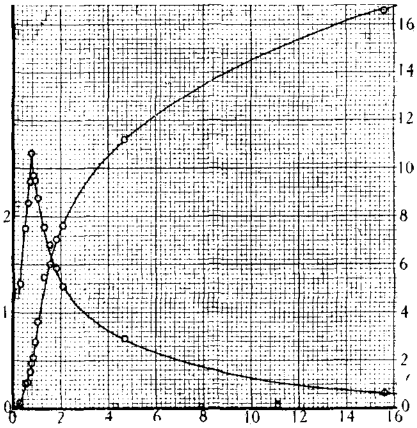

FIGURE 10. Results for slightly carbonized iron at 20°. Abscissae, *H* in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, *B* in thousands of Gauss.

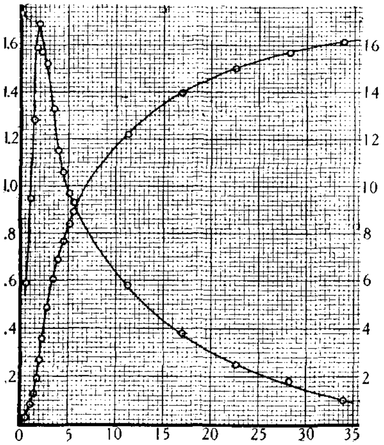

FIGURE 11. Results for .10% carbon steel at 20°. Abscissae, *H* in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, *B* in thousands of Gauss.

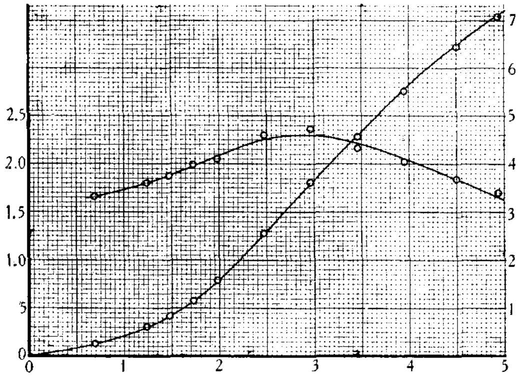

FIGURE 12. Results for .30% carbon steel at 20°. Abscissae, H in Gauss. Ordinates, left hand scale, $-(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, B in thousands of Gauss.

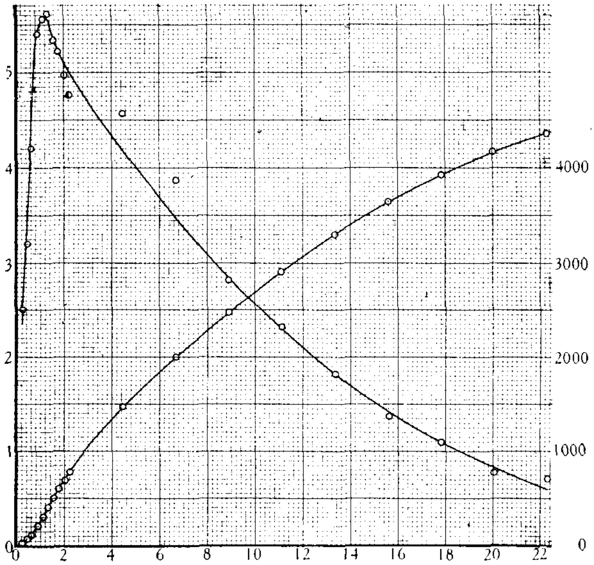

FIG. 13. Results for nickel at 20°. Abscissae, H in Gauss. Ordinates, left hand scale, $+(\Delta B/B_0)$ in per cent per 1000 kg./cm.^{2}; right hand scale, B in thousands of Gauss.

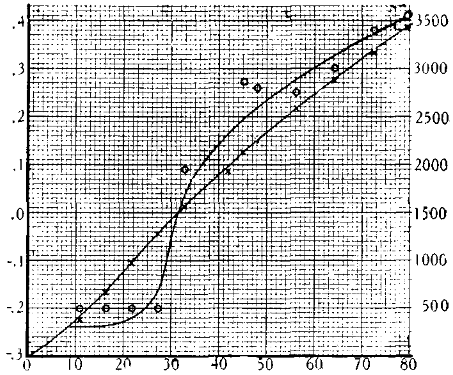

FIGURE 14. Results for cobalt at 20°. Abscissae, H in Gauss. Ordinates left hand scale, $\Delta B/B_0$ in per cent per 1000 kg./cm.^{2} (circles); right hand scale, B in Gauss (crosses).

The Effect of Pressure on the Retentivity of Pure Iron.—From the fact that pressure is able to set free the residual magnetism of iron, it is expected that the retentivity of iron will decrease under pressure. By retentivity as measured here, we mean, when expressed in per cent:—

$$\frac{100\ (\tfrac{1}{2}\ \text{Rev. Defl.—withdrawal deflection})}{\tfrac{1}{2}\ \text{Rev. Defl.}}$$

The withdrawal deflection is obtained by opening the primary circuit. Two kinds of withdrawal deflection are recorded below: one is obtained after many reversals, and the other is obtained after applying and withdrawing the field in one direction several times. We shall call the former cyclic withdrawal deflection, and the retentivity computed from it cyclic retentivity; the latter will be called the minimum withdrawal deflection, and the retentivity computed from it maximum retentivity.

The following data are obtained at $H = 1.62$.

	$P = 0$	$P = 7600$	$P = 8630$	$P = 10340$
1.2 Rev. Defl.	9.74	6.21	2.92	2.46
Cyclic withdrawal defl.	2.36	2.04	1.88	1.68
Min. withdrawal defl.	1.97	1.79	1.79	1.65
Cyclic Retentivity	75.8%	67.1	35.6	31.7
Max. Retentivity	79.8	71.2	38.7	33.0

The run was undertaken merely to obtain some idea of the effect, so only a few points were taken. For some reason the point at $P = 7600$ is bad; there seems to be no reason why there should be a sharp drop of retentivity around 8000 kg./cm².

The Temperature Coefficient of the Pressure Coefficient of Magnetization.—With the data of four magnetization curves, the following four coefficients can be obtained:

(1) temperature coefficient of magnetization under no pressure
(2) “ “ “ “ under pressure
(3) pressure “ “ “ at room temperature
(4) “ “ “ “ at 100° C.

As before, by pressure coefficient we shall mean percentage change per 1000 kg./cm². By temperature coefficient we shall mean the average percentage change per degree averaged over an interval of 75°, from 25° to 100° C. In Table X are presented such a set of coefficients for nickel. In calculating the temperature coefficient under a certain pressure, the value of B at 25° C. and under that pressure is used as the zero level against which percentage changes are figured; in calculating the pressure coefficient at a certain temperature, the value of B under no pressure and at that temperature is used as the zero level.

TABLE X.

Temperature and Pressure Coefficients of Magnetization of Nickel

	Temp. Coef.		Pressure Coef.	
H	At $P = 0$	At $P = 7300$	At 24°.8	At 99°.8
.223	.57% per degree	.647	2.5% per 1000 kg./cm.²	3.1
.446	.595	.618	3.2	3.48
.624	.587	.472	4.2	4.6
.892	.427	.199	5.4	3.04
1.115	.32	.1	5.56	3.06
1.338	.238	−0.053	5.62	2.02
1.561	.172	−0.108	5.34	1.79
1.784	.143	−0.138	5.22	1.59
2.01	.12	−0.147	4.97	1.53
2.23	.099	−0.157	4.76	1.44
4.46	.069			0.63
8.92	.039			0.52

In looking over the table, we are first impressed with the fact that, for nickel, pressure has a large effect on the temperature coefficient. While under no pressure the familiar reversal from positive to negative temperature coefficient does not occur even at $H = 8.92$; it occurs under $P = 7300$ at $H = 1.26$. To confirm these results, two constant field runs at $H = 2.01$ were made:—the run under no pressure gave the temperature coefficient + .118% per degree and that under 8080 kg./cm.^{2} gave — .170% per degree. On the basis of the coefficient — 0.147 for the same field under 7300 (see Table X) the coefficient under 8080 would be — .176. The agreement is fair.

We see that at fields greater than .5 pressure has the effect of decreasing algebraically the temperature coefficient. Since the pressure coefficient is positive for nickel, it is mathematically necessary that temperature has the effect of decreasing the pressure coefficient. This fact is shown in the last two columns of Table X. The reader will also note that $d^2B/dpdt$ is positive at very low fields.

For pure iron, it was found that pressure has the effect of increasing algebraically the temperature coefficient. Since the pressure coefficient is negative for iron, it follows mathematically that temperature has the effect of increasing algebraically the pressure coefficient (i.e. decreasing the absolute value). The data of Table XI illustrate these facts.

TABLE XI.

Temperature and Pressure Coefficients of Magnetization of Pure Iron.

	Temp. Coef.		Pressure Coef.	
H	Under no Pressure	Under $P = 7050$	At 25° C.	At 100° C.
.43	+ .147	+ .177	—3.25	—0.99
.61	.171	.195	—3.93	—2.09
.86	.232	.247	—4.80	—3.69
1.08	.285	.294	—5.48	—4.87
1.30	.263	.264	—5.51	—5.39
1.51	.153	.158	—4.97	—4.65
1.73	.126	.135	—4.36	—3.70
1.94	.097	.106	—3.79	—3.08
2.16	.072	.079	—3.14	—2.61
4.32			—1.20	—1.23

Summing up, we see that the crossed derivative $d^2B/dpdt$ is pre-eminently negative for nickel but positive for iron. The facts seem to indicate that the distance between atoms or molecules is one of the chief factors that control the magnetostriction phenomena. Regard-less of the sign of the pressure coefficient, temperature has the effect of decreasing numerically the pressure coefficient. That is, under the same pressure and the same field, the change of magnetization is less at a higher temperature than at a lower temperature. The underlying reason seems to be that on account of the thermal expansion of the atomic lattice, the same amount of pressure is less effective in producing change of magnetization.

Discussion of the Experimental Results.

For all the fields investigated, the pressure coefficient of magnetization is negative for the different varieties of iron and steel, but positive for nickel. For cobalt, the pressure coefficient is negative, that is, iron-like for fields below about 30 gausses; for higher fields, the pressure coefficient becomes positive, that is nickel-like. The intermediate character of the results for cobalt is very interesting and must have theoretical significance in view of the fact that cobalt is also between iron and nickel when the elements are arranged in the order of their atomic numbers.

When we plot the absolute value of the pressure coefficient against field, the curves for iron, steel, and nickel all show a rather sharp maximum at a certain field. For pure iron, the maximum percentage change is -5.5 per 1000 kg./cm.^{2}, occurring at $H = 1.2$; for nickel, the maximum is $+5.6$, at $H = 1.3$.

It may be asked whether the percentage change also vanishes with a vanishing field. To answer this question experimentally would be a fussy undertaking, on account of the small value of B at very low fields, and consequently, the necessity of winding a large number of secondary turns. From the fact that both permeability and differential permeability (i.e. the derivative dB/dH) are not zero for $H = 0$, it is highly probable that the per cent change also does not vanish for $H = 0$.

With regard to iron, steel and nickel, another interesting question is whether the sign of the pressure coefficient would reverse at very high fields. The question is especially important for iron and steel which exhibit reversal points for both the Joule effect and the Villari effect. For the slightly carbonized iron, at $H = 52$, the percentage decrease was found to be slightly less than .01% per 1000 kg./cm^2. A test was also made at $H = 112$, with pressure put on and released several times. The effect of pressure is unmistakably a decrease of permeability, however minute that may be. The data are, however, still insufficient to answer the question definitely. All we can conclude from the tendencies of the curves and the phenomena of magnetic saturation is that for very high fields, the pressure coefficient of magnetization assumes a very small value, either positive or negative, which is perhaps of the order of the compressibility of metals. The question is fundamentally important and worthy of further attack, either theoretical or experimental. If the limiting value of the pressure coefficient for very high fields were rigorously zero, the saturation intensity of magnetization per unit volume would be independent of pressure, when the change of dimensions due to pressure is taken into account in calculating B and H. Assuming that the atoms are the magnetons, then there would be a decrease of the magnetic moment of the atom under pressure. A conclusion of this sort would involve very important consequences. Incidentally, the reader may note that there is also no experimental data on such a problem as the saturation intensity of an iron rod under tension.

With regard to iron, the data of the present research definitely contradict the positive pressure coefficient of magnetization obtained by Miss Frisbie at low fields. In the earlier course of the work, what Miss Frisbie observed was indeed also observed by the author. The cause was later traced to imperfect demagnetization, the pressure being able to set free a part of the residual magnetism of iron. When demagnetization was perfect, the erroneous effect also disappeared. The reversal obtained by Miss Frisbie might also be due to imperfect demagnetization.

THEORETICAL CONSIDERATIONS.

Thermodynamically, the effect of hydrostatic pressure on magnetization is reciprocally connected with the volume change produced by magnetization, so that from the results of the present research, conclusions with regard to the latter effect can be deduced with a reasonable degree of certainty. But the reader must carefully note that we cannot draw any necessary conclusion in regard to the other types of magnetostriction phenomena. The following classification clearly illustrates the meaning of this statement:—

A. Effect of stress on magnetization	B. Strain due to magnetization
1. Effect of pressure (hydrostatic) on magnetization	1. Volume change due to magnetization
2. Effect of tension on magnetization (Villari effect)	2. Length change due to magnetization (Joule effect)

3. Reciprocals of Wiedemann effect (a) Transient current effect (b) Longitudinal magnetization-effect	3. Wiedemann effect

This classification presents clearly the theoretical connection between the various phenomena. Those in the same horizontal line are reciprocals of each other and are connected by thermodynamics; while those in the same column are connected by mechanisms deeply rooted in the nature of elasticity and magnetism.

The phenomena in the above list are quite complicated. They depend not only on the nature of the magnetized substance, but also on the geometrical form of the specimen. The description of these phenomena is out of place here. In view of the confusion that exists in the literature with regard to the theories of magnetostriction, it will be worth while to outline the general theoretical methods.

There are two methods of treating magnetostriction. The first method may be called the elasticity method. It consists of first finding the force per unit volume acting on the matter when placed in a magnetic field. This is accomplished by equating the change of electromagnetic energy with the work of virtual displacement of the parts against the force. The expression for the body force, in the limit at surfaces of discontinuity, then gives an expression for the surface force acting at the boundary between the two media. The elastic stresses and the strains are then computed by the equations of elasticity, so as to equilibrate the given body and surface forces.

The other method may be called the energy method or the thermodynamic method. The only physical part of the reasoning consists in setting up an expression either for the heat absorbed or for the work done by the matter. All the rest follows the formal work of thermodynamics, assuming reversibility.

Let us now describe the elasticity method more in detail. Let F be the vector force per unit volume acting on matter when placed in the magnetic field. Let X_x, Y_y, Z_z, $X_y = Y_x$, $Y_z = Z_y$, $Z_x = X_z$ be the components of the elastic stress. Then the equations of equilibrium state:

$$\begin{aligned}
\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} &= -F_x \\
\frac{\partial Y_x}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Y_z}{\partial z} &= -F_y \qquad (1) \\
\frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y} + \frac{\partial Z_z}{\partial z} &= -F_z
\end{aligned}$$

The strains are then computed from the stresses by the familiar relations of elasticity involving the elastic constants.

In the literature, another set of stresses has been introduced, namely, that in the ether. We will denote the components of this set by A_x, B_y, C_z, $A_y = B_x$, $B_z = C_y$, $C_x = A_z$. The assumption made by Maxwell and others (6) is simply that

$$\begin{array}{lll} X_x = -A_x, & Y_y = -B_y, & Z_z = -C_z \\ X_y = -A_y, & Y_z = -B_z, & Z_x = -C_x \end{array} \tag{3}$$

This assumption is however not compelling. In the following treatment we will make no use of the stresses in the ether.

With the thermodynamic method, the work amounts to applying, in many cases, only a single general theorem. For an isothermal reversible process, from the two laws of thermodynamics, we know that the work term dA is a total differential. Let $dA = A_\phi d\phi + A_\psi d\psi$, where ϕ and ψ are any two variables that may enter our problem, and $A\phi$ and $A\psi$ are the corresponding coefficients.

If ϕ and ψ are also independent variables, then

$$\left(\frac{\partial A}{\partial \phi}\right)_\psi = A_\phi, \qquad \left(\frac{\partial A}{\partial \psi}\right)_\phi = A_\psi \tag{4}$$

We have therefore

$$\left(\frac{\partial A_\phi}{\partial \psi}\right)_\phi = \left(\frac{\partial A_\psi}{\partial \phi}\right)_\psi$$

The simplest application of (4) concerns the reciprocal relation between the volume change produced by magnetization and the change of magnetization due to pressure. Let V and I be the total volume and the total magnetic moment of a substance under hydrostatic pressure. There will be not only a change of volume due to pressure, but also a change of magnetization due to pressure. The work term consists of two parts: (1) work done by the substance when its volume increases by dV, (2) work done upon the substance when its magnetization increases by dI. $\therefore dA = -HdI + pdV$.

When we take p and H as independent variables, then

$$dA = \left(-H\frac{\partial I}{\partial p} + p\frac{\partial V}{\partial p}\right)\partial p + \left(-H\frac{\partial I}{\partial H} + p\frac{\partial V}{\partial H}\right)dH$$

Whence

$$-\left(\frac{\partial I}{\partial p}\right)_H = \left(\frac{\partial V}{\partial H}\right)_p \qquad \text{by (4)} \tag{6}$$

It is important to note that in this equation I and V mean the total magnetic moment and the total volume of the specimen, not per unit volume or per gram. We now let $I = iV$, where i = intensity of magnetization. Then

$$\left(\frac{\partial V}{\partial H}\right)_p = -V\left(\frac{\partial i}{\partial p}\right)_H - i\left(\frac{\partial V}{\partial p}\right)_H$$

Dividing by V_0, since V/V_0 is practically unity, we have

$$\frac{1}{V_0}\left(\frac{\partial V}{\partial H}\right)_p = -\left(\frac{\partial i}{\partial p}\right)_H - i\frac{1}{V_0}\left(\frac{\partial V}{\partial p}\right)_H \qquad (7)$$

$(\partial i/\partial p)_H$ is the pressure coefficient reported in this paper.

In this deduction, we make no specifications about the form of specimen, so equation (7) is true for any form of specimen provided there is no end effect. This condition is satisfied in the case of a thin toroid.

Let us now see whether equation (7) is in agreement with results obtained in the other way. Since

$$-\frac{1}{V_0}\left(\frac{\partial V}{\partial p}\right)_H = \frac{3}{E}\left(3 - \frac{E}{K}\right) = \frac{3}{E}(1 - 2\sigma)$$

where E is Young's modulus, K, rigidity; and σ, Poisson's ratio, assuming that $(1/V_0)(\partial V/\partial p)$ changes little with magnetization, we obtain from (7) by integrating with respect to H,

$$\left(\frac{\Delta V}{V_0}\right)_p = \frac{3}{E}(1 - 2\sigma)\int_0^H idH - \int_0^H \frac{1}{i_0}\left(\frac{\partial i}{\partial p}\right)_H i_0 dH \qquad (8)$$

This is in agreement with what was obtained by Kolacek by the elasticity method, since the expression $(\partial i/\partial P) + (\partial i/\partial Q) + (\partial i/\partial R)$ in his formula, where P, Q, R are the principal tensile stresses, is exactly our $\partial i/dp$. With (8), we analyse the volume change produced by magnetization into two parts: one part is intimately connected with the pressure coefficient of magnetization, and the other part is still present even when the pressure coefficient is zero. At low fields, the part connected with the pressure coefficient is more important than the other part; but for high fields, the latter becomes increasingly important, since $(1/i_0)(\partial i/\partial p)$ approaches zero asymptotically for high fields.

On the basis of the data of this paper, the volume changes produced by magnetization at fields below 100 gausses can be calculated. But to extend the calculation to much higher fields, further assumptions are necessary.

For iron, whose pressure coefficient of magnetization is negative, both terms work toward an increase of volume. $1/i_0(\partial i/\partial p)_H i_0$ is of the order of 10^{-8}, $3(1-2\sigma)i/E$ is of the order of 10^{-9} for low fields. When the contribution by $\partial i/\partial p$ becomes inappreciable, say after saturation, the volume still increases at the rate of $3(1-2\sigma)i$ saturation$/E$ per gauss. Taking $\sigma = \frac{1}{4}$, $E = 2 \times 10^{12}$, $i_{\text{sat}} = 2 \times 10^3$, this amounts to about 1.5×10^{-9} per gauss.

For nickel, the two terms work in opposite directions. The term connected with $\partial i/\partial p$ will give a decrease of volume. This decrease will become approximately constant when the field is sufficiently high; after that, the volume will continually increase at the small rate of about $.4 \times 10^{-9}$ per gauss, so that the total change of volume eventually passes through zero, and at still higher fields becomes increasingly positive.

For cobalt, there will be an initial increase of volume at low fields, probably too small to be observed. The further course of the volume change will be similar to the case of nickel, being at first a decrease, then a reversal of direction, passing through zero, and eventually becoming positive. Since cobalt has a much greater saturation intensity of magnetization than nickel, the increase of volume at very high fields will be more pronounced in the case of cobalt, and the volume will pass through its initial value after the early contraction at a smaller field.

These qualitative conclusions are in agreement with the facts and the tendencies observed by Nagaoka and Honda (8). The initial volume increase of cobalt has so far not been observed.

Equation (8) gives the volume change produced by magnetization under any pressure. When the i in the first term on the right hand side is i_0, we obtain the volume change produced by magnetization under that pressure. By differentiating (8), we can obtain the pressure coefficient of magnetostriction.

To explain the effect of pressure on magnetization on the basis of the electron theory is not an easy matter. In view of the fact that we yet have no adequate theory of ferromagnetism, it would seem to be premature to attempt any complete explanation of the pressure effect on magnetization. According to current theories, the explanation of ferromagnetism involves at least the following considerations:—

1. The existence of saturation intensity points clearly to the existence of some sort of elementary magnet.

2. Orientation of these elementary magnets must be the principal factor to account for the magnetization curve.

3. The important influences that work against the orienting effect of the external field are thermal agitation and the mutual action between the elementary magnets.

4. It is found that thermal agitation and the orienting effect of the external field are insufficient to explain ferromagnetism. The high permeability of the ferromagnetic substances points towards the existence of some intense molecular field. The nature of this molecular field is yet unknown.

5. The nature of the molecular field presumably depends upon the structure of the atomic lattice and the shape of the atom.

It is this last point that has most intimate connection with the effect we have studied. Conceivably, the existence of both positive and negative pressure coefficients of magnetization is to be explained by the possibility that pressure may either increase or decrease the magnitude of the molecular field, according to differences of its structure. It is interesting to note that while on the one hand we have the characteristic difference between iron and nickel with regard to the pressure coefficient of magnetization, on the other hand we also know that these two metals have different types of crystal structure, iron in the α range being body-centered cubic, nickel being face-centered cubic. Although there is no correlation between ferromagnetism and the type of crystal lattice, it is still quite possible that the characteristic differences in the crystal structure and in the pressure coefficient of magnetization might be consequences of the same primary cause. Attempts to explain ferromagnetism with the molecular field of elementary magnets are so far quantitatively unsatisfactory. The complications in the magnetic behavior of the chemical compounds seem to indicate that the valence electron must play a very important rôle in the ferromagnetic phenomena. This factor, so far, has not been considered in any theory of ferromagnetism. Iron, cobalt, and nickel mark the end of a period, the elements of which, according to Bohr's recent views, suffer a gradual transformation in the inner configuration of the atom. The kernel of the atom is therefore characterized by greater asymmetry. How does this asymmetry influence the valence electron? How does the valence electron influence ferromagnetism? These are interesting problems.

SUMMARY.

1. Within the pressure range 0–12000 kg./cm.^{2}, the change of magnetization at constant H is linear with pressure for iron, cobalt, and nickel.

2. Within the field range 0–100 gauss, the pressure coefficient of magnetization per unit volume is negative for iron, but positive for nickel. It is highly probable on experimental grounds that the sign of the coefficient will not reverse for higher fields.

3. For cobalt, the pressure coefficient of magnetization per unit volume is negative for fields below about 30 gausses, but positive for higher fields.

4. For pure iron at room temperature the percentage change of magnetization has a maximum of -5.5% per 1000 kg./cm.^{2} at H about 1.2. The percentage change decreases quite rapidly on both sides of this maximum, approaching the axis asymptotically for large H. Such a maximum change also exists for nickel, of value $+4.5\%$ per 1000 kg./cm.^{2} at $H = 1.3$.

5. For both pure iron and nickel, (except nickel in very low fields) the absolute value of the pressure coefficient of magnetization per unit volume is less at a higher temperature.

6. For iron, the percentage change of magnetization is a very sensitive function of its carbon content.

7. The retentivity of iron decreases under pressure.

8. By simple thermodynamics, it is shown that the volume change produced by magnetization can be analysed into two terms; the term involving the pressure coefficient of magnetization is important at low fields, while the other term not involving the pressure coefficient becomes increasingly important at high fields.

In conclusion, the writer wishes to voice his gratitude to Professor P. W. Bridgman for his guidance in carrying out this work.

THE JEFFERSON PHYSICAL LABORATORY,
Harvard University, Cambridge, Mass.

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7. **Bridgman, P. W.**—Various Physical Properties of Rubidium and Caesium and the Resistance of Potassium Under Pressure. pp. 385-421. October, 1925. $2.50, with 6 and 8.
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13. **Roebuck, J. R.** The Joule-Thomson Effect in Air. pp. 537-596. December, 1925. $0.50.

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THE JOULE-THOMSON EFFECT IN AIR.

By J. R. Roebuck.

THE JOULE-THOMSON EFFECT IN AIR.*

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This problem is a classical one and is described in all general thermodynamic books, so that only some special points need be considered here. The experimental work so far done on the subject has been excellently summarized by L. G. Hoxton,[1] who also gives a good bibliography. E. S. Burnett's work[2] on carbon dioxide should be added to this list.

The experiment consists in allowing air to expand from a known pressure and temperature through some sort of porous partition to some lower pressure, and in measuring the new temperature assumed by the air. This requires a supply of compressed air, the control of initial and final pressures and of initial temperature, and also the measurement of both pressures and both temperatures. The apparatus used for each of these purposes will be described.

The plan adopted for taking the data is the well known[21] one of dropping from a fixed initial temperature and pressure successively to a series of values of the lower pressure and measuring each resulting lower temperature. The series of pressures and temperatures, including the initial values, are thus the same as would be obtained by a succession of thermometers and pressure gauges along an insulated linear plug. Since the process may be made adiabatic to a close approximation, the only exchange of energy possible is through the pv relations. This is usually stated as

$$u + pv = \text{constant} = h$$

where u is the intrinsic energy and pv the volume energy. This quantity $(u + pv)$ has been called the " enthalpy " and these curves " isenthalpic curves " or " isenthalps." The Joule-Thomson coefficient or porous plug coefficient is then

$$\left(\frac{\Delta T}{\Delta p}\right)_h = \mu$$

where h is the enthalpy, and μ is consequently the slope of these curves.

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Work on this problem has been under way in this laboratory since 1905, first by E. S. Burnett and later by the writer. Many things have interfered, and this report is consequently the result of six or seven years of spare time work. The mere maintenance of a complicated set up such as here described consumed considerable of this time.

THERMOSTAT BATH ARRANGEMENTS.

The apparatus was originally planned by the writer and Mr. Burnett together, and this second construction utilized a number of points which experience had shown to be desirable. It is quite similar to the apparatus used by the writer for porous plug work with water.[3] A vertical cross section of the cylindrical arrangement is shown diagrammatically in Fig. 1. The central part of three superposed cylinders is a single forging of chrom-vanadium steel of the dimensions shown in inches. The outer case is a piece of autogenously welded steel tube which rests on a lead gasket set in grooves at each end and is held in place by screws through riveted-on flanges. The cover is a brass plate held at the center against a lead gasket by a nut running on the central steel tube.

The annular space thus enclosed is divided by a cylinder *aaaa* into two concentric annular spaces. A centrifugal pump *bb* is formed of a group of radial vanes rotating on the steel tube and driven by a gear on a shaft coming down through the cover. The bath liquid is hence forced to circulate through the two annular spaces as indicated by the arrows and the rotation of the pump also serves to displace the fluid continually around the axis. At about 900 r.p.m. the stirring is violent, and the fluid moves rapidly around its path. As a test the kerosene level was set below the top of the flow-dividing cylinder, and under these conditions the stirrer would maintain about 10 cm. difference of level. The bath had to be kept full and the high speed shaft free of protuberances, or air was churned into the fluid and caught by the down flowing column, till the flow almost ceased. An overflow tank was provided.

The compressed air supply entered at *c* in the base. Connections of all kinds were kept off the steel outer case so that it might be easily removed. Inside the bath the air pipe went vertically about half the bath height, then divided into two steel pipes (¼″ inside) wound oppositely in two layers, till at the top they joined each other again and entered the central cylinder at *e*. The coiled pipe in the lower half of the bath was intended for use when working near the saturation

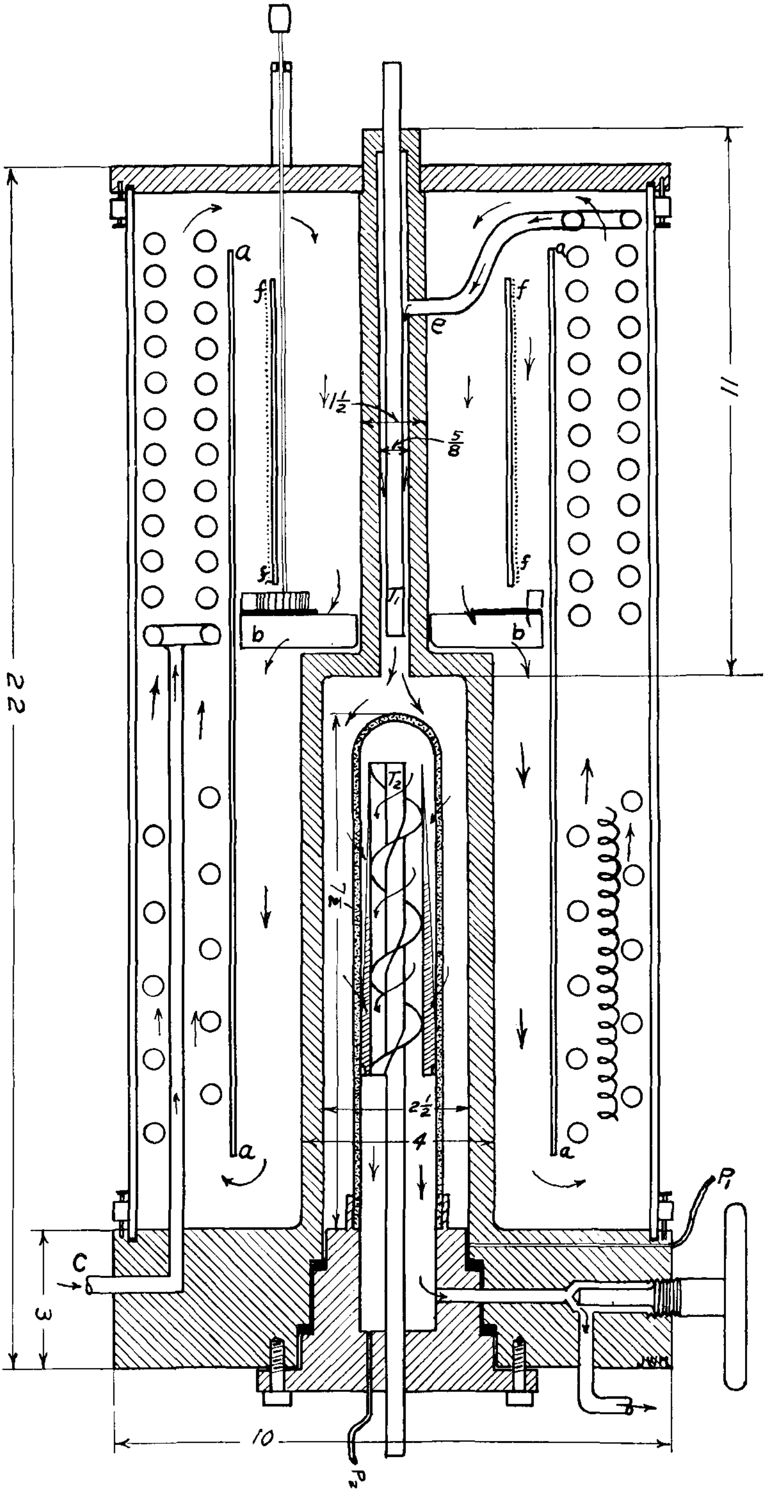
a
f
e
f
b
b
T_1
T_2
$1\frac{1}{2}$
$\frac{5}{8}$
11
22
$7\frac{1}{2}$
$2\frac{1}{2}$
4
a
a
C
P_1
3
10
P_2

FIG. 1. Porous Plug Apparatus.

conditions. The air after passing through the plug could be passed through this lower coil to bring its temperature back to that of the bath. The valves and connections are all in the base. A complete working drawing makes a very complicated picture, since everything has to be readily demountable to permit the almost continuous series of repairs and modifications such work requires.

The heating elements are placed in between the two layers of the lower coil. They are made of nichrome wire wound on glass tubes supported on brass rods forming part of a frame. The three leads go out through the base through mica insulation. The coils are connected so that the two groups may be used in parallel or in series.

The two thermostat coils of nickel wire are wound on the brass cylinder *ffff*, and the four leads go out the top of the bath. The top of the central cylinder is fitted with a union for holding the thermometer case. The coil of the thermometer is thus in the air just before it enters the plug chamber. In most of the work, a steel spiral flow guide was placed in the annular space (diameters 3/8″ and 5/8″) about the thermometer. When difficulty arose at the higher temperature part of the work, this spiral guide was replaced by a thin brass tube into which the thermometer case slipped neatly. Bare copper wire, no. 30, was wound into a coil about 1/8″ outside diameter and the wires were spaced three or four wire diameters apart. This coil was then wound gently around the brass tube, fluxed, and dipped in molten lead. The result was to fill the annular space about the thermometer, through which the air flows, with a net of heat conducting wire loops. The thermometer then responded much more rapidly to change in air temperature, the difference being quite noticeable in the ordinary routine of observations. In case the outer wall is at a slightly different temperature from the air-flow, the temperature read on the thermometer will be but little influenced by this wall difference—at least to a much less extent than with the previous flow guide. Such an arrangement might be an advantage within the plug, to aid the mixing and averaging, and to speed up the response. The increase of viscous friction, and therefore of pressure drop, would, however, certainly introduce other difficulties.

Two small holes were bored horizontally through the base into the central cavity to serve as pressure leads for the barostat and the manometer.

The plug is supported within the large cavity by another chrom-vanadium steel forging. Contact is made on the double shoulders, each supplied with a lead gasket, and they are held together by eleven ¾″ hard steel bolts. The upper gasket serves to retain the compressed air in the chamber outside the plug and the lower gasket to retain the air at the pressure inside the plug and so to force it over into the base proper and to the valve in the line there. This valve was used for restricting the out-flow and raising the pressure inside the plug to the desired value. To require two gaskets to come tight independently is usually not a good arrangement. Here the lead was compressed very severely by the bolts and was retained by good fitting joints. Both sides of the joints being of the same metal, temperature changes did not spoil the tightness. Very small leaks were of no moment for the present purpose, and noticeable leakages gave only a small amount of trouble. A coat of graphite in oil made everything come apart readily after being at the high temperature. All the oil evaporates, but the graphite prevents any sticking. Threaded holes were cut in the flange and a pair of bolts served to push the pieces apart.

The removable part was provided with unions for supporting the plug and the thermometer. Either thermometer could be used in either place. A hole through this part into the chamber at the base of the plug served as a low pressure lead. Tests in the early part of the work showed that while the air was flowing, the pressure in the flow guide was about 7 cm. of mercury lower than in the larger cross section of flow in the plug just below the flow guide. This is small enough not to be of moment except possibly where the pressure drop is small as well as the flow large, and at low pressure—a combination very seldom used—so that it is sufficient to measure the pressure in the chamber at the base of the plug.

INTERCHANGER.

The air escaping from the porous plug is very near the bath temperature. When this temperature is materially different from room temperature it aids greatly in convenience and cost to use this outflow air to bring the inflowing compressed air toward the bath temperature. The compressed air was led through a group of 14 copper tubes in parallel, wound into a coil with the turns separated by copper wire, and the return air was led over the outside of this copper tube. The return air at 200° C. gave up so much of its heat to the compressed air, that the returning air escaped from the interchanger at only 10–15° above room temperature. This lessened very greatly the already severe load upon the heating coils and consequently assisted materially in the bath regulation.

TEMPERATURE CONTROL.

At the beginning, it was hoped that it would be possible to extend the readings over a considerable range of temperature. Thermostats using volume changes are difficult to extend over any considerable range of temperature, as the filling liquids boil or freeze. If a gas, like air, be used, the sensibility is lowered though the temperature range may be greatly increased, but such thermostats are at best cranky, unreliable pieces of apparatus, requiring continual attention

Thermostatic regulators using electrical properties—resistance or thermal e.m.f.—have been described frequently in the literature.[4, 5] It was attempted to incorporate in one of these electrical control devices the principle of the Gouy regulator.[6]

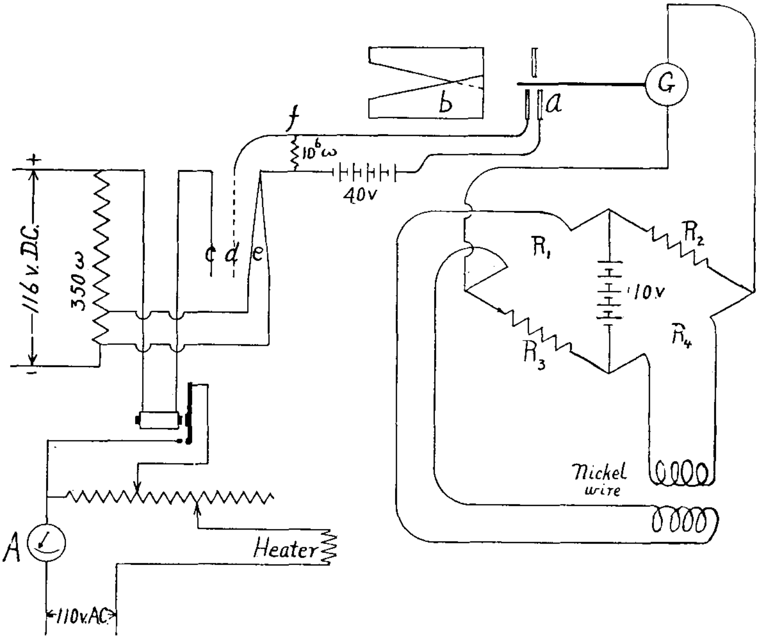

FIG. 2. Arrangement for Temperature Control.

The final form is shown in diagram in Fig. 2. The Wheatstone bridge has arms of about 1000 ohms each and two of its coils R_1 and R_4, are made of very pure nickel wire, 0.003″ in diameter. The nickel wire is some special wire drawn by Driver-Harris Co. from ingots supplied by M. A. Hunter of Troy, N. Y. If the bridge be undisturbed, the setting will return repeatedly on different days, even around 300° C. to usually within less than 0.1° from the previous setting. It has proved very greatly superior to the copper wire which was used at first.

The first coil is wound on thin mica, the bare wire being carefully spaced, coated with Bakelite lacquer, and baked. The second coil is wound on mica laid over the first, lacquered, baked, and covered with mica for protection. The ends of the coils are lead soldered to terminals insulated with mica, and the leads are insulated with glass tubing. Outside the bath, the leads are very carefully insulated with paraffined glass or hard rubber. This careful insulation improved the regulation very greatly.

The resistance R_2 and R_3 are ordinary resistance boxes having coils of manganin wire with brass blocks and plugs for contacts. Since steadiness during a fixed setting was all that was desired, these proved satisfactory. In regular use, the 110 volt D.C. supply was connected directly across the bridge. When working above room temperature, the energy input from the coils in the bath (about 5 watts) was not objectionable, but it made it more difficult to regulate close to room temperature, and it may easily become serious when the attempt is made to go to very low temperatures.

The galvanometer G is a cheap D'Arsonval type of instrument made for student use. It has an iron core held inside the circular coil. The fields were remagnetized and the coil rewound to lessen the magnetic control. A helical spring was put in to save the upper suspension and a pointer of flattened wire terminating in a short piece of platinum wire was attached to the coil in place of the mirror. A good many more expensive instruments were tried out, but when loaded with this pointer and the auxiliary parts, their sensibility was invariably well below that of this rather crude instrument.

Attached to the front of the galvanometer case were the set of jaws represented in vertical section at *a* and in front view at *b*. Both jaws were driven from eccentrics, so that they gripped the platinum pointer without displacing it vertically or disturbing it more than could be avoided. The needle stopped the motion of the jaws till the eccentrics, proceeding with their motion, separated the jaws again. The upper edges of the lower jaws were made of platinum wire, so that the needle made contact between them. The latter were adjusted so that contact was made first at one jaw and then at the other, the second contact bending the flexible needle. Any sparking on contact took place, therefore, at the second contact and the spring of the needle provided sufficient force to separate them. Sticking of these jaws practically never gave trouble.

Continual trouble was experienced, however, because the touching together of the platinum surfaces failed to make electrical contact. If both contact parts were scraped clean, contact would be made without fail for a few hours, then contact would fail with increasing frequency till the thermostat operation was ruined. Voltages up to 10 volts did not help and higher voltages caused too much arcing. The force across the contact also had very little influence. It is possible that a high frequency discharge in the neighborhood would have helped as with the coherer in the early wireless work. At Professor Terry's suggestion, a three electrode vacuum bulb was set up and the galvanometer contact was used to put a 40 volt stopping potential on the grid. The charge carried across the contacts is only that needed to charge the grid, so that the contacts do not stick from arcing and the 40 volts are sufficient to force the required charge across any contact resistance that has yet appeared. The contact has never been observed to fail though they have not been cleaned for a year at a time.

The details of the wiring are given in the diagram, Fig. 2. The 110 volt D.C. is connected between filament and plate, while the filament is heated by shunting it across part of a resistance which is also carrying the 110 volts. The 40 volt stopping potential is connected between the filament and grid, making the grid negative. When the galvanometer circuit was opened, the charge on the grid took about a second to fall sufficiently to operate the relay. This lag was cut to unobservable size by connecting a resistance of a megohm between filament and grid, and this did not increase the charge moving across the contacts sufficiently to affect the sticking.

The small space current of 5 to 8 milliamperes has to operate the relay. As very large resistance can be used without affecting this current materially, the magnet in the relay was wound with No. 40 enameled copper wire till the resistance was about 20,000 ohms. The moving part of the trip is set on knife edges and balanced carefully, so that it operates rapidly and with great certainty.

The relay is used to lift an iron-tipped copper link out of mercury while an auxiliary contact operates a pilot light. For breaking large currents it is necessary to enclose the contacts in an atmosphere of coal gas.[7] The scheme of the arrangement is shown in Fig. 3. A small gasometer was made of two flat bottom cylindrical glass vessels and a heavy lubricating oil was used as the sealing liquid. The electrical connections, the gas inlet, and the trip arm were all carried down through the oil and up inside the central vessel. Coal gas was bubbled out through the oil till the air was practically all removed. This cut down the arcing till 32 amperes could be broken every twenty seconds. The mercury did not foul or become pasty, but the whole interior became covered, after many hours of operation, with a gray coat of finely divided mercury.

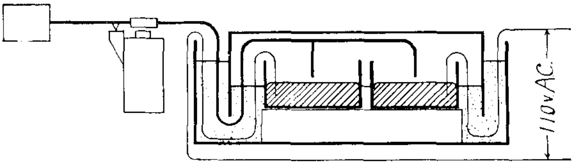

FIG. 3. Trip.

The scheme of the heater control is shown in the lower left of Fig. 2. The heater resistances in the bath were in two groups which could be used in series or parallel. Water cooled resistances, ammeters, and trip were so connected through switches that the heating current could be varied from 0.1 to 65 amperes and the trip could be used to interrupt the whole current or to short any desired part of the external resistance.

With this description in mind, one can go back and discuss the mode of operation of the thermostat. The two jaws shown at *a* and *b*, Fig. 2, approach and recede from each other with a close approximation to simple harmonic motion. The bridge is so adjusted—current and galvanometer—that a rise of temperature of the nickel coils throws the needle to the right. When the jaws are resting on the needle, the stopping grid potential is on, the space current through the tube is stopped and the trip is up. A movement of the needle to the right thus lessens the fraction of the 20 seconds during which the larger heating current is flowing and so allows the bath temperature to fall. With the full sensibility on and a rather carefully adjusted heating current and generally steady conditions, the needle settles down and moves sideways only a few mm. to make the needed continual adjustment. Normally, during a run the heating current requirements lessen slowly, due to the gradual warming up of insulation and outlying parts. The needle consequently tends to drift toward the right, and the necessary rheostat adjustments had to be made occasionally to keep the needle near the middle of its swing. Under these conditions, a thermometer reading to 0.1° C. with rather large divisions gave usually no indications of change of temperature over hours at a time at any temperature between 25° C. and 250° C., which latter was the upper limit of our set of fine mercury thermometers. In a few instances, the temperature as read on the mercury thermometer drifted slightly after it had had sufficient time to become steady. Such drift was very readily offset by slight adjustments of a slide wire included in the thermostat bridge. This slide wire also facilitated the precise adjustment of the bath temperature to any predetermined temperature. In its final form, which covers most of the data presented here, the thermostat controlled the temperature of a point in the bath well within the requirements set by the other experimental difficulties. This requirement is, of course, made lighter by the rapidity with which the second thermometer T_2 follows the variations of the inlet thermometer T_1. It has the effect of reducing the bath variations as they appear in the observed ΔT and consequently the need for smooth regulation. In the latter part of the work and at 240° to 300° C. the ΔT was read to 0.001° and the last figure has some significance as being the actual ΔT, though the experimental uncertainty in the ΔT, for reasons quite distinct from steadiness of bath temperatures, was very much greater than this.

To save time, the working temperature was reached by use of the 65 amp. current, giving 5 to 12° C. per min. rise, depending on the temperature. On approaching the working temperature, the rate of rise was cut gradually to nearly zero, and the thermostat put in control. It normally assumed its duty without any tendency to hunt. This was usually managed by hand regulation, but if the voltage on the bridge was cut from 110 to 20, and the rate of rise was not too violent, the thermostat also took up its duty without hunting. The lack of sensibility as shown by the chance excursions of the bath temperature disappeared as the applied voltage was raised. At the same time, this increase in voltage increased the heating in the nickel wire coils and hence the bath temperature shifted slightly and became steady at a new value.

In most thermostats, as here also, the temperature control is accomplished by making the rate of energy input a very rapid function of the temperature change. To meet variation in energy demand, a change of temperature is required, which may be made small enough, however, to be experimentally negligible. Such high rates give very unstable conditions and hunting becomes very difficult to avoid.

The hand regulation introduces a new principle, in that it allows a wide range of energy input without a corresponding temperature shift. When so supplemented, a relatively insensitive thermostat may be used. This hand regulation may readily be made automatic by utilizing the inequality in the on and off periods to shift a rheostat in the heater circuit, to bring them back toward equality. This would take care of slow variations in the energy demand, while the on and off arrangement takes care of rapid adjustment. It would be possible to rely entirely on this inequality for regulation, but when so used it would probably set up long period hunting. An apparatus for making this automatic was designed and working drawings made some years ago, the desire at that time being mainly to improve the constancy of temperature. As this end was gained by the use of nickel wire and good insulation, it was not built. It offers possibilities, however, in the control of refrigeration in the low temperature range, and it may become advisable to build it.

Thermometers.

The temperature of the thermostat bath was set and followed by the use of two groups of mercury-in-glass thermometers. One group running from 0–200° C. was made up of low priced bath thermometers. The other group running from 0–250° C. was made up of very fine, "normal" instruments. All read to 0.1° C. They were not depended upon except as indicating instruments. Routine work gave occasional chances for comparison with the platinum thermometers and these mercury instruments proved unexpectedly reliable. For work above 250° the only available indicating instrument was an ordinary 0–360° mercury-in-glass thermometer graduated to 1° C. This merely reduced the convenience of control and does not affect the data.

The temperature measurements reported were all made on platinum resistance thermometers of fundamental interval of about 27 ohms. They were inherited from the porous plug work on water[3] and were much more sensitive instruments than required for the present problem. Accidents happening occasionally made it impossible to take all the readings with one pair of thermometers and they were rebuilt as became necessary. The purest obtainable platinum wire, 0.05 mm. diameter, is supported as nearly strain-free as possible on a mica spindle. The details of the construction have been described in another article.[8] The resistance was read by connecting them through a general mercury-cup commutator to an equal arm Wheatstone Bridge of the regular Callendar-Griffiths type. The box has been described in detail in the article on water.[3] The passage of the mercury through and over the copper cups forced the rebuilding of the coils and their connections. A mercury-lubricated plug connection was set up as described in another article.[9] The coils are supported from the ends of the copper bars, and are made of manganin, supported strain-free and enclosed in separate containers with kerosene oil. The coils have been calibrated repeatedly since construction, and both errors and variations are far within the limits needed.

The galvanometer is a D'Arsonval type used with 2.5 meters light arm, and its sensibility at this distance is about 10^{-9}. It is read by a light spot on a translucent scale.

It is customary to arrange these bridges with the slide wire contact in the galvanometer circuit. This introduces two serious difficulties, namely, the contacts even when made of the same metal and immersed in kerosene, disturb the galvanometer erratically when the contact is shifted; and, secondly, when the bridge is near balance a very slight non-conducting film will stop the current passing, simulating a balance. This condition can be recognized, but it takes time and careful attention.

On the other hand, the bridge condition is just as well satisfied with the slide wire contact in the battery circuit. The objection to it has been that the sparking at the contact will ruin the slide wire. If the contacts are under kerosene this sparking is greatly reduced, and such circuits are largely non-inductive. Small shifts may be made without separating the wires, and for large shifts the battery circuit may be broken elsewhere. This making and breaking of the battery circuit does not disturb the galvanometer if the bridge is in balance and the contact thermal e.m.fs. merely add themselves to the battery potential. With this arrangement the light spot follows the slider shift almost like a direct mechanical connection, and the balance may be very readily obtained and kept. The final balance is made by reversing the battery potential.

In the use of such sensitive bridges, it is a very great convenience to be able to control the sensibility while making preliminary trials for balance. Any shunts or series resistances in the galvanometer circuit always introduce troublesome thermals, and the necessary adjustments of the sensibility change these thermals. The desired sensibility variation may, however, be very readily accomplished by the use of a sliding arm rheostat to vary the potential applied to the bridge by means of a group of series and shunt resistances. A combination of this kind can be readily made which will control the sensibility so that in use the light spot may never be thrown off the scale. In this work, where many things called for more or less continuous attention, the time saved by this little device was very important.

The fundamental intervals of the thermometers were determined in the usual steam and ice baths. Check determinations at different times agree to 0.01%. The sulphur boiling point gave 1.49 for the delta of the wire.

COMPRESSOR.

The compressor is a two-stage water lubricated machine of 30 $ft.^3$/min. calculated capacity, and of 120 r.p.m. speed. It was exceedingly badly designed and as badly constructed. It has been continually repaired and rebuilt with a regular improvement in its steadiness of delivery. This erratic variation in the delivery has been a source of constant difficulty in the pressure regulation. The details of the compressor history are too rich and varied to be attempted here.

The output of air at high pressure passed through coils of copper tubing immersed in water for cooling, through a trap to separate the liquid water which was fed in for lubrication, and finally over solid stick potassium hydrate or sodium hydrate to take out the water vapor and the carbon dioxide. The same purifier is used for the liquid air work and the clearness of the liquid air, as well as freedom of the valve from clogging, shows that this purification is reasonably effective. The substitution of castor oil for the mineral oil used to lubricate the piston rod packing abolished the appearance of oil in the liquefier.

PRESSURE REGULATION—BAROSTAT.

The control of pressure is a very vital element in this work. Not only is it necessary for the pressure to be steady in order to measure it satisfactorily, but variation of pressure results in corresponding variation of temperature throughout the mass of the gas, even that which has passed the temperature equalizing coils and is immediately around the inlet thermometer. Small pressure changes are thus recorded very rapidly on the inlet thermometer and on the differential reading. Shifts in bath temperature of similar magnitude show much more slowly on the inlet thermometer and still more slowly on the differential reading. It is not at all difficult to estimate the performance of the pressure regulator by observing the movements of the galvanometer in the thermometer bridge circuit.

Moreover, such a sudden shift of pressure changes the heat content of the material in the plug chamber, which has then to be restored by the air flow. This much slower adjustment is likely to be only partly completed when the pressure shifts in the reverse direction and the adjustment must begin over again. Practically all the experimenters since Joule's time have referred to the difficulty from slow drifts. The matter may be well illustrated by reference to the points taken early in the work for the 50° curve of Fig. 8, as compared with those of the 25° curve taken lately. The major difference is due to the pressure regulation.

The pressure of a moving fluid may be controlled in one of two ways, i.e., by controlling the supply or by wasting an excess. The former was used very successfully in the work on water[3] already referred to. Naturally it was tried out determinedly here, but has been abandoned because of (1) the difficulty of speed control of a 25 H.P. motor; (2) the small effect of speed changes on the delivery, due probably to incomplete filling of the first stage cylinder; (3) the capacity in the stages introducing a large lag, so that these regulators hunted violently. A reasonably steady compressor might have made some of these arrangements usable.

The method used in taking all the data recorded here used a spill valve to waste the excess compressed air while the compressor was run as steadily as possible.

Fig. 4 represents, in a diagrammatic way, the arrangement used. *ab* is a steel cylinder which is entered through a leather packing by a steel cylindrical shaft *cd*. The lower section of this shaft where it enters the stuffing box is of hard steel, ground and polished to a very true cylinder, close to ½-inch in diameter. The cylinder *ab* is connected through a heavy steel tube to another cylinder, the two serving as a U tube to hold mercury which transmits the pressure entering at *e* to the castor oil above the mercury in *ab*. If the air was allowed to come directly in contact with the piston, the stuffing box lacked lubrication and had to be kept much tighter to prevent the air escaping than to prevent the oil. The air under the high pressure dissolves sufficiently in the oil to spray it all through the system when the pressure is reduced. A tube leading through a valve to an oil reservoir *F*, allowed the mercury levels to be equalized when there was no pressure on the system.

The shaft *cd* is kept rotating slowly by a belt driven from a cylindrical pulley, so that the belt produces very little axial force on the piston and it is only necessary that what axial force there is should remain constant. The piston *cd* is loaded by carrying a tank of galvanized iron hung on a vertical shaft through a ball bearing and

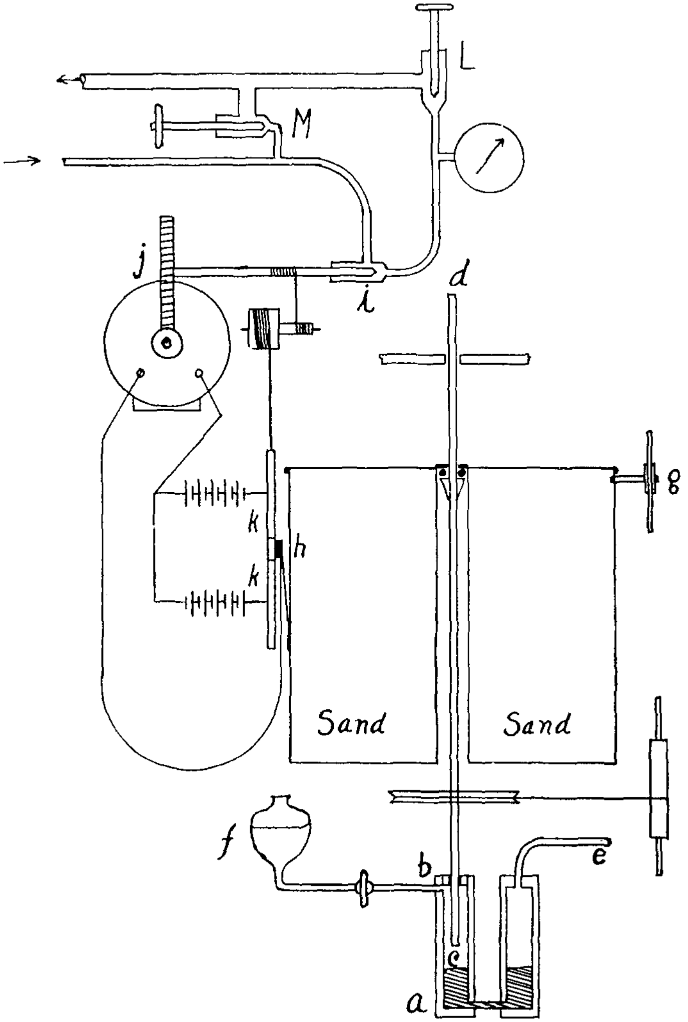

FIG. 4. Arraangement for Pressure Control.

gimbals, so that the load of sand does not rotate and its center of gravity is continually in the axis of the shaft. The tank is kept from rotating by an arm *g* carrying a wheel set between vertical rails. The cylinder and shaft were set carefully vertical. The whole is supported by a pair of pillars held between ceiling and floor, which also supports the electrical control mechanism represented in the upper left of the figure.

A contact block of graphite, *h*, carried by the load tank is connected by a hanging wire to one terminal of the 20 volt, D.C. motor. The two pieces of copper *kk* against which *h* slides are separated by a piece of porcelain, and are each connected through a battery to the second armature terminal. The field of the motor is separately excited so that when *h* touches the upper strip *k*, the motor operates so as to open the valve *i* by the worm gear *j;* and conversely, when *h* touches the lower strip. The two strips *kk* are carried on a frame suspended between vertical guides and supported by a cord which in some cases was wound directly on the shaft of the valve *i;* more generally it went through a multiplying arrangement by which the proportion between shift of *kk* and rotation of the valve could be changed.

Suppose the delivery of the compressor is too large. *cd* rises, carrying *h* into contact with the upper strip *k*. This operates the motor so as to open the valve *i* and to wind up the cord raising the copper strips till the contact *h* goes back on the procelain; conversely, if the compressor delivery is too small. That is, each vertical position of the piston *cd* corresponds to a particular opening of the valve *i*, and therefore a certain amount of waste of compressed air. If the pressure is raised slowly, the piston rises slowly till an equilibrium state is reached and floats at the level required to waste the excess of air being pumped. If, however, the pressure is raised rapidly near the equilibrium pressure, then the lag of the control, due mainly to the inertia of the load, allows the pressure to rise much above the equilibrium pressure, the piston rises to the limit of its path and the waste valve opens to the limit set for it. The waste is therefore large and the pressure falls again rapidly; the piston falls to the lower end of its path and the cycle is repeated indefinitely. It is necessary, therefore, in order to get steady conditions, to approach the equilibrium slowly. This is arranged very readily by using an auxiliary spill valve *M* to waste almost sufficient to take care of the excess when close to the equilibrium pressure. A little practice allows one to get a steady pressure in a few minutes and this is maintained unless some large abrupt change practically reproduces the initial conditions.

At the high pressure it is difficult to get the waste valve *i* adjusted to waste little enough. A very small opening allows a very large quantity of air at 200 atmospheres to escape. To reduce the effect of this too rapid opening, the valve *L* was placed in series with the valve *i* and a gauge was connected with the space between. This arrangement gave complete and ready control of the amount wasted through *i*.

The condition of violent hunting described above arose with all of the arrangements tried, but if the irregularity to be smoothed out was not too large, and the regulator was put in control of only a fraction of the flow sufficient to cover these irregularities and the equilibrium state was set up by some manual assistance, then the piston moved slowly back and forth as required by the irregularities of delivery of the compressor and the pressure regulation became reasonably good. It was greatly improved by the changes in the compressor noted above and also by floating on the line four carbon dioxide cylinders. The effect of this volume capacity was to reduce the pressure variation of a given change in the pump delivery, and so lengthen the period as well as lessen the amplitude of the hunting. These improvements made possible the reading of the temperature difference as shown later to 0.001° C. When it is considered that a 1% variation in the pressure at 250° C. would lead to an adiabatic change of temperature of the air of 1.5° C., it is evident that the pressure variation must have been very slow and very small.

Some considerations advanced by W. Trinks[10] made it appear probable that increasing the stability of the moving piston would improve the regulation. A number of screen-door coiled springs were stretched between the load and the supports above. Hence, as the piston rose, the tension on the springs lessened and the pressure had to increase to support the load. One of these springs was found sufficient to decrease the vertical excursions till they were hardly observable. At the same time, the period fell to 15 to 25 seconds without the four cylinder capacity on the line. This made possible the observation with the thermometers of sufficient of these cycles to fix the average position of balance with great certainty. The only data taken with this arrangement is that for the 25° C. curve of Fig. 8. These are the most consistent data yet obtained, and both from this and from the ease of control, this appears much the best arrangement yet tried.

With regard to this pressure regulator, it may be noted (1) the sensitivity, that is, the ratio of shift of position to shift of pressure, may be set anywhere desired between nearly infinite to values lower than ever desired. On account of the inertia of the load, the pressure shifts during the shift of the regulator, but this persists only during the shift and in the absence of the springs every vertical position of the piston has the same balancing pressure (within the limits of constancy of piston cross section). (2) The energy required to operate the control is drawn almost exclusively from an outside source. (3) Its setting is very steady, that is, what can be called its zero variation is very small. It is of the same order of magnitude as for the rotating piston used to measure the pressure. Consequently, no continual attempt was made to check the steadiness of the pressure by measuring it. Experience has shown that if the piston is holding steady in vertical position, the pressure variation from hour to hour is too small to be measured by any of the ordinary pressure measuring devices save, perhaps, a mercury column. (4) It has a large time lag. (5) It requires a considerable shift in the pressure to shift the piston and load quickly, so that it is not well adapted to take care of large and rapid variation in delivery. The necessity for keeping the load within manageable dimensions restricts the cross section of the steel piston, which limits the obvious way of improving the control over large rapid variations by increasing the volume displacement.

MEASUREMENT OF PRESSURE—MANOMETER.

On account of a series of difficulties, spring gauges are not reliable for careful work, although they are very convenient indicating instruments. Two 3000 lb. Schaffer and Budenberg test gauges of 10″ and 12″ diameter were connected to the low and high pressure spaces during the last part of the work. During most of the work the 12″ gauge could be connected through valves to either high or low pressure. In addition a gauge reading to 50 pounds could be connected to the low pressure so that the difference from atmospheric could be followed closely. While the gauge readings were recorded, as a rule, none are reported. They were used only as indicating instruments to maintain continuous and thorough control over the situation.

The instrument used for measuring the pressure is a modified "dead weight tester,"[11, 12, 13] or free-piston manometer. It was described in detail[14] in an earlier article. Very briefly, it consists of two vertical pistons on the same shaft, one entering a cylinder above and the other one below. The larger pressure was admitted to the lower cylinder and the lower pressure to the upper cylinder and the difference counterpoised by suitable weights. The pistons were rotated slowly while the load did not rotate. The pressure is transmitted through mercury to castor oil in each cylinder. The pistons and connected parts were counterpoised by a weight carrying an iron cylinder which dipped in mercury and served to give the system an equilibrium position. It was always read by adjusting the weights till the pointer set itself at this zero position.

One change was made from the form described in 1913. Then the rotating pistons were driven by cords passing over a grooved pulley on the pistons and over cylinders set parallel with the piston axis. It required frequent adjustment to keep these axes parallel on account of the stretching of the driving cords due to the spring tension. Mr. Burnett pointed out that if the cylinder were on the piston axis and the grooved pulleys in the place of the cylinders, that this adjustment would be continuous and automatic. There was just room enough to make the change. The zero of the cylinders has not shifted since.

The set of weights used were calibrated as described in the previous article by comparing with the State of Wisconsin sub-standards. The diameter of the piston is taken as the value obtained at that time which gave a check to about 0.1 per cent when both this instrument and the open tube mercury manometer were used to measure a pressure of about 34 atmospheres. Most of the readings are of the difference of pressure across the plug, but the high pressure was also read by opening the upper cylinder to the atmosphere. These high pressure readings require, therefore, an addition of one atmosphere to give the actual pressure. This addition has been made in the data reported here.

Flow Meter.

The ΔT measured does not depend directly on the amount of air passing the plug. Possible corrections, however, do depend upon it and control of the experiments is greatly facilitated by such information. A steel block was bored out, tapering both ways to a 5/8″ diameter throat from the 1″ diameter steel tube in which it was mounted and which carried the air from the interchanger back to the compressor. Small holes were bored through the wall at the throat and at the normal size. A ring about the tube at each place served as collector to carry the pressure to a U tube containing water. This meter was calibrated against a good wet gas meter. It was observed, late in the work, that the zero shifted slightly when the compressor was running, even if no air was passing through the apparatus, so that very small readings are particularly unreliable. No particular attention was given to its study, as it appeared quite secondary to the main problem.

PLUG.

The plug used throughout is the radial flow type whose desirable features were described by Burnett and Roebuck,[15] and used later by Roebuck,[3] by Trueblood,[16] by Hoxton,[1] and by Burnett.[2] This plug is a round ended tube of porous porcelain whose length is several times its diameter. Those used were 7½″ long, 1½″ outside diameter, and either 1″ or 1¼″ inside diameter. The edge and about ½″ of the side at the open end were scored in a few places with a file, then rubbed with graphite and a smooth layer of copper half a millimeter thick, plated on the graphite. The copper was lead soldered into a steel collar which was held in place by a hollow nut against a lead or copper gasket. This allowed of ready interchange of plugs and gave no observable trouble from leakage.

The general theory of such a porous plug has been discussed by the authors cited above. In this special case, the external surface can exchange heat only with the fibrous filling of the chamber or with the approaching air. This fibrous packing blocks radiation, being opaque to the effective wave-lengths, and stops convection for even the slow flows, while it makes a trifling addition to the exchange by conduction. The air passing through the plug near its base is led away directly without approaching the thermometer, thus protecting the thermometer from conduction gain from the plug supports. For the major part of the work presented here, the temperature inside the plug is lower than that outside. The directions of the heat flow and of the air flow are therefore alike. Since the drop in pressure is accompanied by a proportionate increase in volume (adiabatic), and the same amount of air must pass every cross section of the flow, it follows that the major part of the pressure drop occurs near the inner surface of the plug wall. If the plug and air had absolutely no heat conductivity, the temperature distribution could be determined from the properties of the air, and the temperature gradient would be much greater near the inner surface. The effect of the conductivity of the air and the procelain will be to move heat forward in the direction of flow. The oncoming air thus drops in temperature faster than called for by its expansion, and its thermal capacity at each point supplies heat for forward conductivity. This limits the fall in temperature of the outer wall which, on account of the smaller pressure gradient in the outer layers, is thus less than might be expected. The disturbance, by the conductivity, of the air-fixed gradient will therefore depend on the heat capacity of the air passing through, being least with the large flows.

The outer surface of the plug is consequently at a lower temperature than either the walls of the enclosure or the fibrous packing or the approaching air. The temperature gradient in the plug wall may therefore be expected to extend out into the fibrous material. Here the heat flow is not parallel to the gas flow because the gas flow has a component directed toward the base of the plug. The result is that heat is transferred across the gas stream, the air entering near the top of the plug acquiring heat at the expense of that entering later. The very poor conductivity of the fibrous packing and the support of the temperature by the heat capacity of the passing air will reduce the temperature gradient rapidly as distance from the plug surface increases. The limit of the guard ring thus appears to be a compromise between guarding against heat gains from the plug support and loss of heat from air being used to air being discarded. Obviously, only a small portion of the air should go through the guard ring; 1/6 to 1/7 was adopted, but allowance must be made for possible uncertainty in the permeability of the wall.

The permeability of the plugs has offered considerable difficulty. The usual fine grained unglazed porcelain is much too permeable for pressures like those used here. The earlier work was all done with plugs whose permeability had been reduced by precipitating barium sulphate from solution right in the wall. Unfortunately, this crystalline precipitate blows out steadily and repeated fillings lead to very erratic performance by the plug, the blowing out probably tending to increase relatively any lack of homogeneity. It was found possible to use a wash with sodium silicate solution to fasten the crystalline precipitate in place, as on drying the sodium silicate goes to a glass. The sodium silicate solution itself made a fairly satisfactory filling material. Plugs so filled change their permeability only slowly. During this work, arrangements* were made for obtaining a set of plugs of suitable range of permeability by burning at a series of temperatures. These have been used for some of the readings and since the permeability is permanent and the most uniform obtainable over the surface, they offer by all means the best solution of the plug difficulty. I have to thank the Rumford Fund of the American Academy for the means for obtaining these plugs, as well as Mr. Montgomery for his interest and care.

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* Montgomery Porcelain Products Company, Franklin, Ohio.

Inside of the plug and extending over about 2/3 of its outside total length is a flow guide (see Fig. 1). This is a steel or brass tube tapered on the outside so that the cross section of flow is about proportional to the quantity of air flowing. The return passage around the thermometer has the same cross section, and the air is forced to follow a spiral path by a strip of metal. This arrangement of cross section assures the same velocity at all points, thus excluding temperature differences due to variation in velocity. The flow guide is held in place and made tight at the contact with the procelain at the lower end of the guide, by packing silky asbestos fibre into the ring cut out of the metal as indicated in the drawing.

The return flow guide was used in the work with water already cited.[3] The conditions were much more definite than with the air, since the temperature difference was controlled almost entirely by the pressure difference. The evidence there was unmistakable that the flow guide made the observed temperature difference much less dependent on the rate of flow and other chance circumstances beside raising it several per cent so as to give a very much closer check with the expected value. The discussion above is in close agreement with this experimental result. If the plug be used without any flow guide inside, the flow of air within the plug is smallest close to the rounded top and increases steadily from point to point down the plug. The air which stays longest in the thermometer zone and has most effect therefore in fixing the thermometer temperature is the air which has been subjected to the greatest gain of heat, while the air which has lost most heat (speaking only of exchange between different parts of the flow) does not pass near enough to the thermometer to influence its reading. The ΔT read is consequently too small.

If, however, the return flow guide be used, almost all of the air passes along the whole length of the thermometer under conditions of considerable turbulence, and hence this error largely disappears. Two experiments on this point were quite conclusive. In the first, use was made of a valve built into the steel forging which carries the plug. It gives passage to the air from the base of the chamber housing the plug to the passage through which the air escapes from the inside of the plug. It was intended to be used to keep the air-flow through the apparatus constant by being opened to offset the retardation produced by raising P_2. This would simplify both the pressure and the temperature regulation by maintaining a constant load. The ΔT

was read with this by-pass valve open and the compressor delivery high but only a small proportion going through the plug. Then this by-pass valve was closed and one outside the bath entirely was used to waste the same amount. The ΔT now measured was greater by possibly 10 per cent. Here, of course, the conditions are extreme. This by-pass was consequently never used.

In the second of the experiments, a piece of cord was wound around the plug in a steep spiral. A cylinder of paper was bound on gently over this. The spiral at the lower end of the plug was closed by bringing the turns of the cord together. The space outside the paper cylinder was filled as usual with cotton. With this a reading was made of ΔT. The spiral was then rearranged to make the block in the flow fall at the level of the division of the flow inside the plug. ΔT was read again with this arrangement, with other conditions the same, and the value of ΔT is materially larger.

PLUG CHAMBER.

As a result of a number of experiments, it appeared to be substantially indifferent whether the air passed through guides outside the plug dividing it at the level of the division inside, or whether the flow space about the plug was filled with cotton wadding and the air allowed to choose its own distribution of flow. In all the experiments it is moving slowly enough on the high pressure side within this large chamber that no great turbulence is present and the flow is probably stream line. In Fig. 5 are given the data for a pair of experiments where the two series of points were taken under these two conditions and the rate of flow differed materially also. The lowest point of the second series is obviously an error; the rest of the points all fall on the curve almost within the error of plotting. In consequence, the data here presented were all taken with the space about the plug filled with loose cotton or long fiber silky asbestos.

A number of experiments were carried out to determine the difference in the ΔT with this cotton insulation present or absent. Fig. 6 gives one case where the two sets of points fall indistinguishably on one curve. This was for a large flow with a maximum pressure drop of only 29 atmospheres giving a temperature difference of 5.5°, while the maximum flow was 8.1 L/sec. This gives a relatively small heat gain to distribute through considerable heat capacity and the resulting effect on the temperature is not observable. Also it should be noted this is true for the whole curve. The curvature is not great, so that in a first approximation the temperature difference can be taken as proportional to the pressure difference. As the lower pressure is raised, the pressure difference becomes smaller and the flow falls off, but not as fast as the pressure difference. In consequence, the relation of the heat gain to the heat capacity passing should result in smaller percentage errors for the smaller ΔT.

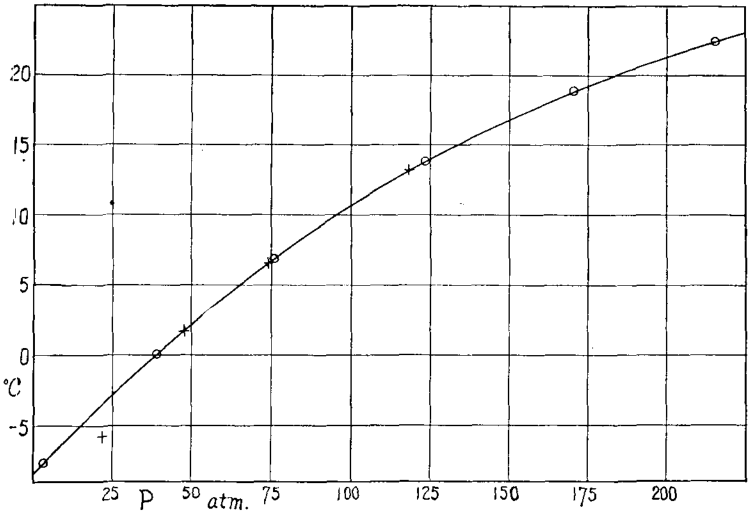

FIG. 5.

The test for an equilibrium condition is that it may be reached from both sides. This second temperature set up by the flowing air should therefore be the same whether approached from a higher or a lower temperature. This was tested repeatedly and in different ways. The set of points forming a curve were determined twice, once with falling and once with rising P_2, as in Fig. 7, Curve A, points ⊙, where the two sets of points all fall on one curve. At another time, the order of taking the points was made erratic without any apparent result on the curve. Again, the point being determined was approached from both sides without any observable difference. This last test served the important practical purpose of proving that the air had been permitted to flow long enough so that the plug interior and its contents had been effectively brought to the temperature of the expanded air. The ordinary proceeding is to follow the ΔT till it becomes stationary but a lingering doubt often remains as to whether a slow drift will not carry the observed ΔT to a value different enough to be of account. This is answered effectively by approaching the value of ΔT from both sides.

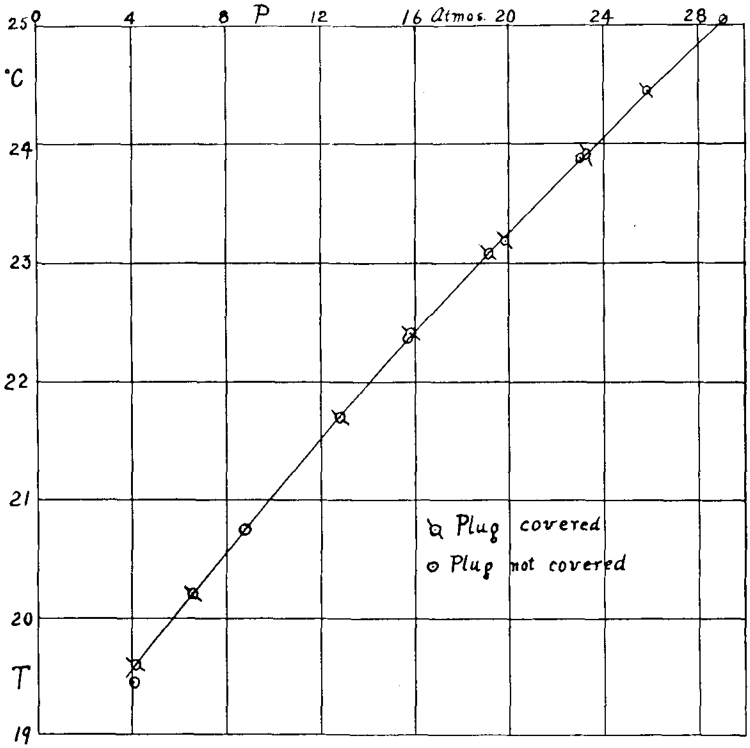

Fig. 6.

Kinetic Energy Effects.

Some observers question whether a considerable fraction of the fall in temperature observed in experiments like these may not be due to turbulence which at the time of passing the thermometer has not become fine grained enough to register as heat. If this should be the case, the observed drop of temperature would depend on the time taken for the gas to pass from the plug to the thermometer. The fact that the data reported here are independent of rate of flow rules out this hypothesis of fine persistent turbulence. It is quite possible, however, that some of the irregularities observed with an occasional plug might have been due to a chance pore being much larger than the average and so giving a streak of violent turbulence which persisted in too large part till past the thermometer. This possibility was tested in one case by comparing the ΔT when the end of the flow guide in the rounded end of the plug was filled with a small wad of cotton, and when not so filled. The ΔT's were closely alike. Had there been any material gross turbulence present, this cotton would certainly have diminished it greatly. Its absence in this case and the general agreement of different plugs makes its presence generally as a material factor very improbable.

The question of the effect on ΔT of the different velocities of flow past the two thermometers has proved a very difficult one, although only of moment where P_2 is less than 10 atm. It is, of course, not difficult to calculate the linear velocity of the air past each thermometer, since the necessary data are readily obtained. But the linear velocity gives no clue to the turbulence of the gross kind which similarly lacks effect on the temperature of the stationary thermometer. This appears quite unamenable to calculation. The velocity past the first thermometer is in all cases too small to affect the temperature appreciably and the same is undoubtedly true of the turbulence. The real problem centers about the conditions surrounding the second thermometer.

The question was attacked from the experimental side in a number of ways. In the preliminary work, where the effort was at first to obtain consistent results, the low pressure P_2 was set at close to atmospheric. That is, the air was allowed to escape from the plug interior with only the rise above atmospheric pressure necessary to force it through the passages. Under these conditions, the velocity past the second thermometer was the largest obtainable and was dependent directly on the quantity of air passing the plug, that is, on the latter's permeability. On account of this high velocity, the ΔT was sensitive to variations in the rate of flow which, not being at first realized, led to much work. When it was realized, the effort to compare individual ΔT's was abandoned, and the regular series of points for each case was taken and the curves compared. This led quickly to very consistent comparisons and to the conclusion that the low end of the curve is its most uncertain region. It is evident also that it is only the low end of the curve about which there is any uncertainty from this cause.

Since the kinetic energy of the air and the turbulence vary some-

TABLE I.

No.	p (atm)	t °C	Flow L/min.	Plug No.
A 1	4.20	19.59	8.1	1
2	6.59	20.27	7.9	
3	9.21	20.89	7.5	
4	12.87	21.69	6.8	
5	15.84	22.35	6.0	
6	19.93	23.22	4.9	
7	23.27	23.93	3.9	
8	23.34	23.92	3.7	
9	25.89	24.43	3.5	
10	19.33	23.09	5.0	
11	16.36	22.46	5.8	
12	29.07	25.04		1
B 1	4.21	19.64	5.1	2
2	3.20	19.33	5.1	
3	8.91	20.76	5.0	
4	15.65	22.27	4.5	
5	29.07	25.04		
C 1	4.19	19.64	5.1	2
2	21.73	23.68	3.8	
3	14.28	21.94	4.6	
4	29.07	25.04		
D 1	18.07	22.86	9.7	3
2	23.76	24.05	5.8	
3	29.07	25.05		
E 1	3.95	19.60	7.7	4
2	8.36	20.70	7.4	
3	12.53	21.62	6.7	
4	17.66	22.72	5.7	
5	23.12	23.87	4.2	
6	29.07	25.04		
F 1	4.20	19.82	4.3	1
2	2.97	19.45	4.3	
3	6.43	20.35	4.1	
4	9.52	21.04	3.8	
5	18.04	22.84		
G 1	4.19	19.84	3.5	1
2	3.32	19.62	3.5	
3	2.85	19.47	3.5	
4	5.72	20.21		

TABLE I.—*Continued.*

No.	p (atm)	t °C.	Flow L/min.	Plug No.
5	7.28	20.80	3.4	
6	12.21	21.58	3.4	
7	14.32	22.04		

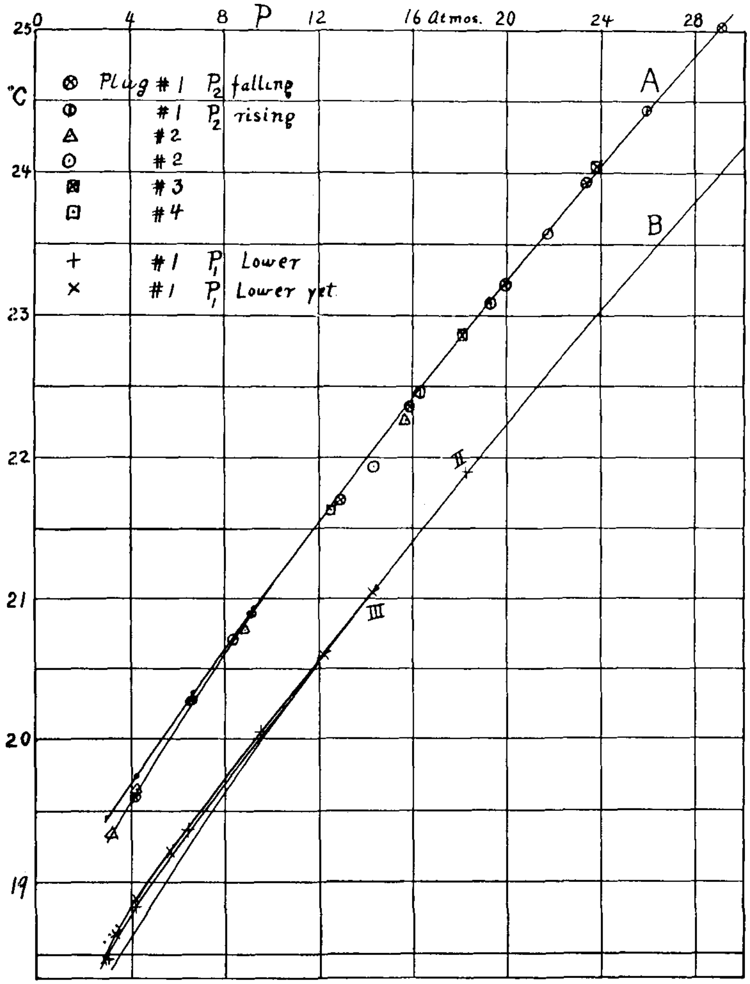

FIG. 7.

what similarly, it was next attempted to plot a series of curves for decreasing velocities of flow, expecting that as the kinetic energy falls with the square of the velocity, the slower flows would have a negligible kinetic energy effect on the ΔT and the curves would become coincident. In Fig. 7, Curve A is a set of points distinguished according to the legend there. The data are given in Table 1. They were taken with four different plugs over which the flow varied by more than a factor of two. The plot is to a large temperature scale and the divergence of the experimental points from the curve is in only one case 0.1° and all the others less than 0.05. The point at 4 atmospheres was taken in all four runs and they fall so close together that only two of them are plotted—and this in spite of the twofold variation of velocity of flow.

At the low pressure end of curve *A* the kinetic energy corrections for the directed velocity were calculated and this gave the slightly raised line diverging from the original curve at 10–12 atmospheres. The corrected curve loses the sharp bend down and now maintains about the same curvature.

Curve *B* is curve *A* transferred down one degree on the scale. Another set of values of T and P were measured using the values at the point II as the initial conditions and one of the plugs used in taking curve *A*. The effect of this decrease of the initial pressure operating across the same permeability is to decrease the rate of flow. On account of the increase of volume resulting from this decrease of initial pressure, the mass flow falls off much more than proportional to the decrease of pressure drop. Hence the velocity inside the plug falls off sharply and the kinetic energy correction decreases with the square of the velocity. The temperature difference across the plug decreased nearly in proportion to the decrease in the pressure drop, and if the flow did so also, the effect on ΔT of the heat gains would remain the same and the kinetic energy effect could be estimated. But since the flow falls off more rapidly, one cannot distinguish the decrease of kinetic energy effect and the increasing effect of heat gains. The third curve was measured using the point III *B* as the initial point and the points are distinguished by appropriate symbols. The envelope so drawn falls a little bit higher than the corrected branch of *A*. How much of the difference is due to heat gains, making *B* high, and how much to neglecting turbulence in *A*, it is impossible to estimate. It is hardly possible, however, that the effect of the turbulence is equal to that of the directed flow, so that the correction remaining is smaller than the correction just made. This is confirmed by the trend of the envelope B where the directed velocity correction is negligible except for the lowest three points.

In another experiment the cross section of flow past the second thermometer was about ½ filled with a metal strip and the divergence from smooth curvature at the lower end compared with a curve taken without this filling. The correction for the filled case is about twice the other. Another comparison, where the passage around the thermometer was about twice the cross section, shows the corresponding variation in the correction. Both of these observations show that the turbulence is not far from proportional to the other factor; and the other facts above show that it is not very large.

Since the correction is as uncertain as a simple extrapolation of the curve, the correction for the lowest 10 atmospheres has not been computed. The correction is necessary only in a very small part of the field covered and will have to be determined by a lot of careful experimental work. By keeping the same length of plug and increasing the cross-section, the velocity can be reduced while the flow per unit area of surface is not changed, and this can be used to separate the effect of velocity of flow and of heat gains. While the question is interesting and important, it has had to be left incompletely solved in this paper.

HEAT LEAKS.

The most important heat leak is that to the air between the thermometers. With a uniform bath temperature and air which has reached that temperature, the magnitude of this leak will be largely determined by the ΔT set up across the plug. With a given ΔT, the effect of this leak on the observed ΔT will be inversely proportional to the heat capacity in the passing air. With sufficiently large flows the effect on ΔT may be quite negligible. But as the flow is decreased with the same ΔT a condition must finally arise where the heat gain error will reach undesirable proportions. Some of the preliminary experiments showed such errors amounting to 3 to 4% with unprotected plugs, and give ground for the belief that with the flows and protection finally used these errors are below 1%.

The question was investigated by taking cooling curves. The air was kept flowing till T_2 became steady, when ΔP was reduced rapidly to zero, and the variation of T_2 with time followed for half an hour. The expansion of the air in the system produced a sharp fall in T_2

and T_1 which was recovered from quickly. The plot of T_2 against time gave a good logarithmic curve. The constant for the curve and the estimated heat capacities involved were used to calculate the rate of input of heat during normal use, and this input distributed over the heat capacity, 54 cal/min., going through, gives about a 2% correction. This is a large flow but on the other hand the heat gains for no flow must be much larger than for a flow, as interpreted by the thermometer. Probably the main path for heat to get to the thermometer in the no-flow state is along the thermometer case. During flow the heat coming in along this case is largely, if not entirely, caught up by the passing air and carried out again. This would mean that the 2% correction above is a maximum not often approached in the experiments. This agrees with the observations reported where the observed ΔT is independent of the rate of flow.

All this discussion serves to emphasize the principal condition which must be satisfied by data of this kind. It must be proved to be independent of the rate of flow of gas. A change of plug is the most effective way of varying the flow and also serves to exclude the effect of irregularity in the plugs. Shift of the initial conditions will serve somewhat less satisfactorily to shift the relation of rate of flow and drop of temperature. In the published work preceding this, such a condition has never been seriously considered. This situation should change and in the future all such data should be subject to this most obvious and most fundamental test.

The *procedure in an experiment* may be outlined as follows: With the whole apparatus in place, the temperature is raised as rapidly as possible to close to that desired. Time is saved if the final step to this temperature is made slowly so that the thermostat may be put into service and take up its duty smoothly. The heating current required gradually falls off for some time as the insulation warms up. When the major part of the rapid change is past so that the temperature control becomes satisfactory, the compressor is started and the pressure raised to that desired. By operating different waste valves, the flow may be gradually established, accompanied by the necessary but gradual increase in the heating, without disturbing the thermostat balance materially. P_1 and T_1 are read and the adjustments made for reading ΔP and ΔT. Conditions are held as steady as possible till ΔP and ΔT have been steady for four or five minutes when the settings are recorded along with the flow reading.

The lower pressure is then changed by manipulating the valve in the outflow from the plug and the heating current shifted to match.

With experience, this can be done without upsetting the pressure or temperature balance. ΔT, ΔP, and flow are again read after they become steady. And so on for the rest of the points. The difference of resistance of the thermometers when they are both at the initial temperature is always measured by immersing both thermometers directly in the bath fluid, and always checks closely with the calculated values.

EXPERIMENTAL RESULTS.

The data for the main group of experiments are given in Tables II to IX. The data in each table are grouped in runs, distinguished by the letters. The last reading of the temperature and the pressure in each run is the temperature of the bath and the initial pressure, respectively. The air flow expressed in liters per minute measured at

TABLE II.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	84.8	8.83	8.6	14
2	98.9	11.42	7.7	
3	113.9	13.74	7.2	
4	127.9	15.81		
5	142.4	17.75		
6	154.9	19.30		
7	167.8	20.79		
8	215.2	25.19		
B 1	2.0	—10.01	7.2	18
2	13.1	— 6.97	7.2	
3	28.1	— 3.11		
4	41.7	+ 0.14		
5	55.8	3.31		
6	69.0	6.04		
7	84.6	9.02		
8	99.9	11.68		
9	114.1	13.92		
10	129.4	16.15		
11	142.8	17.90		
12	154.4	19.40		
13	168.5	20.93		
14	181.7	22.31		
15	215.2	25.20		

TABLE III.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	149.1	44.42	5.7	4
2	168.8	46.32	4.9	
3	184.7	47.69	4.0	
4	215.9	50.10		
B 1	123.3	41.90	6.1	3
2	180.1	47.50		
3	197.8	48.89	4.1	
4	215.9	50.10		
C 1	55.3	31.47	6.8	4
2	84.0	36.16	6.1	
3	113.7	40.30	5.2	
4	140.1	43.42	3.9	
5	141.0	43.51	4.1	
6	180.3	47.40		
D 1	1.9	20.68	5.9	4
2	13.1	23.28	5.8	
3	35.2	27.84	5.5	
4	56.3	31.80	5.2	
5	84.5	36.41	4.3	
6	123.1	41.28	2.8	
7	156.0	45.34		
E 1	1.3	20.47	3.5	10
2	6.7	21.66	3.3	
3	21.1	24.67	2.9	
4	34.7	27.52	2.1	10
5	43.2	29.12	8.3	7
6	51.1	30.65		
F 1	1.9	20.95		10
2	12.9	23.25		
3	28.1	26.38		
4	42.3	29.13		
5	55.7	31.45		
6	76.0	34.93		

TABLE IV.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	97.1	65.61	7.2	3
2	133.2	69.24	5.9	

TABLE IV.—*Continued.*

No.	p (atm)	t °C.	L/min.	Plug No.
A 3	149.8	70.69	5.6	
4	178.3	72.88	4.3	
5	193.3	73.83	3.6	
6	215.7	75.25		
B 1	20.5	53.96	9.2	2
2	44.3	57.93	8.8	
3	70.7	61.77	8.2	
4	107.0	66.21	7.0	
5	215.7	75.25		
C 1	28.7	55.68	7.7	4
2	57.4	60.15	7.6	
3	85.0	63.79	7.3	
4	122.3	67.95	6.3	
5	160.3	71.25	4.9	
6	215.8	75.25		
D 1	1.6	51.02	5.8	4
2	8.0	52.28	5.8	
3	24.3	55.08	5.6	
4	63.9	61.04	5.0	
5	92.8	64.86	4.2	
6	115.4	67.46	3.5	
7	156.0	71.20		
E 1	1.8	51.18		10
2	13.2	53.24		
3	27.9	55.65		
4	41.2	57.86		
5	55.9	59.72		
6	76.0	62.60		

TABLE V.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	28.4	84.92	9.2	2
2	63.1	89.12	8.7	
3	106.0	93.43	7.1	
4	140.2	96.25	5.6	
5	165.7	97.97	4.2	
6	197.3	99.65	3.1	
7	215.7	100.8		

TABLE V.—*Continued.*

No.	p (atm)	t °C.	L/min.	Plug No.
B 1	89.9	92.15	6.9	4
2	125.2	95.16	6.0	
3	162.2	97.75	4.7	
4	215.7	100.37		
C 1	1.6	81.21	2.5	4
2	6.2	81.96	2.6	
3	12.9	82.92	2.7	
4	20.4	83.95	2.7	
5	34.8	85.82	2.6	
6	55.6	88.27	2.2	
7	76.8	90.46	1.6	
8	109.4	93.82		
D 1	1.8	80.90	5.9	10
2	13.1	82.74	6.0	
3	28.4	84.83	5.5	
4	41.8	86.57	4.8	
5	55.8	88.25	3.8	
6	76.0	90.35		

TABLE VI.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	1.1	137.52	4.1	4
2	13.0	138.83	4.1	
3	28.8	140.25	4.0	
4	49.4	142.01	3.9	
5	69.5	143.58		
6	90.9	145.05	3.6	
7	114.4	146.50	3.2	
8	137.1	147.72	2.9	
9	155.1	148.54	2.9	
10	176.9	149.45	2.8	
11	187.6	149.84	2.7	
12	215.1	150.71		
B 1	78.9	144.28	8.6	5
2	112.5	146.36	7.5	
3	142.5	147.94	6.2	
4	166.8	149.00	5.1	
5	215.1	150.71		

TABLE VI.—*Continued.*

No.	p (atm)	t °C.	L/min.	Plug No.
C 1	1.8	137.42	5.7	10
2	6.6	137.96	5.6	
3	20.0	139.26	5.5	
4	34.6	140.60	5.0	
5	51.9	142.05	3.9	
6	76.0	143.97		

TABLE VII.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	120.0	200.81	6.9	4
2	146.5	201.56	5.8	
3	162.2	201.94	5.0	
4	177.2	202.19	4.2	
5	189.2	202.34	3.3	
6	215.4	202.96		
B 1	1.8	195.89	4.6	9
2	13.2	196.62		
3	27.9	197.43	4.6	
4	51.3	198.57	4.4	
5	92.1	200.18	4.1	
6	145.9	201.67	3.5	
7	215.4	202.96		
C 1	2.2	195.79	6.9	4
2	4.0	196.02	6.9	
3	7.1	196.25	6.8	
4	14.1	196.70	6.6	
5	28.0	197.47	5.2	
6	52.0	198.57		

TABLE VIII.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	1.9	249.54	5.0	9
2	11.9	249.97		
3	29.5	250.54	4.8	
4	49.0	251.12	4.7	
5	71.2	251.65	4.5	
6	107.9	252.29	4.1	

TABLE VIII.—*Continued.*

No.	p (atm)	t °C.	L/min.	Plug No.
A 7	143.1	252.66	3.5	
8	175.5	252.85	3.1	
9	191.6	252.82	2.9	
10	215.4	253.17		
B 1	19.9	250.03	9.5	4
2	57.0	251.18	8.8	
3	91.9	251.99	7.8	
4	130.9	252.61	6.3	
5	160.8	252.83	5.0	
6	197.6	252.79	3.5	
7	161.9	252.83	4.8	
8	215.4	253.17		
C 1	2.3	249.45	6.5	10
2	7.0	249.99	6.5	
3	14.0	250.23	6.3	
4	24.5	250.60	5.4	
5	52.0	251.18		

TABLE IX.

No.	p (atm)	t °C.	L/min.	Plug No.
A 1	84.1	282.65		4
2	119.1	282.95		
3	145.9	282.98		
4	183.2	282.83		
5	197.2	282.53		
6	205.5	282.35		
7	215.4	282.76		
B 1	2.2	281.83		9
2	39.8	281.87		
3	68.5	282.51		
4	106.4	282.89		
5	215.4	282.76		

atmospheric temperature and pressure, is given in the fourth column while the last column gives the serial number of the plug used to get the data. The data and curves are referred to for convenience by the bath temperature used at the highest initial pressure.

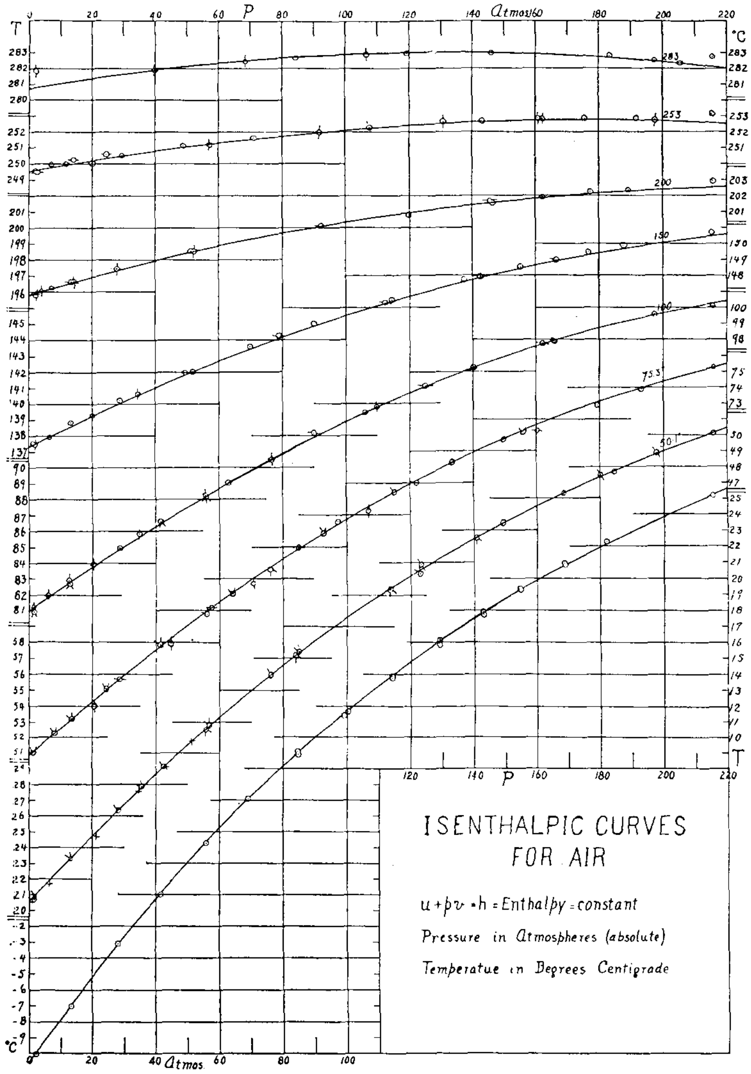

Fig. 8.

This data is plotted in Fig. 8. As far as possible the different points of the different runs are distinguished by different plotting symbols, and the different symbols fall equally well on the curves, excluding therefore any effect on the data due to rate of flow or particular plug. The curves are plotted with breaks in the temperature scale so that a larger scale may be used in order to emphasize the slope and curvature. The curves are drawn to zero pressure with a steady curvature disregarding a few low pressure points which fall below the curves.

The data were taken in the order from low to high temperature except that the early 25° data had to be discarded, so that the 25° curve shown was measured last. It will be observed, both here and in Fig. 12, that the regularity of the data improves in this same order which is due partly to improvements in the compressor, giving steadier delivery, and to improvements in the pressure regulator, both leading to steadier pressure.

Consideration of the curves shows that with rising temperature they grow rapidly less steep and also decrease somewhat in curvature. The slope $(\Delta T/\Delta p)_h = \mu$ is the Joule-Thomson coefficient, and it is consequently a marked function of both temperature and pressure. The effect of temperature is so marked that it has been observed by all workers, but the effect of pressure has been more in doubt.[2, 18]

$$\text{Locus of Maxima, } \left(\frac{\Delta T}{\Delta p}\right)_h = \mu = 0$$

From the form of all the curves, it appears that this slope must go to zero at sufficiently high pressure on any one curve or at sufficiently high temperature at any pressure. Originally it was intended to limit the work to 250° C., and to 100 atmospheres above 200°. But when it appeared that the maxima were within experimental reach, the range was increased somewhat. The two upper curves show the maximum, and the locus of these maxima therefore cuts across the upper right hand corner of Fig. 8. The location of the maximum on curves showing so little curvature is a matter of considerable difficulty. The graphical treatment can be improved by increasing the temperature scale, but only profitably to a limit set by the consistency of the data. Since the experimental conditions were so very steady it was decided to get these points by special experiments.

The values of P and T for maximum were estimated from these curves and P_1 and T_1 for a curve chosen from a point a few atmospheres to the right of this maximum. Using a very permeable plug so that the flow was large for a small pressure drop, a series of P_2 and T_2's were taken where T_2 covered the maximum. The estimates were near enough that all but one of the four points were obtained on the first try. The curves formed by plotting the observed ΔT against Δp are given in Fig. 9, from which the pressure for the maximum (marked with a cross) may be read. The corresponding temperature is calculated from the bath temperature and the ΔT. The bath was pushed up to 300° C. which is quite close enough to 327° C., the melting point of pure lead, which was used to make several joints tight. It will be observed that the curves are consistent to a few thousandths of a degree and P_{max} may be located to about one atmosphere. This shows a remarkably steady state, especially in view of another factor to be discussed later.

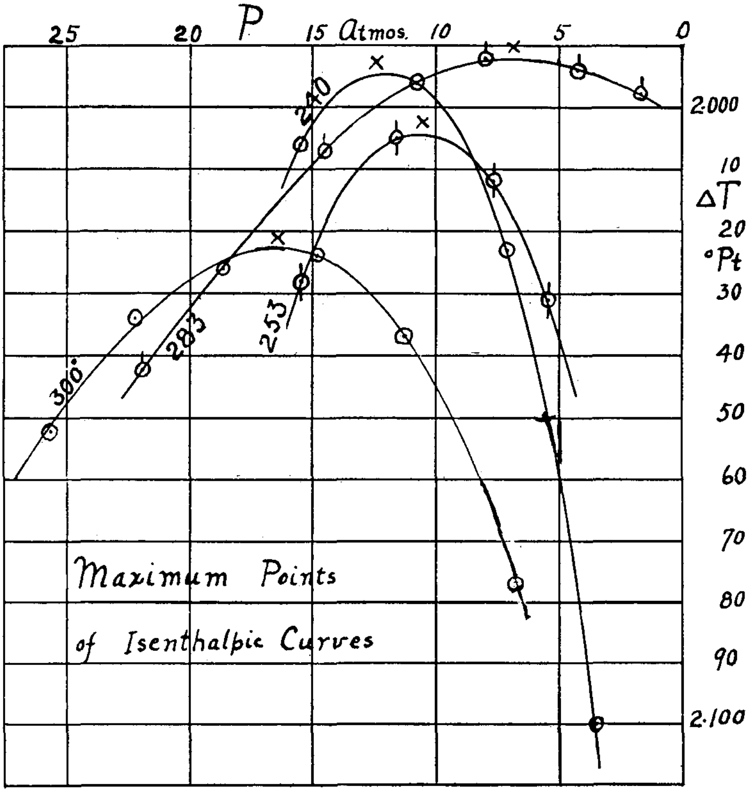

Fig. 9.

The four values so obtained for the shift of the maximum are plotted as small circles in Fig. 10, while the crosses are from the curves of Fig. 8. It is evident that while the accidental errors in an individual experiment are very low, the systematic errors from one experiment to another are very much larger. The circle at 283° has much the slowest flow, so that the other three circles are probably more reliable. It should be observed, however, that these four special experiments were done only once and without any precautions to show that the results did not depend on the rate of flow.

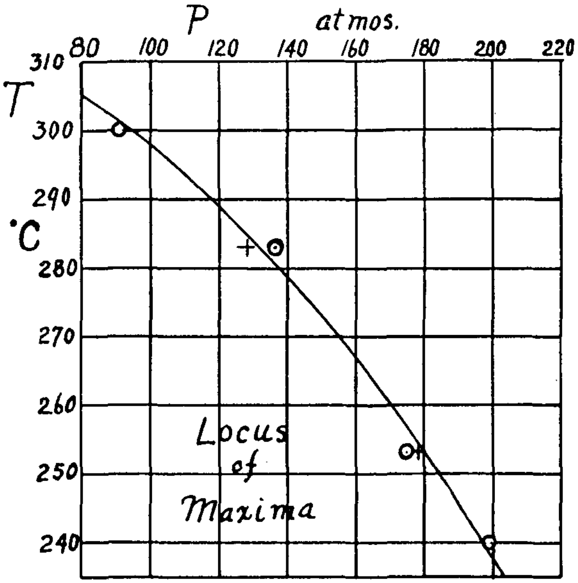

FIG. 10.

The curve of Fig. 10 may be calculated from most equations of state and the comparison of the results with experimental data serves as a peculiarly sensitive test of the equation. Porter[17] and others[18] have carried out such computations for Van der Waals' and Dieterici's equations. Fig. 11 is reproduced here from Noell's article,[18] with the data from Fig. 10 added as circles. To these are added the points lately reported by Knoblauch[19] at — 140° C. at 125 atm. and at — 133° C. at 150 atm. These two groups of points may be connected by a continuous curve somewhat after the manner shown. Both calculated curves are very far from the experimental facts.

Although Dieterici's equation generally gives better correspondence with the properties of the gas, it is the farther off in this case. Noell's extrapolation from his experimental data is also a long way off, which is not surprising in view of the extent of the extrapolation and the uncertainty of the original data. If the extrapolation in Fig. 11 is to be trusted, the whole curve is within experimental reach.

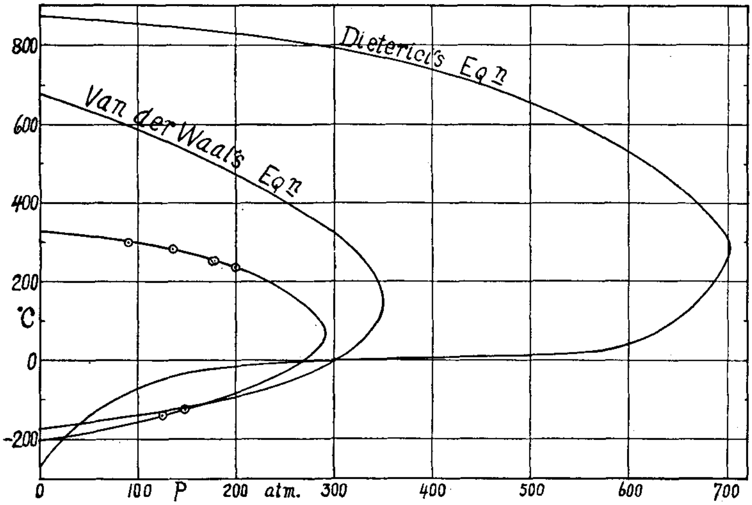

FIG. 11.

Somewhat similar data have been obtained by Olzewski[20] who measured an integrated zero-cooling effect for hydrogen at $-80°$ C. and under a drop from about 113 to 1 atmosphere, but hydrogen liquefier performance raises some question as to its correctness. Jenkin and Pye[21] observed zero slope for liquid CO_2 for one curve where the observed curves are straight lines.

The most closely related data, however, are in Olzewski's[22] work on the "inversion temperature" for air. In his earlier work, hydrogen at a high pressure was allowed to expand through a valve into a space at 1 atmosphere. An exposed platinum thermometer coil was suspended in the space, and the approaching gas and the space were maintained at the working temperature. Air was apparently treated essentially the same way, except to replace the platinum thermometer by thermocouples. The temperature and pressure were adjusted till the opposed thermocouples in bath and air gave no deflection, the sensibility being perhaps 0.1° C. In terms of the curves of Fig. 8, if chords be drawn parallel to the pressure axis from the 1 atmosphere point, they will cut the curve again at some higher pressure. What he observed is therefore the zero value of the integral,

$$T_2 - T_1 = \int_{p_2}^{p_1} \mu dp = 0 \qquad \text{where } \mu = f(p, T)$$

This is a different thing from that discussed above, namely $(\Delta T/\Delta p)_h = 0$. His values for P as related to T are reproduced as Table X. From the curves of Fig. 8, the horizontal chord at 283° C. would cut the curve again at about 260 atmospheres. The chord at 300° C. cuts the curve again about 180 atmospheres. From the work presented here, it would appear that Olzewski's values of the temperature for P = 160 atm. is at least 50° too low.

TABLE X.

Air

P	T
160	+259
100	249
90	244
80	240
70	235
60	226
40	198
20	124

Moreover, it is evident from the curves of Fig. 8 that as P is reduced, the temperature at which the horizontal chord runs from 1 atm. to p atmospheres is higher, not lower as he reports (compare Dalton[33]). The form of the whole group of curves makes this necessary and any question of minor errors hardly enters the case.

There are many doubtful points in the experimental arrangements he used. The air escaping from the jet gathers very great velocity and cools itself in proportion. In consequence, it picks up considerable heat from the metal of the valve itself and the walls of the box, so that when the kinetic energy has gone back through gross turbulence to heat, the enthalpy $(u + pv)$ of the gas has increased and the temperature is higher than it should be for the interpretation he puts upon it. This would require that some cooling effect should be left to offset the heat gain and maintain the initial and final temperatures alike. The observed temperatures for his zero effect would therefore be too low, as seems to be the case.

There is no means of assuring oneself that this kinetic energy has gone entirely back to heat at the time it is in contact with the measuring thermocouple and it seems very doubtful whether it has. If the gas gained no kinetic energy in the valve, the temperature of the expanding gas would first rise and then fall, so that it would be difficult to estimate whether it would gain or lose heat to the valve. The kinetic energy effect is doubtless the dominant source of difficulty, though one would enjoy some experimental proof that the gas actually reached the bath temperature, and as to the bath's uniformity of temperature.

UNIFORMITY OF BATH TEMPERATURE.

If the group of curves of Fig. 8 are examined, it will be noted that the initial point determined, of course, by the inlet thermometer, falls on the smooth curve up to $T_1 = 75°$. At 100° it is about 0.2° low. At 150° about 0.3° above, about 0.4° at 200°, about 0.5° at 250° and about 0.6° at 283° C. The low end of the curve is separately determined by an experiment with P_1 and T_1 set at values picked from the curve already drawn. These low end curves for 150°, 200°, and 250° showed the same irregularity, that is, the points all fell below the smooth curve already drawn when the initial point was put on this curve. This points to an error in comparing the readings of the thermometers when both are at the same temperature. A number of leads were tried but no experimental work gave any clue till a very porous plug was used, one so porous that the pressure drop is practically zero. This showed a ΔT too large by about 0.7° to 0.8°, which is of the right sign and size to put the low end curves for 200° and 250° on the previous curves. This correction is dependent to not more than 0.08° on the rate of flow but does depend on it. So that it is evident that the temperature of the air changes between the thermometers. At these temperatures the input of energy required to hold the bath at constant temperature ran up to about two kilowatts. It is all put in from the heating coils, Fig. 2, located at one part of the circulation. As an approximation, it was assumed the heat was all added at one point in the circulation, and the rise in temperature calculated. If the oil circulates once per second, this gave about 0.5° C. It circulates fast, but the period is probably nearer 5–10 seconds. In consequence, the air coming from the upper part of the bath is probably above the temperature of the oil about the plug chamber. The whole base of the apparatus has to be heated from the oil, so the base is necessarily below the temperature of the oil. That is to say, the air will give up heat to the walls of the plug chamber and the observed ΔT will be too large. This agrees exactly with the observation and there is no reasonable doubt but it is the true explanation.

Up to 100° or even 150° its effect on the data is negligible. Also the small influence of rate of flow makes it an almost negligible correction for the other curves except possibly that at 283°. The same holds for the special experiments for fixing the maxima.

This difficulty can be minimized in future work above room temperature, by placing the heating coils on the exterior of the sides and ends of the apparatus, and on the air lead from the interchanger. This latter should be used to bring the air temperature close to that of the bath. The interior coils having very small lag should be used for the control.

There should be pointed out here the definite advantages which this method of obtaining μ, through the isenthalpic curves, has over the differential method used by Joule and Thomson and most of the succeeding workers. This differential method requires that the pressure difference employed be small and consequently also the temperature difference will be small. Any errors in the thermometer zeros, or change of temperature of the air between the thermometers, comes in as a first order effect and is magnified relatively by the small ΔT employed. On the other hand, in the isenthalpic curve method, only one temperature difference among those used depends upon the inlet thermometer. Consequently the effect on μ of any errors in the difference of resistance of the thermometers when both are at the working temperature, does not enter materially in the results. Also if the air gains or loses heat between the thermometers, the total temperature difference so produced does not enter μ, but only its variation between successive readings. From the experimental point of view, these are very material advantages. The isenthalpic curves themselves contain valuable information and while they may be built up by integrating μ (cf. Davis as reported by Keenan[34]), the resulting curves cannot be given the certainty of the directly determined curves.

JOULE-THOMSON COEFFICIENT μ.

The values of μ may be obtained from this data in a variety of ways. It is necessary always in the reduction of data covering a considerable field, to introduce arbitrary smoothing somewhere. It was desired to make its introduction and arbitrariness as obvious and reasonable as possible. If one uses graphical methods to get the slopes from the curves, the effect of the process on the results is very difficult to estimate. If equations be written for the curves and the derivatives taken for the slopes, the smoothing is done in part in fitting the equations to the experimental points, and since the slope is the more sensitive function, further smoothing is necessary to make μ consistent over the field. The best procedure appeared to be to calculate μ from successive differences of the numerical data and to draw a set of consistent curves, μ against P, one for each isenthalp. This also allowed of working into the result a lot of fragmentary data, quite reliable but, for various reasons, not suitable for plotting isenthalpic curves. This was done to a large scale and the figure is reproduced here as Fig. 12. The curves were first drawn in as indicated by the points alone. These were then shifted slightly as shown to be advisable by the cross plotting described below, but these small adjustments have still left the curves a fair representation of the data.

It will be noted immediately that beginning with the 50° curve, proceeding down the figure, the consistency of the data improves steadily, with the 25° curve probably the best. Attention should be called to the fact that this successive difference treatment is a very severe test for experimental data. If the values of μ along a curve be used to draw the original isenthalpic curve, it is reproduced usually within the width of the line on the large drawing. The value of μ falls rapidly with both rise of pressure and rise of temperature. Also the curvature as well as the slope of these curves decreases steadily with rising temperature. The same condition holds for these as for the original curves, so that they are also isenthalps.

The curves are carried directly to the zero pressure with steady slope. A number of the points between zero and 10 atmospheres fall well above the curve and are to be attributed probably to kinetic energy effects. Grindley's work[31] with steam indicates that μ keeps its regular value to ½ atm. Gases below 1 or 2 atm. obey Boyle's law very closely, that is, the change of pv with p approaches zero, and hence the contribution of the external work to the temperature change here considered disappears. Depending on the law of force between the molecules, the contribution of the intermolecular work may produce a greater or less fall of temperature on expansion. Any internal molecular contribution is probably very small. So that there appears no obvious reason for expecting these curves to change their direction suddenly at low pressures.

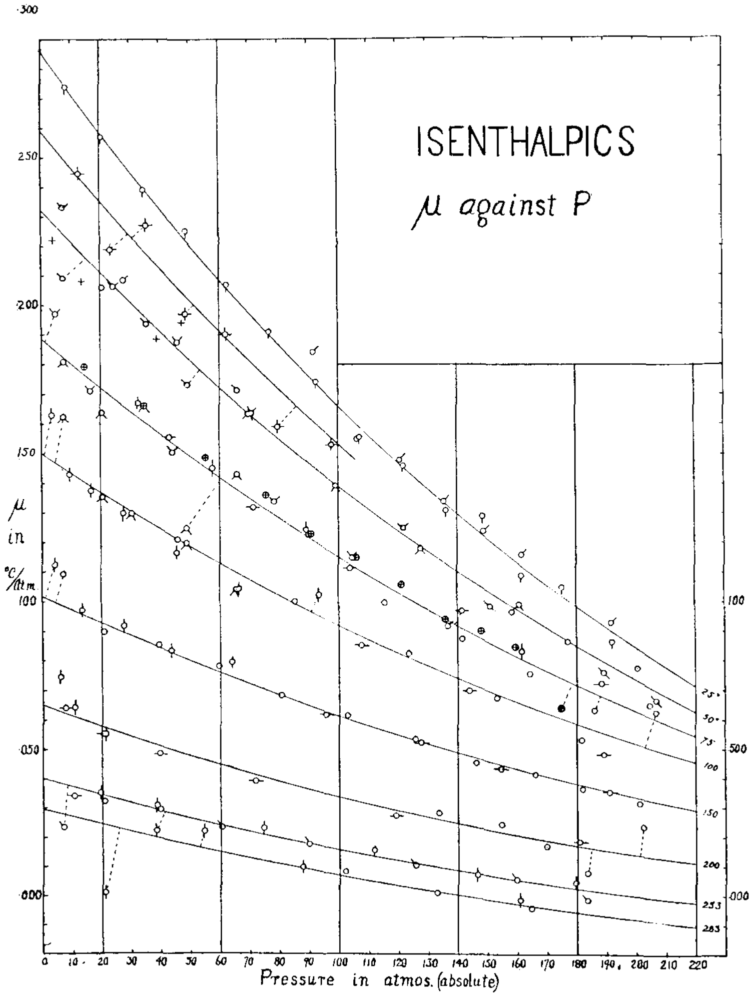

Fig. 12.

The curves of Fig. 12 were next changed to isopiestics by picking off the values of μ for a set of pressures—1, 20, 60, 100, 140, 180, and

220 atm.—and from the original curves picking the temperature corresponding to the μ and p. These values of μ and T are plotted in Fig. 13, and smooth curves—isopiestics—then drawn through the points. The points fall with one or two exceptions (the 100° curve) readily on smooth curves within the error of plotting. These two groups of data are really independent, so that this excellent fitting of the data into this second set of curves adds confirmation at least to the consistency of the data.

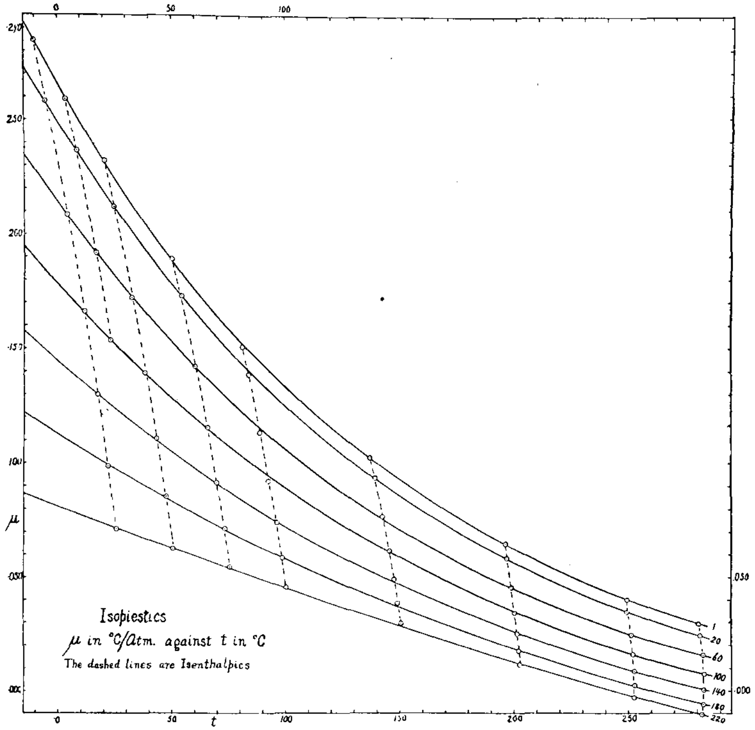

FIG. 13.

The curvature and slope of the isopiestics of Fig. 13 fall off rapidly with rising pressure, till the 220 atm. isopiestic is almost straight. The values of μ for a group of integral values of p and t were then picked off and are tabulated in Table XI.

TABLE XI.

μ AS A FUNCTION OF T AND p

Press.	0°C.	25	50	75	100	150	200	250	280
1	.2663	.2269	.1887	.1581	.1327	.0927	.0625	.0402	.0297
20	.2494	.2116	.1777	.1490	.1244	.0856	.0564	.0346	.0246
60	.2143	.1815	.1527	.1275	.1057	.0708	.0447	.0251	.0161
100	.1782	.1517	.1283	.1073	.0890	.0587	.0347	.0164	.0078
140	.1445	.1237	.1047	.0875	.0723	.0467	.0258	.0093	.0011
180	.1125	.0974	.0833	.0800	.0578	.0366	.0185	.0027	—.0054
220	.0812	.0718	.0627	.0542	.0452	.0286	.0127	—.0020	—.0110

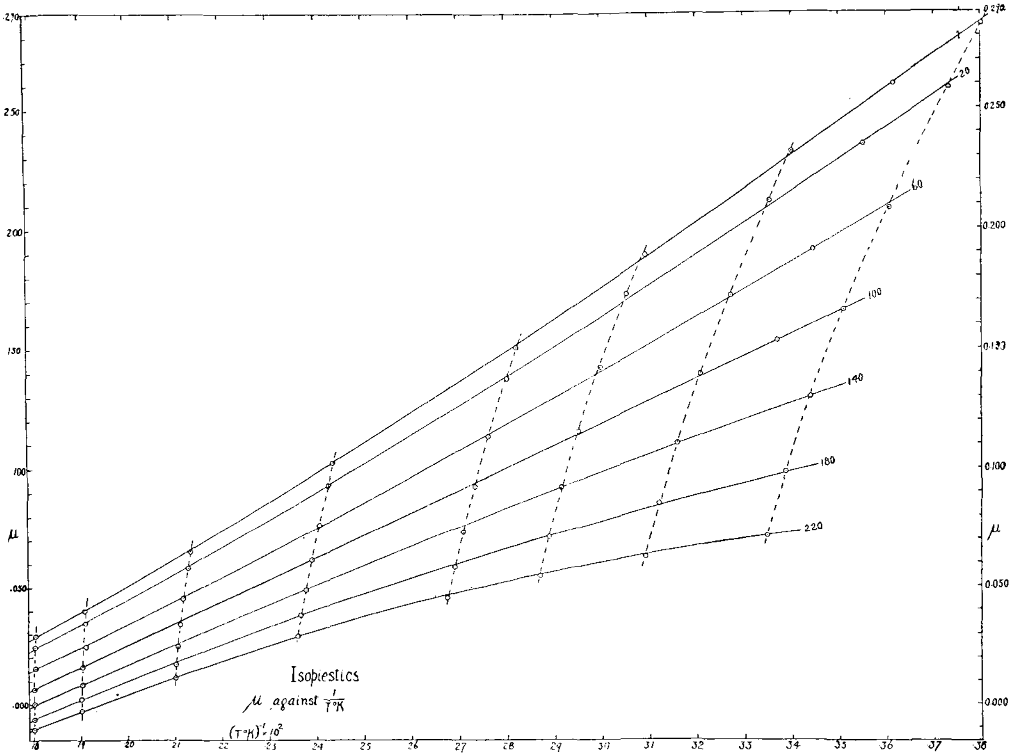

FIG 14.

These isopiestics were next replotted, now however as μ against the reciprocal of the absolute temperature, and this is reproduced in Fig. 14. The 100 atm. isopiestic is a straight line and the other curves are convex toward it. The curvature of the 1 atm. line is very small, so that the equation proposed by Rose-Innes,[23] by Keyes,[22] and by Buckingham[24] make good approximations at 1 atmosphere. The curvature increases however quite markedly at the higher pressures.

SPECIFIC HEAT.

The original isenthalpic curves of Fig. 8 are characterized by the condition that $h = (u + pv)$—the intrinsic plus the pressure volume energy—is a constant. If heat energy be added at constant pressure to the gas at a point on one curve to carry it to a point on the next curve, then

$$h_2 - h_1 = \int_{T_1}^{T_2} C'dt = C\Delta T^*$$

The value of C' changes very slowly with pressure, so that for not too large temperature changes, C may be set as the average value of C' for the interval and will approximate very closely to the actual value of C' at $(T_1 + \frac{1}{2}\Delta T)$. Hence, between a given pair of isenthalpics

$$\frac{C_1}{C_2} = \frac{\Delta_2 T}{\Delta_1 T}$$

Hence, if the specific heat C be known at one pressure over the range in temperature, this relation may be used to calculate it over the pressure range covered by the curves.

To use this relation required a long numerical calculation, which may be summarized as follows. The values of T at 10 atm. intervals along the curves of Fig. 8 were calculated, using the values of the slopes picked off the large scale plot (Fig. 12), the values being calculated in two series, one each way from a point about the middle of the curves of Fig. 8. From these the values of ΔT between each pair of curves were obtained, as also the average value of T for each ΔT. The ratio of the ΔT at 1 atm. to the other ΔT's between the same pair of isenthalps, give the factors by which C for 1 atm. is to be multiplied. These factors were then plotted and smoothed, the maximum uncertainty being about 1%, but averaging 0.2 to 0.3%. From these curves were read off the factors used to multiply the C at 1 atm. of Table XII to fill the following columns. These values for C at 1 atm. are taken from the Reichsanstalt Tabellen[25].

---

* Since C_v is not mentioned, C is used here for C_p.

TABLE XII.

C AS A FUNCTION OF T AND p

Press.	0°C.	25	50	75	100	150	200	250	280
1	.2405	.2410	.2415	.2419	.2424	.2434	.2443	.2453	.2458
20	.2492	.2487	.2480	.2475	.2470	.2466	.2463	.2468	.2471
60	.2656	.2627	.2603	.2581	.2562	.2532	.2512	.2500	.2492
100	.2804	.2760	.2717	.2681	.2650	.2602	.2565	.2536	.2519
140		.2873	.2816	.2767	.2725	.2658	.2607	.2566	.2544
180		.2960	.2898	.2840	.2790	.2707	.2644	.2596	.2569
220		.3020	.2956	.2893	.2838	.2748	.2678	.2622	.2593

C shows a uniform increase with pressure over the range covered but the increase falls rapidly, so that probably it would reverse at still higher temperatures. At one atmosphere C increases with temperature, but at 20 atm. there is a minimum at about 200° C. and at the higher pressures C decreases with rising temperature. The values at 0° C. at the higher pressures could not be obtained without a marked extrapolation and are not recorded.

The Reichsanstalt[25] publishes an equation for the variation of the average C (between 20° and 100° C.) with pressure. This equation was used to calculate C_{60} recorded in Table XIII; while the factors and C were calculated as in Table XII, using 0.2419 as the value at 1 atmosphere for both C's. The two values of C show increases with increasing pressure, but C_{60} increases more slowly than C at low pressures and more rapidly at high pressures, till about 0.6% different. The agreement is about what could be expected.

TABLE XIII.

COMPARISON OF C'S AT 60° C.

Press.	1	20	60	100	140	180	220
Factor	1.0	1.025	1.0725	1.118	1.157	1.189	1.2125
C	0.2419	0.2480	0.2595	0.2705	0.2799	0.2876	0.2933
C_{60}	0.2419	0.2473	0.2590	0.2699	0.2805	0.2884	0.2950
Diff. %	0	+0.28	+0.19	—0.22	—0.21	—0.28	—0.59

THE PRODUCT, μC

The values of μ of Table XI and of C of Table XII were combined to give Table XIV where μC is tabulated as a function of T and p, since this product is useful for many calculations. The value μC

falls with rising pressure as well as with rising temperature, being controlled by the larger variation in μ.

TABLE XIV.

$\mu C \times 10^2$

Press. Atm.	0°C.	25	50	75	100	150	200	250	280
1	6.404	5.464	4.557	3.825	3.216	2.256	1.527	0.986	0.730
20	6.215	5.261	4.407	3.688	3.073	2.111	1.368	0.854	0.608
60	5.691	4.768	3.975	3.291	2.708	1.785	1.123	0.627	0.404
100	4.997	4.188	3.462	2.877	2.359	1.527	0.890	0.416	0.196
140		3.564	2.949	2.421	1.970	1.241	0.673	0.239	0.028
180		2.883	2.414	2.272	1.612	0.991	0.489	0.070	—0.138
220		2.169	1.853	1.518	1.293	0.786	0.340	—0.052	—0.285

THE ABSOLUTE TEMPERATURE OF THE ICE POINT.

The fundamental thermodynamic equation for the porous plug experiment, namely

$$\mu C = T\left(\frac{dv}{dT}\right)_p - v \qquad \text{(A)}$$

may be readily rewritten[24] as

$$\frac{v}{T} - \frac{v_0}{T_0} = \int \frac{C\mu dT}{T^2} = -\int C\mu d\frac{1}{T} \qquad \text{(B)}$$

where T is in the absolute scale. For the present purpose, we have $T - T_0 = 100$ and $v = v_0(1 + \alpha\, 100)$ both by definition.

A plot of uC for 1 atm. between 0° and 100° C. shows it to be so nearly a straight line that the difference produces less than 1% in the integral. That is, we may write

$$\mu = A\frac{1}{T} + B$$

and therefore (B) may be written

$$T_0 = \frac{1}{\alpha} + \frac{T_0(T_0+100)}{100\,\alpha\, v_0}\left[\frac{A}{2}\left(\frac{1}{T_0^2} - \frac{1}{T^2}\right) + B\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \qquad \text{(C)}$$

From Equation C it is evident that the large term on the right is a correction term, since T_0 is approximately equal to $1/\alpha$. This complicated term for air at 1 atm. comes out as less than 1% of T_0, so that the even value 273 may be used for T_0,

$$1/v_0 = 1.293\ g/L \text{ at } 0^\circ \text{ C. and } 76 \text{ cm.}$$

and its value must be adjusted by Boyle's Law to suit the pressure under which α is measured. The values of the integral are not materially different at these slightly different pressures. The values so obtained for T_0 are given in Table 15.

TABLE XV

100α	$1/\alpha$	$p(mm)$	Observer[26]	T_0
0.36730	272.26	1000	Chappuis	273.18
0.36732	272.24	1002	"	273.16
0.36708	272.12	760	Eumorfopoulos	273.12
			Average	273.15

The value used by Griffiths[29] is 273.10 and by Henning [26] 273.09. Buckingham picks 273.13 from the data then (1907) available and Keyes[22] (1920) 273.14. Some of the available data is collected in Table XVI for comparison here. Since Keyes' summary the beautifully consistent work of Henning and Heuse[25] has appeared. It should be weighted heavily in comparison with previous work, and the probable value of T_0 may be set at 273.17 ± 0.02.

TABLE XVI.

Observer	Air	H	N	He	CO_2	Method	Date
Berthelot[28]	273.19	273.05			273.10	J-T data	1907
Buckingham[24]	273.27	273.06	273.25		273.12	"	1907
Buckingham[27]	273.27	273.05	273.29		273.27	"	1908
Hoxton[1]	273.35					"	1916
Roebuck	273.15					"	1925
Berthelot		273.07	273.09		273.12	$\alpha_v = \alpha_p$ for $p \doteq 0$	1907
Henning and[35] Heuse		273.17	273.18	273.22		"	1921

In connection with the present work, it is of interest to note that Hoxton's correction is undoubtedly too large as discussed below. Buckingham's value from air is also high on account of Joule-Thomson's data being somewhat higher than that in the present paper. Berthelot's value from air (J-T basis) is much lower than the other workers have obtained from the same data. All these values have been based on Chappuis' value for the average coefficient.

It should be emphasized that not sufficient attention has been paid to the absorption of gas (cf. Keyes[32]) on and in the wall of the container. It is a simple matter to vary the ratio of volume to exposed surface and extrapolate to zero surface, as well as to free the surface layers from all other gases. The effect of variation in the absorption between 0° and 100° C. will enter differently in the two methods. To assume its effects zero in the $p \doteq 0$ method is hardly justified, though it might be expected to be of less importance. In a comparison of the two methods, this is hardly an advantage, as its effect must be followed to low pressures. In consideration of the greater ease of measuring the coefficient at ordinary pressures, and the small accuracy required of the Joule-Thomson data, this method would seem to be preferable. The meagerness and uncertainty of the available Joule-Thomson data have been much against it, though it now appears as if uncertainty in the coefficient has had a somewhat equal effect on the results.

Coefficient of Expansion.

With $T_0 = 273.15$ as determined above, the equation C may be rewritten as

$$\alpha = \frac{1}{273.15}\left[1 + \frac{273.15 \times 373.15}{100\, v_0}\int \mu C d\frac{1}{T}\right]$$
$$= \frac{1}{273.15}\left[1 + \frac{4.213 \times 10^4 I}{v_0}\right] = \frac{D}{273.15}$$

and used for calculating α, the average coefficient between 0° C. and 100° C., as a function of the pressure. The integral I and specific volume v_0 (Witkowski's data[30]) are both functions of the pressure. These values are collected in Table XVII with the resulting values for α. The fractional part of the factor D is calculated from this data and should be trustworthy to about 1%. The uncertainty in α from this cause only is listed under E. The way that the integral of μC enters the coefficient makes it a very advantageous means for determining α especially for its smaller values, though even at 100 atm. the advantage still amounts to a factor of 5.

TABLE XVII.

Coefficient of Expansion.

p	v_0	$I_p \times 10^5$	D	E%	$\alpha \times 10^3$	$\alpha_1 \times 10^3$	$\alpha_2 \times 10^3$
0	∞		1.0		3.6610		
1	773.4	4.725	1.0025	0.003	3.6704		3.666
20	38.28	4.568	1.0503	0.05	3.845	3.83	3.826
60	12.54	3.869	1.1300	0.1	4.137	4.18	4.166
100	7.49	3.571	1.2009	0.2	4.397	4.41	4.424

For comparison the values of α determined by Witkowski[30] are given under α_1, and by Holborn and Schulze[30] under α_2.

OTHER WORK ON μ FOR AIR.

For comparison, Table XVIII gives the previous data over the range of temperature 0°–100° C. at one atmosphere. It will be noted that Hoxton's data are largest, followed in order by Joule-Thomson, Noell, and the author.

TABLE XVIII.

μ FROM DIFFERENT OBSERVERS.

t °C.	J-T	Noell	Hoxton	Roebuck
0	0.275	0.278	0.303	0.266
50	0.204	0.185	0.226	0.189
100	0.147	0.125	0.170	0.133

Hoxton's[1] data are quite materially higher. In connection with this, it might be pointed out:

(1) That the inlet thermometer in Hoxton's set up is some distance from the plug chamber which gives opportunity for heat transfer to or from the air after it has passed the first thermometer. The velocity of flow of the air and the opportunity for its temperature equalization with the bath were such that this difficulty was probably not of a magnitude to account for the shift of his curves off the zero or for his uniformly large values.

(2) That the carrying of the whole air flow down to the base of the plug by the use of the radiation shield is undoubtedly the main cause of the difficulty. The plug supports are subject to a continual withdrawal of heat by the expanded and cooled escaping air, so that the region about the base of the plug is certain to be somewhat below the bath temperature, and the air, in passing it, loses heat. Changes in the rate of flow affect the expanded air, as well as the approaching air, so that beside the shift of the line off the zero there should also be a shift of slope. The author's experience has been that the ΔT observed is rather surprisingly sensitive to the treatment of the air in the plug chamber. On the basis of this experience, Hoxton's arrangement seems quite sufficient to account for the difficulty.

(3) That uncertainty in the zeros of the thermometers, especially for his very small observed ΔT, may also account for part of the trouble. Reading the differential zero with one thermometer inside

the plug, while the other was inside the inlet tube, places the two thermometers under radically different conditions for equalizing with the bath, both as to the main temperature equilibrium and as to the dissipation of the current heating.

Hoxton's work was greatly superior to preceding work in his careful temperature and pressure regulation, and careful temperature measurement. In the light of my experience, it was unfortunate for his objective that in limiting his problem he chose the much more difficult low pressure part of the field.

Joule and Thomson worked under such different conditions of temperature and pressure regulation, as well as measurement, especially of temperature, that comparison of their pioneer work with work done now is difficult and seems unfair. Outside of these considerations, it was unfortunate that they did not measure the temperature of the air which was entering the plug. One can point out a good many probable sources of error, and it is a great tribute to Joule's experimental genius that his data were correct enough to hold the field for so long.

Noell's[18] work was done under modern conditions, but takes less advantage from it than would be expected. The regulation of both temperature and pressure appears to have been entirely manual and in view of the persistent drifts regularly encountered, this would explain part of the large irregularity in the data. The author's whole experience shows that it is almost impossible to get consistent data without the finest regulation, particularly of the pressure. A slowly rising pressure, for example, adds heat to the air between the thermometers, so cutting down the observed temperature difference, and if this drift of pressure is steady the ΔT is steady. A temperature drift acts similarly, the second thermometer lagging behind the first on account of the heat capacity between them, and if this drift in the bath temperature is steady, the observed ΔT is also steady. Before a steady ΔT can be interpreted as therefore the ΔT desired, it must be known independently that the initial pressure and bath temperature are steady to the limit required by their effect on ΔT. For example, the ΔT in my work was regularly read to 0.01° C. and the bath temperature was held steady over long intervals to 0.01 to 0.02° C.—which is certainly within the requirement. The requirement in the pressure regulation is much more severe experimentally, so that even with the best regulation attained in my work, it was still easily possible to observe the effect of pressure variations actually present, in the rise and fall of ΔT. The best which could be done has been to make this cyclic variation rapid so that a good average could be taken. The pressure variations in question cannot be observed with certainty on a good test gauge, so that they are quite a different order of magnitude from those present under hand regulation guided by spring gauge indications. Under these latter conditions, it will be quite impossible to tell when the ΔT is free from the effect of pressure variation. The phenomenon is made still more complex by the ΔT being the difference in two temperatures read in connecting chambers, in both of which the pressure must be constant while the second chamber always lags behind the first in pressure changes, as also in temperature changes. When the employed ΔT is made small, as in Noell's work, the effect on μ of these pressure and temperature variations is exaggerated, so that a large variation is to be expected.

The heat leak conditions around the plug are open to strong objection. The conditions of the experiment require that there shall be a temperature gradient between high and low pressure surfaces of the plug. His plug is a wad of asbestos compressed in a steel tube. The air in passing across the asbestos will spread over its whole cross section, the flow being limited all around by the steel tube. This tube will doubtless have a temperature gradient, but that it will be the same as the air flow is hardly possible. The air will probably gain heat from the metal wall, whose temperature is supported somewhat by the flow in the annular space outside. From the diagram it appears as if all of this contaminated air must pass the thermometer, though the text indicates that the double wall of insulator is to divide the flow. It would be better still if the inner insulating tube ran right through the plug and so kept the flows really separated. Noell seems to be concerned only with heat conducted parallel to the direction of flow, though such conductivity cannot affect the ΔT once a steady state has been set up.

For comparison, Fig. 15 gives Noell's data plotted with crosses and circles to keep the series distinct and help relate the points to the companion curve. Noell's curves representing his equations are plotted as dashed lines, and the author's curves as solid lines.

It will be observed first that his curves and points fall together very imperfectly. Any one series of points could be represented very much better by a line drawn through them, but the set of such lines varies so irregularly in both position and direction that they cannot be fitted into any orderly system. The particular system chosen to represent such points is therefore very arbitrary. As a matter of fact, the author's curves represent the points almost as well as Noell's.

This chaotic irregularity is very strong evidence of experimental uncertainty, such as might be expected to result from uncertainty in pressure and temperature regulation. On the other hand, the regularity of any one series and the agreement of single points indicate rather systematic than chaotic errors, such as might arise from heat leaks.

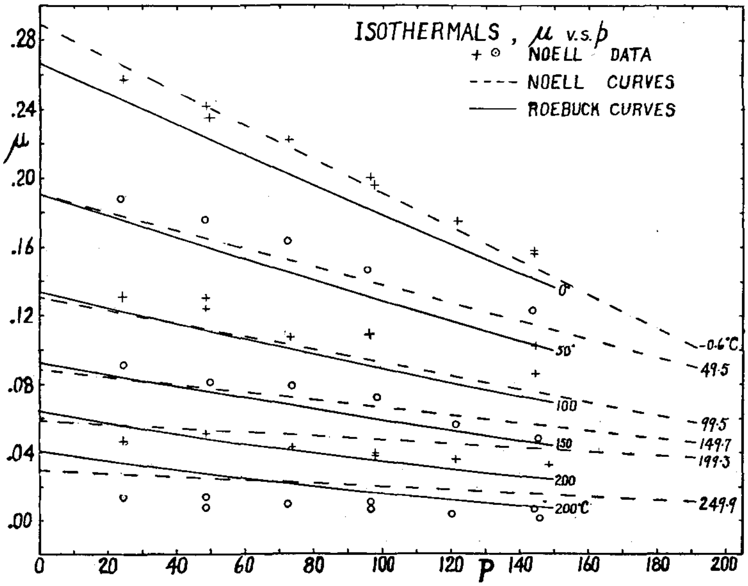

FIG. 15.

CONCLUSION.

The data in this paper cover the readily available temperatures above that of the room. A bath liquid above 300° C. is a serious problem. It also covers the available and useful pressure range. It would be interesting to extend both these ranges to fill in the inversion curve points. But the more profitable field appears to be the low temperature range, and these measurements will be extended as quickly as available time, and to as low a temperature as the available facilities allow. These facilities include a carbon dioxide ice machine and a compressor which may be used for an ethylene cycle, so that it is hoped to get well down into the liquid-vapor region.

Meanwhile, there are also quite a number of related facts which may be extracted from the above data, such as the correction to the Kelvin scale, the volume temperature coefficient of expansion, the free expansion effect, and so forth. Such computations will be carried out and extended to the new data as they become available.

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Physical Laboratory,
University of Wisconsin.

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2. Barnett, S. J., and Barnett, L. J. H.—New Researches on the Magnetization of Ferromagnetic Substances by Rotation and the Nature of the Elementary Magnet. pp. 125–216. July, 1925. $1.50.
3. Baxter, G. P., and Cooper, W. C., Jr.—A Revision of the Atomic Weight of Germanium. II The Analysis of Germanium Tetrabromide. pp. 219–237. October, 1925. $0.75.
4. Dana, L. I.—The Latent Heat of Vaporization of Liquid Oxygen-Nitrogen Mixtures. pp. 241–267. October, 1925. $0.75.
5. Pierce, George W.—Piezoelectric Crystal Oscillators Applied to the Precision Measurement of the Velocity of Sound in Air and CO_2 at High Frequencies. pp. 271–302. October, 1925. $0.75.
6. Bridgman, P. W.—Certain Physical Properties of Single Crystals of Tungsten, Antimony, Bismuth, Tellurium, Cadmium, Zinc, and Tin. pp. 305–383. October, 1925. $2.50, with 7 and 8.
7. Bridgman, P. W.—Various Physical Properties of Rubidium and Caesium and the Resistance of Potassium Under Pressure. pp 385–421. October, 1925. $2.50, with 6 and 8.
8. Bridgman, P. W.—The Effect of Tension on the Transverse and Longitudinal Resistance of Metals. pp 423–449. October, 1925. $2.50, with 6 and 7.
9. Boschma, H.—On the Symbiosis of Certain Bermuda Coelenterates and Zooxanthellae. pp. 451–460. October, 1925. $0.50.
10. Kennelly, A. E., and Slepian, Arthur.—Computation of the Behavior of Electric Filters Under Load. pp. 461–483. October, 1925. $0.75.
11. Franklin, Wm. S.—Indeterminism in the Physical World. pp. 485–502. November, 1925. $0.50.
12. Yeh, Chi-Sun.—The Effect of Hydrostatic Pressure on the Magnetic Permeability of Iron, Cobalt, and Nickel. pp. 503–535. December, 1925. $0.50.
13. Roebuck, J. R. The Joule-Thomson Effect in Air. pp. 537–596. December, 1925. $0.75.
14. Records of Meetings; Biographical Notices; Officers and Committees; List of Fellows and Foreign Honorary Members; Statutes and Standing Votes, etc. pp. 597–610. December, 1925. $0.50.

Proceedings of the American Academy of Arts and Sciences.

Vol. 60. No. 14.—December, 1925.

RECORDS OF MEETINGS, 1924–25.

BIOGRAPHICAL NOTICES.

OFFICERS AND COMMITTEES FOR 1925–26.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

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(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

One thousand one hundred and thirty-third meeting.

OCTOBER 8, 1924.—STATED MEETING.

The Academy met at its House at eight P. M.

The PRESIDENT in the Chair.

There were present fifty-three Fellows and Associates and sixteen guests.

The Records of the Annual Meeting, of May 14, were read and approved.

The following letters were presented by the Corresponding Secretary: from E. G. Boring, Ingersoll Bowditch, E. A. Boyden, J. B. Conant, C. A. DeCourcy, Raymond Dodge, C. L. Edgar, W. E. Fernald, W. C. Graustein, Paul Heymans, S. J. Holmes, J. A. Miller, E. W. Taylor, and C. H. Van Tyne, accepting Fellowship; from Albert Einstein, accepting Foreign Honorary Membership; and from E. H. Abbot, F. N. Balch, C. E. Fay, F. R. Hart, A. J. Peters, A. J. Philpott, and E. S. Webster, accepting Associate Membership.

The Corresponding Secretary reported the receipt of biographical notices of Abraham Jacobi, by Simon Flexner; Jacques Loeb, by W. J. V. Osterhout; and Lincoln Ware Riddle, by Bruce Fink.

The Corresponding Secretary announced the following appointments made by the President of delegates to the following conferences or celebrations:

Alfred C. Lane, to the Fiftieth Anniversary of the Founding of the Société Géologique de Belgique, July 27–30;

R. G. D. Richardson, to the meeting of the International Mathematical Congress, at Toronto, August 11–16;

Henry P. Talbot, to the Centennial Celebration of the Rensselaer Polytechnic Institute, October 3–4;

Harvey Cushing, to the Inauguration of President Vinson and the Dedication of the new building of the School of Medicine of the Western Reserve University, October 9.

The President announced the deaths of six Fellows:—Charles Ambrose DeCourcy (Class III, Section 1); James DeNormandie (Class III, Section 1); Oliver Whipple Huntington (Class II, Section 1); Robert Williamson Lovett (Class II, Section 4); Allan Marquand (Class III, Section 4); Robert Simpson Woodward (Class I, Section 4).

The newly elected Fellows and Associates were then presented.

The following communication was presented:

Mr. Reginald A. Daly: "Studies on Volcanic Islands of the Deep Sea," with lantern illustrations.

Five papers were presented by title:

"Contribution towards a Monograph of the Laboulbeniaceæ, Part 3, by Roland Thaxter.

"The Geology of Ascension Island," by Reginald A. Daly.

"Certain Physical Properties of Single Crystals of Tungsten, Tellurium, Cadmium, Zinc, and Tin," by P. W. Bridgman.

"The Effect of Tension on the Transverse and Longitudinal Resistance of Metals," by P. W. Bridgman.

"Various Physical Properties of Rubidium and Cæsium, and the Resistance of Potassium under Pressure," by P. W. Bridgman.

The Meeting was dissolved at 9.35 P.M.

One thousand one hundred and thirty-fourth meeting.

November 12, 1924.—Stated Meeting.

Held in Connection with the Autumn Meeting of the National Academy of Sciences.

The Academy met at its House at 8.10 P.M.

The President in the Chair.

There were present sixty-two Fellows and thirty-five guests, including ladies.

The Records of the Meeting of October 8 were read and approved.

The Corresponding Secretary reported the receipt of the following biographical notices:—Louis Derr, by H. M. Goodwin; Harold Clarence Ernst, by S. B. Wolbach; Theophil Mitchell Prudden, by S. B. Wolbach; and Wallace Clement Sabine, by E. H. Hall.

The President announced the deaths of Henry Cabot Lodge, Fellow in Class III, Section 3; and of Sir Archibald Geikie, Foreign Honorary Member in Class II, Section 1.

The following communications were presented:

Mr. Charles H. Haskins: "The Spread of Ideas in the Middle Ages."

Mr. Henry Osborn Taylor: Discussion of Mr. Haskins' paper.

Mr. Arthur L. Day: "Some Causes of Volcanic Activity," with lantern illustrations.

At 9.35 the Meeting was dissolved.

One thousand one hundred and thirty-fifth meeting.

DECEMBER 10, 1924.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

There were present twenty-seven Fellows and Associates and one guest.

The Records of the Meeting of November 12 were read and approved.

The Corresponding Secretary reported the election by the Council of Mr. Ingersoll Bowditch as Treasurer to fill the unexpired term of Mr. Harold Murdock, resigned.

It was

Voted, To authorize the Council to transfer the unused balance of the appropriation for Mrs. Holden, late Assistant Librarian Emerita, to the appropriation for General and Meeting expenses.

The President announced the deaths of Walter Elmore Fernald, Fellow in Class II, Section 4; and of George Chandler Whipple, Fellow in Class I, Section 4.

The following communication was presented:

Mr. Ephraim Emerton: "Bartolus *de Guelphis et Gebellinis.* The Italian Party System."

The following paper was presented by title:

"On the Symbiosis of certain Cœlenterates and Zooxanthellæ, by H. Boschma." Presented by E. L. Mark.

At 9.20 the Meeting was dissolved.

One thousand one hundred and thirty-sixth meeting.

JANUARY 14, 1925.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

There were present twenty-two Fellows and three guests.

The Records of the Meeting of December 10 were read and approved.

The President announced the death of Alfred Marshall, Foreign Honorary Member in Class III, Section 3.

The following communication was presented:

Mr. Francis H. Williams: "Measurements of X-Rays and of Electrons and Gamma Rays from Radium by Fluorescence, with Illustrations of the Usefulness of these Measurements in Medicine."

The following paper was presented by title:

"Computation of Behavior of Electric Filters under Load," by A. E. Kennelly and Arthur Slepian.

The Meeting was dissolved at 9.10 P.M.

One thousand one hundred and thirty-seventh meeting.

FEBRUARY 11, 1925.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

There were present twenty-six Fellows and two guests.

The Records of the Meeting of January 14 were read and approved.

The Corresponding Secretary reported the receipt of a biographical notice of George Chandler Whipple, by A. E. Kennelly.

The President announced the deaths of Willard Bartlett, Fellow in Class III, Section 1, and of Joseph Clark Hoppin, Fellow in Class III, Section 2.

The following communications were presented:

Mr. William J. Cunningham: "Recent Tendencies in Policies of Railroad Administration."

Mr. Harvey N. Davis: "Recent Developments in Railroad Equipment."

One paper was presented by title:

"The Effect of Pressure on the Magnetic Permeability of Iron, Cobalt, and Nickel," by C. S. Yeh, presented by P. W. Bridgman.

The Meeting was dissolved at 10.15 P.M.

One thousand one hundred and thirty-eighth meeting.

MARCH 11, 1925.—STATED MEETING.

The Academy met at its House at 8.10 P.M.

The PRESIDENT in the Chair.

There were present thirty-eight Fellows and Associates and six guests.

The Records of the Meeting of February 11 were read and approved.

The Corresponding Secretary reported the receipt of a biographical notice of Walter Elmore Fernald, by C. Macfie Campbell.

On recommendation of the Council, the following appropriations were made for the ensuing year:

From the income of the General Fund, $7,700, to be used as follows:

for General and Meeting expenses	$ 800.00
for Library expenses	1,800.00
for Books, Periodicals and Binding	1,200.00
for House expenses	3,000.00
for Treasurer's expenses	800.00
at the disposition of the President	100.00

From the income of the Publication Fund, $2,995.02, to be used for publication.

From the income of the Rumford Fund, $3,432.35, to be used as follows:

for Research	$1,000.00
for Books, Periodicals and Binding	200.00
for Publication	600.00
for use at the discretion of the Committee	1,632.35

From the income of the C. M. Warren Fund, $1,069.46, to be used at the discretion of the Committee.

The President announced the deaths of one Fellow, Joel Hastings Metcalf (Class I, Section 1); and of two Foreign Honorary Members, Sir Thomas Clifford Allbutt (Class II, Section 4) and Oliver Heaviside (Class I, Section 2).

The President appointed the Nominating Committee as follows:

William S. Franklin, of Class I.
Walter B. Cannon, of Class II.
Albert Matthews, of Class III.

The following communication was presented:

Mr. Henry B. Bigelow: "Recent Oceanographic Problems as Illustrated in American Waters," with lantern illustrations.

The Meeting was dissolved at ten o'clock.

One thousand one hundred and thirty-ninth meeting.

APRIL 8, 1925.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

There were present forty-five Fellows and Associates, and six guests.

In the absence of the Recording Secretary, the Corresponding Secretary was requested to assume his duties.

The Records of the Meeting of March 11 were read and approved.

The following communications were presented:

RECENT PROBLEMS IN ASTRONOMY.

The Milky Way

1. Mr. Willem J. Luyten: "The Distribution and Brightness of the Stars."

2. Mr. Solon I. Bailey: "Dark Nebulæ along the Milky Way."

3. Mr. Harlow Shapley: "On the Dimensions of the Galactic System."

Illustrated with lantern slides.

The Meeting was dissolved at ten o'clock.

One thousand one hundred and fortieth meeting.

MAY 13, 1925.—ANNUAL MEETING.

The Academy met at its House at 8.10 P.M.

The PRESIDENT in the Chair.

There were present thirty-six Fellows and Associates and one guest.

The Records of the Meeting of April 8 were read and approved.

The Corresponding Secretary reported the receipt of a biographical notice of John Trowbridge, by Theodore Lyman.

The President announced the death of two Fellows:—Arthur Lord (Class III, Section 1), and John Singer Sargent (Class III, Section 4).

The following report of the Council was presented:

Since the last report of the Council there have been reported the deaths of fourteen Fellows:—Willard Bartlett, Charles Ambrose DeCourcy, James DeNormandie, Walter Elmore Fernald, Joseph Clark Hoppin, Oliver Whipple Huntington, Henry Cabot Lodge, Arthur Lord, Robert Williamson Lovett, Allan Marquand, Joel Hastings Metcalf, John Singer Sargent, George Chandler Whipple, and Robert Simpson Woodward; and of four Foreign Honorary Members:—Sir Thomas Clifford Allbutt, Sir Archibald Geikie, Oliver Heaviside, and Alfred Marshall.

Fourteen Fellows and one Foreign Honorary Member, as well as ten Associates, were elected by the Council and announced to the Academy in May 1924.

The roll now includes 571 Fellows, 61 Foreign Honorary Members, and seven Associates (not including those elected in May 1925).

The annual report of the Treasurer, Ingersoll Bowditch, was read, of which the following is an abstract:

GENERAL FUND.

Receipts.

Income on hand April 1, 1924		$ 2,771.95
From Investments	$4,826.06	
" Assessments	3,320.00	
" Admissions	170.00	
" Sundries	341.45	8,657.51
		$11,429.46

Expenditures.

Assistant Librarian Emerita	$ 350.00	
Expenses of Library	1,982.30	
Treasurer's Expenses	903.41	
Books and Binding	1,190.96	
General Expenses	920.07	
House Expenses	2,663.41	
Restoring Pictures	145.00	
President's Expenses	103.82	$ 8,258.97
Interest on Bonds, bought	13.61	
Transferred to Publication Funds	250.00	
Income transferred to Principal	382.93	646.54
		$ 8,905.51

RUMFORD FUND

Receipts.

Income on hand April 1, 1924		$ 6,520.78
From investments	$3,995.14	
" Unexpended Grants returned	254.69	4,249.83
		$10,770.61

Expenditures.

Purchase and Binding of Books	$ 201.52	
Research	1,425.00	
Tables of Constants	200.00	
Sundries	6.75	$ 1,833.27
Income transferred to Principal		175.45
		$ 2,008.72

C. M. WARREN FUND.

Receipts.

Income on hand April 1, 1924	$ 1,064.83
From Investments	$ 1,136.75
	$ 2,201.58

Expenditures.

Research	$2,004.75	
Vault Rent, part	3.00	$ 2,007.75
Income transferred to Principal		56.34
		$ 2,064.09

PUBLICATION FUND.

Receipts.

Income on hand April 1, 1924		$ 4,853.23
From Income Appleton Fund	$ 915.00	
" " Centennial Fund	2,448.34	
" Authors Reprints	206.77	
" Sale of Publications	479.48	
" Use of Plate	75.00	
" General Fund Income	250.00	4,374.59
		$ 9,227.82

Expenditures.

Publications	$4,695.39	
Vault Rent, part	10.00	$ 4,705.39
Income transferred to Principal		179.80
		$ 4,885.19

FRANCIS AMORY FUND.

Receipts.

From Investments		$ 1,810.00

Expenditures.

Publishing Statement	$ 67.55	
Interest on Bonds, bought	16.67	$ 84.22
Income transferred to Principal		1,725.78
		$ 1,810.00

The following reports were also presented:

Report of the Library Committee.

The Librarian begs to report for the year 1924–25, as follows:

During the year 110 volumes and 28 unbound numbers of periodicals have been borrowed by 22 Fellows and 6 libraries. Many books have been consulted at the Academy, though not as many as the Librarian could wish. All books taken out have been returned or satisfactorily accounted for.

The number of books on the shelves at the time of the last report was 39,625. During the year 423 volumes have been added, making the number now 40,048. This includes 48 purchased from the General Fund, 20 from the Rumford Fund, and 355 received by gift or exchange. The number of pamphlets added was 170.

The expenses charged to the Library during the financial year ending April 1, 1925, are:

Salaries	$1,800.00
Binding:	
General Fund	728.25
Rumford Fund	80.45
Purchase of periodicals and books:	
General Fund	411.31
Rumford Fund	121.07
Miscellaneous	182.30

The Committee wishes to add a word of appreciation of the many years of devoted service to the Academy of Mrs. Agnes H. Holden, whose death occurred on October 18, 1924. From October, 1903, to October, 1922, she filled most ably the position of Assistant Librarian, succeeding her husband, the late Dr. Austin Holden, and after ill-health forced her to resign in 1922, the Academy made her Assistant Librarian Emerita.

H. M. Goodwin, *Librarian.*

May 13, 1925.

Report of the Rumford Committee.

The Committee held six meetings during the year (Oct. 8th, Nov. 5th, Jan. 14th, Feb. 11th, Mar. 11th, and Apr. 8th).

Theodore Lyman, as newly-elected President of the Academy, resigned the chairmanship of the Committee on October 8th, and Arthur E. Kennelly was elected to succeed him, as Chairman, with P. W. Bridgman as Secretary.

The Committee made the following grants during the academic year 1924–25:

Jan. 14, 1925. To S. A. Mitchell, Director, Leander McCormick Observatory, University of Virginia, towards printing of observations on variable stars	No. 250.	$ 500
Feb. 11, 1925. To S. Stillman Berry, Redlands, Calif., towards researches on light production in cephalopods,	No. 251.	300
Mar. 11, 1925. To F. A. Saunders, Harvard University, towards equipment in continuation of researches in mapping spectra	No. 252.	200
Apr. 8, 1925. To G. Shannon Forbes, Harvard University, towards equipment in photochemical researches .	No. 253.	250
		$1,250

Since the date of the last Committee report, information concerning researches completed with aid from the Rumford Fund has been received from the following persons:—A. L. Clark, W. W. Campbell, W. C. Hayes, J. R. Roebuck, H. Shapley and R. W. Wood.

Reports concerning researches being continued with aid from the fund have also been received from P. W. Bridgman, N. A. Kent, and S. A. Mitchell.

The following papers covering researches aided from the Rumford Fund have been published since the date of the Committee's last report:—

"Viscosity of Ethyl Ether near the Critical Point," by A. L. Clark, *Transactions of the Royal Society of Canada*, Vol. 18, 3rd Ser., pp. 329–338.

"Controlled Orbital Transfers of Electrons in Optically Excited Mercury Atoms," by R. W. Wood, *Proc. Royal Soc.*, A, Vol. 106, 1924, pp. 679–694.

"The 3883 Cyanogen Band in the Solar Spectrum," by Raymond T. Birge, *The Astrophysical Journal*, Vol. 59, No. 1, Jan. 1924, pp. 45–60.

The mss. of a paper by J. R. Roebuck on the "Joule-Thomson Effect" has been received covering a research aided by the Rumford Fund. This paper has been accepted and forwarded to the Academy's Publication Committee for printing in the Proceedings.

On March 11th, the committee reported for the first time, unanimously, and on April 8th, for the second time, unanimously, to recommend to the Academy the award of the Rumford Premium to Professor Henry Norris Russell of Princeton University for his Researches in Stellar Radiation.

A review of the history and aims of "The Rumford Fund" was published on behalf of the committee in "Science" for December 19th, 1924, Vol. 60, No. 1564, pp. 558–559.

Respectfully submitted,

A. E. Kennelly, *Chairman.*

May 13, 1925.

Report of the C. M. Warren Committee.

The Committee had at its disposal at the beginning of the fiscal year 1924–1925, $2,124.61. During the year ending March 31, 1925, grants to the amount of $2,040 were made. The balance on hand on that day was accordingly $84.61. The appropriation of the Academy for the coming year is $1,070. The amount at the disposal of the Committee for the year 1925–1926 is $1,154.61.

Since the last annual report of the Committee, awards have been made as follows:

April 3, 1924. To Professor A. C. Lawson, National Research Council, $150, to be used in connection with the work of his committee on the measurement of geological time on the basis of atomic disintegration.

April 3, 1924. To Professor G. P. Baxter, Harvard University, $500, to cover the cost of a standardized meter bar.

April 3, 1924. To Professor W. V. Evans, of Northwestern University, $300, to aid him in his research on chemiluminescence.

April 3, 1924. To Dr. Roland R. Read, of the University of Vermont, $125, to be used in the study of the reduction of aldehydes.

June 19, 1924. To Professor Charles Kraus, of Brown University, $500, for his research on intermetallic compounds.

July 3, 1924. To Professor D. A. MacInnes, of the Massachusetts Institute of Technology, $200, to be used in his work on liquid junctions.

December 2, 1924. To Dr. Charles F. H. Allen, of Tufts College, $200, to assist in his study on delta ketonic nitriles.

December 2, 1924. To Dr. Arthur F. Scott, of Reed College, $75, to be used in connection with his work on the atomic weight of chlorine.

Reports of progress have been received from the following:

Professors Conant, Evans, Worrall and Allen.

JAMES F. NORRIS, *Chairman*.

May 13, 1925.

REPORT OF THE COMMITTEE OF PUBLICATION.

NOTE: At the meeting of May 13, 1925, the chairman made a general oral report to the Academy and requested, that he be permitted to submit his report later as of date when the work of the Committee should be turned over to the chairman to be appointed, Mr. W. S. Franklin. The request was granted.

The Committee of Publication reports as follows for the period April 1, 1924 to October 13, 1925:

The Committee and the Academy have lost the valuable services of Mr. A. A. Howard, representing Class III, who had so long served as a member of the Committee and at times as its Acting Chairman, through his death the past summer.

During the period of one year and 5 months there have been printed No. 5 of Vol. 14 in the Memoirs and Nos. 1–3 of Vol. 15; Nos. 10–17, of Vol. 59 of the Proceedings and Nos. 1–8 of Vol. 60.

The financial statement of the Committee for the fiscal year can be seen in the report of the Treasurer of the Academy. The receipts, for the year ending March 31, 1925, were $4,374.59, the expenditures for the year ending March 31, 1925, were $4,885.19, thus drawing down the surplus accumulated in past years by about $500. This statement is so favorable merely because of an unfortunate delay at the press. Three manuscripts were given to the printer for the Proceedings in December 1924, which had not been received in galley in the middle of May 1925 and for which the printer's bill, which normally should have come in before the middle of May, has only been received in the early fall. The balance on hand at the present date, for the payment of printer's bills on the Proceedings from No. 6 of Volume 60, forward, is $2,596.83, and if the press handles manuscripts in its charge with normal dispatch, it will be exhausted considerably before the end of the current fiscal year.

In view of this situation, the Committee may be permitted to quote from its report of May 14, 1924: "Even with a continuation of contributions by the Rumford Committee toward the expenses of publication of articles from the field of that Committee, it is safe to say that there will come a time within the next three or four years when the publications of the Academy must be considerably reduced in volume unless there is some way to reduce the cost per page now paid for printing. . . . Your Committee will direct every effort towards the reduction of costs of the publications, but believes that it is none too soon for the Academy seriously to consider where it may look for some additional endowment for publication, because it is too much to hope that costs can be reduced to anything like the pre-war figure and it will be unfortunate both for the Academy and for its members if any steps shall have to be taken to reduce the space available for publishing material of the caliber that has distinguished these publications for many years."

Owing to the delays which we experienced with the printer with whom the Committee had done business for a great many years, it was determined that the unfortunate necessity had arisen for seeking a new printer. After a great many inquiries it was decided to try out the service and the rates of the Intelligencer Printing Co., of Lancaster, Pa. The first manuscript sent to them was that of No. 5 (the concluding number) of Volume 14 of the Memoirs. They have printed that and Nos. 1, 2, 3 of Vol. 15. They have taken over the printing of the Proceedings beginning with No. 6 of Volume 60. It is too early to say whether the change will result in the long run in either a better service or lower rates. The Committee desires to register at this time its appreciation of the uniform courtesy that it has received during the long period from the Cosmos Press with which it has done business, and to express its regret that that press found itself unable during the past year to continue its previous satisfactory service.

The Committee will continue to do all in its power to keep down costs of publication. With the approval of the President the Committee has, during the past summer, addressed to a committee of the National Academy, charged with the disbursement of a certain amount of money to aid in scientific publication, a request that the emergency which now exists in the financial situation of your Committee of Publication be carefully considered, and that if possible an appropriation be made to aid in ensuring the publication of the volume of the Proceedings (Vol. 61) of the current year. The Committee again urges that steps be taken to ensure that an adequate income is available to the Committee each year so that the publication of the valuable contributions offered to the Committee may not have to be restricted.

On account of the delay in publication during the past year a number of authors have become justifiably restive, but it is the hope of the Committee that the situation will soon clear up and the normal promptness of publication be resumed. The Committee believes, however, that it is only fair to point out that a great deal of material has been published. The 4 Memoirs make a total issuance of about 455 quarto pages plus 25 plates, which is a very large volume in itself, and represents an amount of material as great as has been previously printed in the Memoirs since June 1908, or more than 15 years. The amount of material is in comparison even larger than the number of pages would indicate. The Academy has been accustomed to print the Memoirs in 11 pt. on a 16 pt. body. As a part of its responsibilities in lowering the costs the Committee had to consider whether this style of printing was not unnecessarily elaborate and after an examination of the whole question decided that to follow the example of the National Academy and print with 10 pt. type on 12 pt. body was of adequate dignity for the American Academy, and consequently with the exception of certain introductory parts these 4 Memoirs have been so printed. The result is a saving in space and cost of something like 25%. In addition to the work on the Memoirs there have been printed during the period of 17 months in the Proceedings, 470 pages of Vol. 59 and 450 pages of Vol. 60, making a total of 920 pages, which is at a rate of only little less than usual. The total printing program accomplished in the 17 months is therefore very largely in excess of what the Committee is ordinarily expected to accomplish and the delays which have been annoying alike to the Committee and to the authors have been attributable in a very considerable measure to the fact that the Committee has been called upon to print more than is usual and has merely failed in the attempt to print still more.

The present status of the publications is as follows: There are no manuscripts in hand to be printed as Memoirs and there are no Memoirs in the course of publication. On the Proceedings, Nos. 9 and 10 of Vol. 60 are in page proof, No. 11 is at the printer's for composition. Three others, whose serial numbers have not been assigned and some of which will be put into Vol. 61 instead of Vol. 60, are also at the printer's for composition and of these galleys have already been received for one. Three manuscripts are in the hands of the Committee for editorial consideration and are expected to go to the printer within the next few weeks.

Respectfully submitted,
EDWIN B. WILSON, *Acting Chairman.*

Oct. 13, 1925.

REPORT OF THE HOUSE COMMITTEE.

The House Committee submits the following report for the year 1924–25:

With the balance of $137.24 left from last year, an appropriation of $2,800, and $205 received from other societies for the use of the rooms, together with $47.83 received from the Friends of Medical Progress for telephone service, the Committee had at its disposal the sum of $3,190.07. The total expenditure has been $2,916.24, leaving an unexpended balance on April 1, 1925, of $273.83. The expenditure has been as follows:

Janitor		$ 968.00
Electricity	Light	186.96
	Power	68.40
Coal		1,009.14
Care of elevator		49.38
Gas		70.56
Water		15.40
Telephone		118.56
Ash tickets		34.65
Upkeep		266.00
Furnishings and equipment		84.05
Janitor's supplies and sundries		44.14
Total expenditure		$2,916.24

This amount minus the $252.83 contributed by other societies leaves the net expense of the House $2,663.41.

It was necessary to replace a cable in the elevator at a cost of $47.55 and to purchase a new vacuum cleaner. The Lecture Hall was painted during the summer.

This coming summer we expect to have the Reading Room and the halls of the upper floors painted, as well as the outside window frames and sashes all over the building. An appropriation covering the estimate for the work was included in our budget for next year.

Meetings have been held as follows:

The Academy	8
American Antiquarian Society	1
Colonial Society	4
Geological Club of Boston	1
Harvard-Technology Chemical Club	7
National Academy of Sciences	1
New England Botanical Club	9
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On November 12, the National Academy of Sciences held an all-day meeting at the House of the American Academy as the latter's guest.

There were morning and afternoon sessions, followed by a reception, and in the evening a joint session of the two academies.

The Council Chamber has been used for Academy Council and Committee meetings, and also by the Colonial Society, the Trustees of the Children's Museum, and the Thursday Evening Club.

The society of Friends of Medical Progress, which had occupied the Committee Room on the first floor since July 1923, removed from the building in September 1924.

Respectfully submitted,
W. H. Lawrence, *Chairman.*

May 13, 1925.

On the recommendation of the Treasurer, it was

Voted, That the Annual Assessment be $10.00.

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Professor Henry Norris Russell, of Princeton University, for his researches in stellar radiation.

The annual election resulted in the choice of the following officers and committees:

Theodore Lyman, *President.*
Arthur E. Kennelly, *Vice-President for Class I.*
William M. Wheeler, *Vice-President for Class II.*
Arthur P. Rugg, *Vice-President for Class III.*
Norton A. Kent, *Corresponding Secretary.*
Charles B. Gulick, *Recording Secretary.*
Ingersoll Bowditch, *Treasurer.*
Harry M. Goodwin, *Librarian.*

Councillors for Four Years.

Harvey N. Davis, *of Class I.* Herbert V. Neal, *of Class II.*
Edward W. Forbes, *of Class III.*

Finance Committee.

Thomas Barbour Paul J. Sachs Frederick P. Fish

Rumford Committee.

Arthur E. Kennelly

Elihu Thomson
Percy W. Bridgman
Harry M. Goodwin
Charles L. Norton
Harlow Shapley
Frederick A. Saunders

C. M. Warren Committee.

James F. Norris

Henry P. Talbot
Gregory P. Baxter
Walter L. Jennings
Arthur D. Little
Lawrence J. Henderson
Frederick G. Keyes

Publication Committee.

William S. Franklin, *of Class I.*
Herbert V. Neal, *of Class II.*
Albert A. Howard, *of Class III.*

Library Committee.

Edwin B. Wilson, *of Class I.*
Thomas Barbour, *of Class II.*
William C. Lane, *of Class III.*

House Committee.

William H. Lawrence, *Chairman*
Robert P. Bigelow
S. Burt Wolbach

Committee on Meetings.

The President
The Recording Secretary
George H. Parker
Gregory P. Baxter
Edward K. Rand

Auditing Committee.

George R. Agassiz
John E. Thayer

The Council reported that the following gentlemen were elected members of the Academy:

Class I, Section 1 (Mathematics and Astronomy):
Willem Jacob Luyten, of Cambridge, as Fellow.
Alfred North Whitehead, of Cambridge, as Fellow.
Class I, Section 4 (Technology and Engineering):
Vannevar Bush, of Chelsea, as Fellow.
Maurice d'Ocagne, of Paris, as Foreign Honorary Member.
Class II, Section 1 (Geology, Mineralogy, and Physics of the Globe):
John Horne, of Edinburgh, as Foreign Honorary Member.
Esper Signius Larsen, Jr., of Cambridge, as Fellow.
Kirtley Fletcher Mather, of Cambridge, as Fellow.
Frank Bursley Taylor, of Fort Wayne, Ind., as Fellow.
Class II, Section 2 (Botany):
Nathaniel Lord Britton, of New York City, as Fellow.
Albert Spear Hitchcock, of Washington, D. C., as Fellow.
Class II, Section 3 (Zoölogy and Physiology):
Samuel Randall Detwiler, of Cambridge, as Fellow.
Herbert McLean Evans, of Berkeley, Cal., as Fellow.
Alexander Grant Ruthven, of Ann Arbor, Mich., as Fellow.
Class II, Section 4 (Medicine and Surgery):
Nathaniel Allison, of Boston, as Fellow.
Robert Bayley Osgood, of Boston, as Fellow.
Benjamin White, of Boston, as Fellow.
Class III, Section 2 (Philology and Archæology):
William Chase Greene, of Cambridge, as Fellow.
Karl Friedrich Geldner, of Marburg, Germany, as Foreign Honorary Member.

Associates:—Charles Francis Adams, Frederick W. Allen, Charles F. Batchelder, Samuel F. Batchelder, Henry W. Cunningham, Philip L. Hale, Charles H. Hawes, Charles Hopkinson, William James, Everett Morss, Herbert Parker, Ralph A. Stewart, Charles H. Taylor, B. Loring Young.

The following communication was presented:

Mr. George H. Chase: "The Restoration of Ancient Monuments," with lantern illustrations.

Professor C. R. Lanman then introduced H. R. H. the Prince of Chandaburi, with remarks on the relations between Siam and the United States.

The following paper was presented by title:

"A Further Study of the Polyhedral Shapes of Cells. I. The Stellate Cells of *Juncus effusus;* II. Cells of Human Adipose Tissue; III. Stratified Cells of Human Oral Epithelium," by Frederic T. Lewis.

The Meeting was dissolved at 9.50 P. M.

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LOUIS DERR (1868–1923).

Fellow in Class I, Section 2, 1908.

Louis Derr, Professor of Physics at the Massachusetts Institute of Technology and a member of this Academy since 1908, died after a short illness at his home in Brookline on May 11, 1923. He was born in Pottsville, Pennsylvania, August 6, 1868. His father, Simon Derr, and his mother, Sarah Ann Sieger Derr, were descended from early settlers who came to Pennsylvania before the time of the Revolution. He prepared for college in the Pottsville High School, and through the influence of one of his teachers, an Amherst graduate, decided to go to Amherst College for his collegiate education. He entered in 1885 and graduated with Phi Beta Kappa honors in 1889. Having a decided bent towards science and engineering he then matriculated at the Massachusetts Institute of Technology with the intention of becoming an electrical engineer. He graduated with the degree of Bachelor of Science in 1892, receiving also in the same year the degree of Master of Arts from Amherst College.

Instead of entering at once upon the practice of his profession as electrical engineer, Derr accepted an appointment as assistant in the Department of Physics at the Institute of Technology, and finding the work of teaching highly congenial, decided to follow an academic career. He remained a member of the instructing staff of the Institute until his death, thus rounding out a period of continuous service of thirty-one years. At the time when young Derr was first appointed assistant, the course in Electrical Engineering was administered by the late Professor Charles R. Cross as a part of the Department of Physics, and his work was at first chiefly in the field of applied electricity. Later, in 1902, when the department of Electrical Engineering was separated from that of Physics, Derr decided to remain in the latter department, and to devote himself henceforth to pure science rather than engineering. He had long been interested in photography, and now turned his attention to the development of the instruction in theoretical and applied photography, and geometrical optics. In these subjects he became a recognized authority. He was appointed to a full professorship in Physics in 1909. He was an excellent teacher, and his lectures were models of clear exposition and skillful manipulation.

Professor Derr was elected to the American Academy of Arts and Sciences April 8, 1908. He was devoted to its interests, and gave unstintingly of his time to the work of the various committees to which he was appointed. He served on the house committee from May 1909 to May 1920, acting as chairman of the committee during the last year. In 1920 he assumed the chairmanship of the publications committee, which position he held at the time of his death. His membership in other scientific societies included the American Physical Society, the American Astronomical Society, Optical Society of America, and the Royal Photographic Society of London. He was editor for many years of the Encyclopedia of Engineering, and the author of a well-known treatise on photography, besides being a contributor to various engineering and scientific journals.

Professor Derr was a man of broad culture; he traveled extensively and was something of a bibliophile, his library containing many rare first editions of the works of early scientists. He took an active part in civic and church affairs in his home town of Brookline, and was held in high esteem by his colleagues and associates in the community in which he lived. In 1893 he married Miss Jane E. Coy, of Little Rock, Arkansas, who with his son, Thomas S. Derr, survive him.

H. M. Goodwin.

HAROLD CLARENCE ERNST (1856-1922).

Fellow in Class II, Section 4, 1889.

Dr. Ernst was born in Cincinnati, Ohio, July 31, 1856. His grandfather, John Zacharias Ernst, who was Amtbauermeister of the Sandltown Roringen, Hanover, fled to this country in 1804 with his eldest son, Andrew Henry Ernst, the father of Dr. Ernst, because his principles forbade compliance in enforcing the harsh exactions under Napoleon.

Andrew Henry Ernst was successful in business in Cincinnati and in horticulture as an avocation. He was one of the founders of the Horticultural Society of Ohio. Dr. Ernst's mother, Sarah H. Otis, a direct descendent of General Otis of the Revolution, was a person of initiative, a great worker during the Civil War, an abolitionist and a pioneer advocate of suffrage for women.

Harold Ernst graduated from Harvard College in 1876, from the Medical School in 1880. He received the degree of A. M. from Harvard in 1884. During his student days he was famous as a member of the baseball nine. He introduced curved-ball pitching and the great speed of his delivery led to the invention of the catcher's mask. During the four years he pitched, Harvard was supreme in college baseball.

After graduation from the Medical School, and a hospital service, Dr. Ernst entered the practice of medicine, taking residence in Jamaica Plain. In 1883 he married Ellen Lunt Frothingham of Boston.

From the outset he was interested in bacteriology. His first papers, published in 1883 and 1884, were upon the tubercle bacillus, and in 1885 he went abroad to study the bacteriology of tuberculosis with Koch. In that year he was made an Assistant in Bacteriology in the Harvard Medical School, and his lectures were the first in this country on bacteriology as a part of the regular instruction to medical students.

The importance of the new science of bacteriology in the service of medicine and surgery was steadily maintained by Dr. Ernst in the face of considerable opposition on the part of some of his older colleagues, who wished to deny him laboratory space and teaching privileges. In 1888, laboratory work became a part of the bacteriological instruction to medical students. In 1889, Dr. Ernst was promoted to be Instructor, in 1891 to be Assistant Professor, and in 1895 he was made Professor of Bacteriology.

Dr. Ernst's researches are represented by publications on tuberculosis, suppurative bacteria, rabies, diphtheria, photomicrography with ultra-violet light, and subjects in immunology. In 1895, he conducted, for the Massachusetts Society for Promoting Agriculture, an extensive research upon the "Infectiousness of Milk from Tuberculous Cows with no Lesions of the Udder." His report was published in book form by the Society and contained the conclusion, as important today as then, that milk from tuberculous cows "with no appreciable lesion of the udder, may, and not infrequently does, contain the bacillus of the disease."

His services to medicine cannot be estimated from his published work. While Dr. Ernst possessed the true spirit of investigation and the abilities of a great investigator, his temperament, true to the Ernst-Otis inheritance, left him constantly engaged in efforts to accomplish work of immediate utility. He was a pioneer in placing laboratory resources at the command of practising physicians and he was frequently consulted by physicians and surgeons about problems having a possible solution through bacteriology. He introduced the sterilization of milk for infant feeding. He was the first to advise the use of and to prepare dry sterilized (baked) surgical dressings, which were used by Dr. J. C. Warren at the Massachusetts General Hospital. His laboratory for a time served in the capacity of a city bacteriological laboratory for Boston and a number of other cities and he first manufactured the tuberculin and diphtheria anti-toxin for the City of Boston.

In 1890 he again visited Koch's laboratory, for the purpose of learning about tuberculin and later at the Massachusetts Hospital, where he was physician to out-patients until 1900, he gave the therapeutic use of tuberculin a cautious and careful trial.

Upon the entry of this country into the recent war, he immediately offered his services and was placed in charge of laboratory work of the Northeastern Division with the rank of Major.

From 1914 until his death he was Visiting Bacteriologist to the Children's Hospital, a position which gave him great satisfaction because he loved the sense of being directly useful to patients.

Dr. Ernst for many years presided at the meetings of the Boston Society of Medical Sciences. In 1896 he founded the Journal of the Boston Society of Medical Sciences, which in 1901 was continued as the Journal of Medical Research, of which he was editor until his death, at which time volume forty-three was just completed. Dr. Ernst was active and largely instrumental in the founding of the American Association of Pathologists and Bacteriologists, which in 1901 took over the support of the Journal of Medical Research. Until 1920 Dr. Ernst was Secretary of the Association and it is not an exaggeration to say that until that time he was its presiding genius and benefactor. In all affairs at the Harvard Medical School he played an active and thoroughly loyal part. For many years he conducted the opposition to attempted legislation in Massachusetts against animal experimentation. As Chairman of the Committee on Public Lectures, he secured and arranged the admirable programs of Sunday lectures with which the public is familiar. He accumulated an important library relating to bacteriology, which is now the property of the Medical School.

His recreations were found out of doors,—golf, fishing and gardening were his favorite pastimes. He was particularly successful in horticulture and in growing pond-lilies.

Any account of Dr. Ernst's life demands a tribute to his character and personality. His high ideals, direct speech, promptness to action and punctiliousness in personal relationships and everything pertaining to the amenities of life, made him an outstanding personality in each of the groups he served. Throughout his professional life he gave freely of his time to his colleagues and his assistance and encouragement are gratefully remembered by many practitioners of medicine as well as by laboratory workers.

S. B. Wolbach.

WALTER E. FERNALD (1859–1924).

Fellow in Class II, Section 4, 1924.

Dr. Walter E. Fernald died at his home in Waverley, Mass., on November 26, 1924, after a brief illness which came on him in the midst of full activity.

He was born at Kittery, Maine, in 1859, and after the usual preliminary education he taught for a while in a country school and graduated from the Medical School of Maine (Bowdoin) in 1881. From 1882 to 1887 he gained experience in mental disorders in a state hospital in Wisconsin. In 1887 he took up his life task as superintendent of the Massachusetts School for the Feebleminded. In 1893 he transferred the school from its situation in South Boston to the new institution at Waverley, which had been constructed under his supervision.

In the new institution he devoted himself to the organization of the study and care of the feebleminded and elaborated an organization which became a model and attracted visitors from all over the world. He supervised with scrupulous care every detail of the training of his pupils, he collected around him an enthusiastic group of teachers. He instructed physicians, medical students, psychologists, social workers, nurses. He collaborated with others in detailed studies of the pathological anatomy of the brain in the feebleminded. He published a series of papers on the practical problems of the management of the feebleminded and on the social aspects of this subject. He devoted much time to instructing the public and to developing that community atmosphere of enlightened sympathy without which the adequate management of the feebleminded is impossible. For Dr. Fernald the mentally retarded child was always a concrete individual with his or her own special needs and limitations and assets, for whom he felt a profound sympathy.

During his life time the study of the feebleminded developed great prominence as the rôle played by this factor in such social problems as delinquency, alcoholism, venereal disease, etc. became more fully appreciated. An additional impetus to the study of the subject was given by the elaboration of special intelligence tests, with which the names of Binet and Simon are so closely associated. Both in appreciating the value of these new methods of measuring mental retardation and in recognizing the wider problems of the social bearing of feeblemindedness, the sound judgment and the rich experience of Dr. Fernald were of the greatest value.

In the early years of his work he, himself, had emphasized the relationship of feeblemindedness to delinquency and had been in favor of segregation as the most important method of dealing with the problem. As the years went on and as the application of intelligence tests disclosed the number of mentally retarded who get along placidly in a tolerant environment, he came to feel that the solution of the question was not in segregation but in the education of the community to institute measures for early diagnosis, suitable training and adequate social supervision. Thanks to his advocacy, the Massachusetts Legislature, in 1919, passed a law for the state-wide examination of mentally retarded children, and with this law on the statute book and the necessary machinery set in motion to give it full effect, Dr. Fernald might well say that he had brought to completion the main task of his life.

It was while he was giving a series of courses to the physicians, on whom devolved the responsibility for carrying out the actual survey of retarded children in the public schools, that the first symptoms of his fatal illness manifested themselves. His ability as a teacher was at its height; his mature judgment in all matters relating to the problem of mental defect was eagerly sought after by those working in this field; his influence on the community was profound; years of productive activity still seemed in front of him.

Although it was in the special field of mental defect that the work of Dr. Fernald was so outstanding, the whole Mental Hygiene movement found in him most cordial support. His influence on the community was based not only on his technical mastery of his subject nor on the efficiency of the institution which he had organized in such a marvellous way, but it was largely due to the very great charm of his personality with its outstanding simplicity and uprightness.

C. Macfie Campbell.

ABRAHAM JACOBI (1830–1919).

Fellow in Class II, Section 4, 1904.

Doctor Abraham Jacobi, the founder of American pediatrics, was born in Hartum, Westphalia, on May 6, 1830 and died at his summer home on Lake George on July 10, 1919, in the eighty-ninth year of his life.

Doctor Jacobi was an original and outstanding figure in medical, and in its wider sense the social life in the United States during a period of more than sixty years. A liberal by nature and conviction, he became a victim of the unsuccessful revolution in Prussia in 1848, was confined in prison, and finally escaped to the United States which became his adopted country, and which he was enabled to serve through a long, eventful life with rare professional and social zeal.

A product of the German gymnasial and university systems, Doctor Jacobi was fortunate during his course of medical study to come under several outstanding teachers and leaders of medical thought of his day. As a student in Göttingen he sat under the chemist Woehler and the clinician Frerichs, whose advanced views must have acted powerfully upon the acute and receptive mind of the young pupil.

Woehler will be remembered as the associate of Liebig in his investigations of uric acid, the cyanogen compounds, the oil of bitter almonds, the synthesis of sugar, morphine, and salicin, and especially by reason of his epoch-making discovery of the artificial synthesis of urea from ammonium cyanate. Finally, it may be said that the discovery made by him in 1824 and confirmed in 1842, that benzoic acid taken in with the food appears in the urine as hippuric acid, banished the current idea that plants only and not animals can synthetize their complex materials; it also may be regarded as the starting point of the modern chemistry of metabolism.

Frerichs, the clinician, on the other hand, first achieved a reputation as an ophthalmologist, and then a little later as the exponent of scientific internal medicine and, as it were, of experimental pathology, in which branch he became a leader of German thought. He was also a phenomenal teacher, whose studies under Liebig had provided him with a chemical proficiency deeply affecting his studies and views. He will be recalled as the discoverer of leucin and tyrosin in the urine of acute yellow atrophy of the liver, studies on cirrhosis and other diseases of the liver, brought together in a remarkable treatise on that organ, on malarial fever, and Bright's disease of the kidneys.

After many vicissitudes, Doctor Jacobi settled in 1853 in New York City where he opened an office and began, in the most modest manner, the practice of medicine. With his rare natural gifts of personality and mental vigor and his, for the period in the United States, extraordinary professional training, his rise to prominence and then to eminence could not long be delayed. It may perhaps be regarded as an incident of the influence of Frerich's teachings that in 1854 Doctor Jacobi invented a laryngoscope which antedated by a year the appearance of the Garcia instrument. The variety and breadth of his writings are other evidences of the fundamental discipline which he had imbibed.

How quickly his talents became obvious to his colleagues may be gathered from the fact that by 1857 Doctor Jacobi was lecturing in pediatrics at the College of Physicians and Surgeons of New York, and in 1860 was called to the first special chair of diseases of children in the New York Medical College. In 1865, Doctor Jacobi transferred to the clinical chair of his subject in the University of New York, and in 1870 he became clinical professor of pediatrics at the College of

Physicians and Surgeons. The last chair he held until his retirement from active teaching in 1899. In the meantime he engaged in teaching pediatrics in New York for nearly half a century and in establishing by his own superb efforts and the work of his many pupils that specialty on the broad lines along which it moves to-day. In this connection it is informing to recall that Doctor Jacobi established a pediatric clinic in 1862 which ran for two years and proved to be the first example of bedside teaching in internal medicine in the United States.

Doctor Jacobi's activities quickly made themselves felt in literary fields. In 1859 he took part in publishing a volume on The Diseases of Women and Children, and in 1862 in founding the *American Journal of Obstetrics*. It is not possible in brief compass to refer by name even to the stream of shorter and monographic papers and books which appeared from his busy pen. They cover a very wide territory and variety of topics; they are observational, critical, historical, and withal stimulating, wise and often whimsical and witty. Taken together they form a valuable, almost monumental collection which has had, and still continues to exert, a profound influence on the teaching and practice and advancement of pediatrics not only in the United States, but throughout the world. In 1909 Doctor Jacobi's miscellaneous papers were assembled in eight volumes (Collectanea Jacobi).

In 1873, Doctor Jacobi married Miss Mary C. Putnam, one of the first women doctors in the United States, who became herself an outstanding figure in medicine.

Doctor Jacobi possessed striking physical and personal traits. Short, slender, virile, with a large noble head surmounted with a splendid crown of hair, he arrested attention everywhere. His features were pronounced but finely formed, his face lighted by brilliant, searching eyes and moulded by a play of expressions responding to emotions and thoughts the most diverse. He was a delightful companion and devoted friend and, as everyone could see, the kindest as well as the most skillful physician.

It was one of Doctor Jacobi's genial habits to attend all manner of medical gatherings. To do this was with him a duty as well as pleasure and benefit, and the practice was continued until the very end of his life. He was a conspicuous, outstanding, welcome personality at innumerable meetings, conventions, and congresses of local, national, and international scope. To these he was often a contributor, in these he often held high office: every official honor was in time shown him by his admiring colleagues; but whether as officer or private, he attended sessions, listened closely, and at times and always temperately participated in the discussion.

Doctor Jacobi befriended all manner of persons, professional and otherwise, but he had an especially warm place in his capacious heart for the striving youth aiming at a real goal in medicine. To him he never failed to hold out a helping hand, but with discrimination. His words were always heartening if sometimes, though rarely, corrective. Many men now at, or even beyond, the zenith of honorable and distinguished careers recall with appreciation and affection the words of good cheer, the gently hinted caution, so happily and so wisely administered at just the right moment by Doctor Jacobi.

There is something noble in the life of a great physician. His presence at and participation in those profound primal events in which life is ushered in and assisted out of the world, with all the attendant emotions extending from brightest hope through love to utter despair, bring him nearer to unadorned human nature than is vouchsafed other men. Hence it is that the name of such an exemplar of that ancient profession as Doctor Jacobi, who was at once pioneer practitioner and investigator, teacher, good citizen, friend, benefactor, will long endure in the annals of American medicine.

SIMON FLEXNER.

JACQUES LOEB (1859–1924).

Fellow in Class II, Section 3, 1914.

The death of Jacques Loeb at the zenith of his career came as a great shock to all who knew him. His loss will be keenly felt not only among biologists but among men of science throughout the world.

I shall not here attempt to disclose the qualities which quickly made him a recognized leader in biology or the personal traits which endeared him to an ever widening circle of devoted friends. It need only be said that his goodness is an open book to all who knew him and his greatness will loom even larger with the lapse of time.

Of his influence on biology it would be difficult to give an adequate account. All his fellow-workers felt its spell and his death will not impair it: many who never knew him will be unconsciously guided by it in ways little guessed.

His conceptions were often so bold and original as to startle conventional thinkers. Fearless in attacking difficult questions of fundamental importance, he showed almost uncanny insight into the most obscure and baffling matters. His discoveries often had a dramatic quality in their unexpected and beautiful solutions of perplexing problems. His results were reached by methods so simple as to compel admiration. His papers were always clear, cogent and convincing. It is no wonder that he was a powerful stimulus to his fellow-workers.

His mechanistic view-point profoundly affected biology and medicine and had an important influence on psychology and philosophy.

It is possible only to recall some important facts in his career, which to so many has served as example and inspiration.

He was born in Mayen, Germany, April 7, 1859. He attended the Askanisches Gymnasium in Berlin, 1877–1880; the Universities of Berlin, 1880; Munich, 1880–1881; Strassburg, 1881–1885 (M.D., 1884; Staatsexamen, 1885), and in 1885–1886 he was again at the University of Berlin.

He was assistant in physiology at the University of Würzburg, 1886–1888, and at the University of Strassburg, 1888–1890. From 1889 to 1891, at the Naples Zoological Station, he laid the foundation for his work on marine biology which was afterward continued at the Marine Biological Laboratory at Woods Hole, Massachusetts, and at Pacific Grove, California. In 1891 he came to America and became associate in biology at Bryn Mawr College. From there he went to the University of Chicago as assistant professor of physiology and experimental biology in 1892; later he became associate professor, and then professor of physiology. In 1902 he accepted the professorship of physiology at the University of California. In 1910 he became Member of The Rockefeller Institute for Medical Research and remained there until his death.

Attracted in his student days by certain aspects of metaphysics, he became especially interested in problems connected with the freedom of the will. The idea that certain brain functions are localized in definite centers and that human conduct may be profoundly affected by disturbances in these centers led him to study medicine to gain the technical knowledge needed for experiments in order to learn to what extent apparently volitional acts can be controlled by physical and chemical agencies. These studies led him, before the age of thirty, to the revolutionary conception that the actions of animals may be largely explained on a physicochemical basis. He took from the botanist, Julius Sachs, the idea of tropisms and applied it to animals. He sought a mechanistic explanation of animal conduct which should drop the question of purpose and reduce the reactions of animals to quantitative laws. His researches in this field are summarized in English in two volumes, "Comparative Physiology of the Brain and Comparative Psychology" (1900) and "Forced Movements, Tropisms and Animal Conduct" (1918).

He next undertook to control the growth and form of animals by physical and chemical means. The studies in this field, which he named "Physiological Morphology," covered a very wide range and continued to receive his attention up to the time of his death. The goal at which he aimed was to secure the same degree of control over living matter that the chemist and physicist have over their material and he felt that the best prospect of success lay in applying their methods to biology.

It was characteristic that in the pursuit of these researches he was prompt to utilize recent discoveries in physical chemistry, particularly those connected with the ionic theory and the theory of osmotic pressure. From 1897 on he published papers applying these theories to biological phenomena and they may be said to form the leitmotif of his subsequent work. Out of them arose, almost at once, two important discoveries, antagonism and artificial parthenogenesis. Both resulted from experiments of the simplest kind, carried out on marine organisms.

It was found that the fish, *Fundulus*, can grow and develop in distilled water but soon dies if sodium chloride is added. The addition of other salts, particularly those of potassium and calcium, which are themselves toxic when they alone are present, produces a harmless solution. Loeb called this a balanced solution, that is, one in which the toxic effects of one substance are offset by the antagonistic action of other substances. This conception proved to be a fruitful one.

As the result of subsequent experiments he concluded that these facts may be accounted for by the effects of the antagonistic substances on the permeability of the protoplasm. It may be added that his experiments on permeability, which covered a wide range, led to the conclusion that Overton's hypothesis is untenable.

His experiments on artificial parthenogenesis may be truly called epoch-making. In 1913 he reviewed them in a volume entitled "Artificial Parthenogenesis and Fertilization" (following the publication of two volumes on this subject in German). He continued work in this field for some years after the publication of this volume. He aimed at a complete analysis of the mechanism of fertilization, development and heredity. His work on fertilization indicated that the principal function of the sperm in stimulating development is to carry into the egg a substance which produces a surface change leading to the production of the fertilization membrane. This effect could be brought about, independently of sperm, by a variety of physical and chemical agencies.

At the same time he succeeded in finding means to bring about crosses which never occur in nature. By a slight change in the composition of the sea water, eggs could be fertilized by sperm of other species which could not normally enter the egg.

In the course of these studies a great number of questions presented themselves which excited his keen interest. The variety and extent of these problems can only be realized by a careful consideration of the contents of the volumes entitled "Studies in General Physiology" (1905), "Dynamics of Living Matter" (1906), "The Mechanistic Conception of Life" (1912) and "The Organism as a Whole" (1916).

Among these subjects may be mentioned the rôle of oxygen in metabolism, toxicity and development. Another in which he was deeply interested is the cause of natural death and the means of lengthening life. In the course of these studies he found that in certain cases low temperature prolongs life to a remarkable degree. In this connection it may be recalled that the first studies on the temperature coefficient of the heart beat and of the transmission of stimuli in nerves were made by his students.

Another investigation which he initiated, and which grew naturally out of these studies, showed that the electric potentials existing in the organism can, in many cases, be accounted for qualitatively and quantitatively by relatively simple means, and that they can be imitated to a considerable extent by artificial models.

As he truly said, many biologists accept a mechanistic explanation of various functions of the organism but fail to employ mechanistic conceptions in dealing with the larger problems of organization and adaptation. It was, however, precisely these problems which fascinated him and he did not hesitate to attack them from his point of view. He showed that many characteristics of the organisms which are regarded as adaptive may be explained on a mechanistic basis. He felt that here, as elsewhere in biology, progress requires quantitative investigation and with this in view he began the quantitative studies on regeneration upon which he continued to work until his death. His latest book, "Regeneration, from a Physico-Chemical Viewpoint" (1924), is devoted to this subject.

His work began, as was said, with questions concerning the freedom of the will. But he found that in order to study these he must attack the simpler problems involved in the behavior of lower organisms. These in turn required for their understanding a study of the physical and chemical reactions on which they are based. It is, therefore, not surprising that in the closing years of his life he came to devote his attention almost wholly to the properties of colloids, upon which life phenomena so largely depend. In a recent volume on "Proteins and the Theory of Colloidal Behavior" (1922) he contends that the behavior of colloids may be explained by the ordinary laws of chemistry without recourse to theories based on adsorption. As in earlier researches he had found a clue to the solution of many problems by applying the theories of electrolytic dissociation and osmosis, so in the work of his later years he discovered a guiding principle in the theory of the Donnan equilibrium. By applying this he was able to give quantitative explanations of some of the most important properties of proteins and to reduce them to simple mathematical laws. These studies, important for chemistry as well as for biology, form a fitting termination of his activity.

Thus closed a career rich in the joy of pioneer adventure in fresh fields of thought, abounding in brilliant discoveries, and splendidly stimulating far beyond the boundaries of biology. It will always stand out as a prominent feature of the progress of biology toward the status of an exact science. It is a career which reveals everywhere a creative imagination and capacity found only in minds of the highest order.

W. J. V. OSTERHOUT.

THEOPHIL MITCHELL PRUDDEN (1849–1924).

Fellow in Class II, Section 4, 1904.

Theophil Mitchell Prudden was born in Middlebury, Connecticut, July 17, 1849. His father, George Peter Prudden (1816–1872), Yale 1835, a clergyman living in Medina, New York, Middlebury, Southbury, and Watertown, Connecticut, was a descendant of the seventh generation of Reverend Peter Prudden, founder and pastor of the First Church in Milford, Connecticut. His mother, Eliza Ann Johnson (1819–1889) of South Britain, Connecticut, was a daughter of Eben Johnson and Sally (Mitchell) Johnson.

His early boyhood was spent in country parsonages and he attended public and private schools in several New England towns. He prepared for Yale at Wilbraham Academy, Wilbraham, Massachusetts. He received from Yale the degree of A. B. in 1872, M. D. in 1875, and in 1897 the honorary degree of LL. D. In college he took the biological course and from 1872 to 1874 instructed in chemistry. His education was paid for largely by his own earnings and throughout his early years simple living was a necessity. None the less, he received several honors and a prize in mineralogy.

Attracted by several fields in science, he chose the study of medicine because he thought that he could contribute most there to mankind. He spent a year in hospital service in New Haven and two years in post graduate medical work in Germany. In 1878 he was brought to Columbia University by its late Professor Francis Delafield as his assistant in the newly formed Department of Pathology. In 1882 he worked with Robert Koch in Germany upon the bacteriology of tuberculosis, and in that year was made Director of the Laboratory of the Alumni Association of the College of Physicians and Surgeons.

In 1892 he became Professor of Pathology in the College of Physicians and Surgeons in Columbia University, New York City. In 1909 he resigned because of poor health and became Emeritus Professor.

Professor Prudden was one of a few pioneers noted for their influence in stimulating and guiding the development of scientific medicine in this country. This type of service rather than any outstanding discovery distinguishes his career, though his own scientific contributions to medicine were numerous and of no little importance. Had he lived in an older country and been one of many engaged in research in pathology, instead of a very few, his energies might have been expended more completely in the laboratory. The character of his earlier researches indicate the probability of truly great achievements, for he possessed ideas which have been developed only in very recent years, as are indicated by his publications upon living structures of cartilage, of blood, and the action of bacteria in the living body.

As it was, his most important work was in organizing and developing research and public health service. He was largely responsible for the development of the New York City Board of Health, especially in its laboratory aspects, an institution that antedated Koch's Hygienic Institute in Berlin and which has contributed enormously to the methods of preparation of antitoxins, vaccines and to the control of rabies, tuberculosis, diphtheria and the venereal diseases. As Vice-President and member of the Board of Directors of the Rockefeller Institute of Medical Research since its establishment and as member of the International Health Board of the Rockefeller Foundation, he had extended opportunities for useful influence. The organization of the New York State Board of Health, with all its ramifications, county laboratories and the resulting State Association of Public Health Laboratories, is largely the result of Dr. Prudden's endeavors.

Dr. Prudden's personality will be suggested by a quotation from a private letter. "His erect, slender figure had an air of distinction in any company. He was dignified, quiet, and too modest to allow many to know him intimately, but because of these qualities there was a rare charm in his conversation as in his writings. So strong was his friendly interest in the problems and needs of his friends that he was always ready to take endless pains to help them with advice, suggestion or criticism, if asked."

In the classroom and in the direction of his laboratory he was rigid and exacting in maintaining a high standard of work. Every manuscript from his laboratory, and the list of publications is a large one, passed through his hands and was carefully read and criticized, and his own high standards applied as a test. The publications of his laboratory have had a permanent value and stand today as an important part of the foundations of pathology in this country. The Text Book of Pathology by Delafield and Prudden has gone through twelve editions. In its present form it is largely the work of Professor Prudden and is perhaps the most widely used book of its sort in this country. He was also the author of a number of popular books, among them "Dust and its Danger" in 1894, which is regarded by sanitary engineers as a classic of its kind.

Outside of the medical sciences, American anthropology and archeology occupied Prudden's interest. One summer, 1873, he spent in Nebraska searching for fossils with Professor O. C. Marsh. He spent eight summers on the western deserts, in Colorado, Utah, Arizona and New Mexico, with pack train, exploring the habitations of the cliff dwellers and the desert structures of the prehistoric races in the neighborhood of the Grand Canyon of the Colorado River. Some of his observations were published in 1907 in a book entitled "The Great American Plateau." The objects and information he collected were presented by him to Yale University and to the American Museum of Natural History.

Professor Prudden was a member of the National Academy of Sciences, the American Anthropological and Ethnological Associations, the Archeological Institute of America, and of many medical societies. He died April 10, 1924. After his retirement as Professor in Columbia University his active work was on the Board of Directors of the Rockefeller Institute and the Public Health Council of the State of New York.

He was unmarried. In his early life he displayed rare initiative, unusual scientific interests and abilities. During what should have been his productive period, he gave his energies and talents to the development of teaching, the training of investigators, and application of his subjects to the public welfare. In these fields he evidently felt that he could accomplish more of real service than by his individual efforts in the laboratory.

By nature modest, shy and sensitive, absolutely devoid of political guile, the best measure of his ideals and abilities is his successes towards the solution of great problems of public health service.

S. B. WOLBACH.

LINCOLN WARE RIDDLE (1880–1921).

Fellow in Class II, Section 2, 1915.

Doctor Lincoln Ware Riddle was born at Jamaica Plain, Massachusetts, October 17, 1880, and died at his home, 123 Walker Street, Cambridge, Massachusetts, January 16, 1921. He was fitted for college at Roxbury Latin School and a private school in Boston. While pursuing his work in the former school he became interested in plant life and decided to make botanical science his life work. With this object in view, he entered Harvard University, from which institution he graduated in 1902. Four years later he received the doctorate in philosophy from his Alma Mater and began his career as an instructor in Botany at Wellesley College. He was made full professor at Wellesley in 1917 and held the position but two years when he was recalled to Harvard as assistant professor of Cryptogamic Botany. A year's leave of absence from Wellesley, in 1913, he spent in Europe, where he worked with several eminent botanists in a number of institutions and herbaria. He became an associate editor of the Bryologist in 1911 and retained the position until the time of his death in 1921. In this position he contributed several papers on lichens and had general charge of the work on lichenology.

I first learned of the young botanist, when during his first year at Wellesley, a letter was received in which inquiry was made regarding a number of matters lichenological. The correspondence begun at this time extended through 15 years and was terminated by Doctor Riddle's death in 1921. More than 40 letters were exchanged, and the letters received by me are still on file in my office. In these letters are discussed the plans and ambitions of the young, enthusiastic, well-trained, and rapidly growing lichenist, whose life work was, very unfortunately, cut off before he could accomplish more than a small portion of the research which he had planned, to say nothing of more which would doubtless have been considered, had he been spared to many years of usefulness. Many fine qualities are reflected in his published writings, which showed plainly the promise of yet better results to follow.

The first letter contains a statement of interest in lichens, extending back through several years, and an expression of the hope that, with the collections of Clara E. Cummings, whom he succeeded at Wellesley, and the great collections of lichens at Harvard, including the Tuckerman Herbarium, he might be able to do some valuable work on lichens. The early letters consisted largely of solicitations of advice and aid; and there was always an expression of keen appreciation. But he had the advantage of the Tuckerman Herbarium and other large collections of lichens at Harvard, and his facilities for work were superior, so far as authentic herbarium material for comparison was concerned. Consequently, he grew rapidly into a lichenist of excellent ability; and in the last years of his life the flow of specimens for determination ran about equally in two directions, each man sending specimens from groups of lichens in which the other was especially interested. So broad had become his knowledge of lichens and so discriminating his judgment regarding specific characters that it was a great advantage to be able to send to him herbarium specimens from genera which he was studying for publication.

It is the lot of scientists who correspond widely with other workers to know some of their co-workers only through the exchange of letters; and yet a thoroughly cordial friendship often springs up, even with workers in foreign lands, the correspondence often dealing with matters quite foreign to the common scientific interest. Thus, for more than a decade after the correspondence between Doctor Riddle and the present writer began, there was no personal acquaintance. Yet the letters retained give ample evidence of the cordiality of the friendship of the young lichenist. It was my privilege, while studying at Harvard in 1915, to meet my hitherto long-distant friend, both in the herbarium and in his home. From what I learned through personal touch, and earlier and later through correspondence, I can appreciate fully the words of three Harvard botanists, Thaxter, Osterhout, and Fernald, in a short sketch in Science, for July 1, 1921. These eminent botanists wrote as follows regarding his character:— "In his relations with his fellows he was the soul of honor and loyalty, with a personality that drew all men to him. In the class room his sympathy and friendliness, as well as his clarity of style, made his teaching attractive. His devotion to his students was noteworthy and his influence great and lasting. In the circle which mourns him, his careful scholarship was widely esteemed; he was honored by all for his inspiring ideals, and, beyond the lot of most men, he was sincerely beloved."

Early in my sojourn at Harvard in 1915 came an eagerly accepted invitation to a week-end visit in the Riddle home at Wellesley, where Mr. and Mrs. Riddle made me feel very much at home. This visit was filled with enthusiasms such as scientists know very well, the particular field of interest varying of course. In this instance, the interest centered upon lichenology, a branch of science in which workers are comparatively few so that one must seek his companionship largely through correspondence with men in various parts of his own country and in foreign lands as well. With workers so scattered, it requires more determination to keep at work; and those who are doing research appreciate very greatly the opportunity for interchange of ideas at close range. On the occasion of my visit, specimens and methods were gone over enthusiastically, Mrs. Riddle being a much-interested listener and often taking part in the discussions. Besides these professional interests, the town and the college furnished ample opportunity for exercise and diversion. While at the college, I discovered something of Doctor Riddle's interest in teaching, his methods, and his success. While walking through the streets of Wellesley and meeting various citizens, I learned of his interest in local matters both civic and religious; and it became apparent that he was a very useful citizen, much appreciated in the community as well as in the college.

During my sojourn at Harvard, my co-worker made several trips to Cambridge to consult material in the Cryptogamic Herbarium of Harvard University; and it was here and in his home, where he did his microscopic work and made notes, drawings, keys, and descriptions, that my high opinion of his methods and results, obtained through his letters and his published papers, was verified by first-hand observation. While at Harvard, I was working on the *Graphidaceae*, and it was my privilege to determine a considerable number of specimens of this family sent to him from Mexico, Bermuda, Jamaica, Porto Rico, and elsewhere. For this I was amply repaid by the help that I received from him on lichens of other groups.

The first letter mentioned above, came to me from the young enthusiast in 1907. In this letter Doctor Riddle made the following statement regarding his plans:—"As you know Miss Cummings' lichen collection has been added to the Cryptogamic Herbarium of Wellesley College. Having been interested in lichens for several years, I desire to acquire as good a knowledge of systematic lichenology as I can, that proper use may be made of the rich collections left by Miss Cummings." In a letter received early in 1909, a statement occurs regarding the helpfulness of "practice in the determination of fungi under Prof. Thaxter and Prof. Farlow"; and mention is made of a realization of the dangers of "hasty publication." The young lichenist's first work of note was done on the genus *Stereocaulon* and published in 1910. My specimens of the genus were studied by him in 1909. In returning the specimens he said—"the separation of '*paschale*' and '*tomentosum*' is the hardest problem in the whole genus," a statement which will doubtless seem reasonable to all who have worked on species of *Stereocaulon*.

Early in 1912, he wrote that he expected to sail for Europe in June, and November brought a letter from Geneva, Switzerland. In this letter, I was informed that he had worked during August and September at Kew, spending some time also in the British Museum and at the Linnaean Society. He wrote especially of the excellent condition of the lichens at Kew, these having been studied and arranged by Dr. O. V. Darbishire. From London he went with his family to Geneva to spend the winter. Part of the time here was spent with Professor R. Chodat "learning his methods of studying the green algae in pure cultures." The remainder of the time was spent in studying lichens in the herbaria, especially the Müller-Argau collection at Herbier Boissier. From Geneva the itinerary passed to Paris, and Helsingfors and Upsala were reached later. Leyden was to be reached if possible to study the Persoon types and Stockholm to study the Swartz types; but my correspondence does not tell me whether this part of the plan was realized. In closing a long letter written in Geneva, Doctor Riddle says very interestingly—"I do not know whether you mix sentiment with your Botany or not. But I have a great veneration for the historical side of Systematic Botany and much interest in the lives and voyages of the botanists of the past. These feelings have been fostered by an intimacy, during nearly twenty years, with the 'Letters of Asa Gray.' One of the treats, therefore, of this sojourn in Geneva has been working at the De Candolle Herbarium and library. I have enjoyed it all the more because I have been reading at the same time the 'memoirs' of the first De Candolle."

A considerable number of things were projected before going to Europe, while there, and after the return, which were never completed. One of these was a study of the *Lecideaceae*, mentioned in 1908, before young Riddle had published his first paper on lichens. In 1909, while writing about the work on *Stereocaulon*, he said:—"The only other genus in which I should care to express a critical opinion is *Pannaria*. I have been studying that genus and have recently been over the material in the Tuckerman Herbarium and in the Herbarium of the New York Botanical Garden. If you have any doubtful specimens of Pannaria, I should be glad to have a try at them." In 1910 he wrote that he was studying *Ramalina* and *Cetraria* and asked to see material which I had recently collected on the islands of Puget Sound. Another project which never came to light was "a set of analytical keys" to North American lichens.

In 1912, he wrote regarding the work on Miss Cummings' Jamaican lichens, which was published in the same year in a 15-page paper, containing 11 or 12 new species. In conclusion he said:—"Now I have another set of about 200 numbers of Jamaican lichens, as well as all of Pringle's Mexican Lichens to work up, so fate seems to have decided that the lichens of tropical America are to be my life-work." Following this the tropical lichens collected by Doctor Britton and others who went to tropical America from the New York Botanical Garden were examined. In 1914 came information regarding a paper on *Megalospora;* and in the same letter was the statement that he was working on the lichens for Doctor Britton's Flora of Porto Rico.

In 1917, Doctor Riddle agreed to prepare the manuscript for the *Arthoniaceae* for the North American Flora, and I sent to him the members of the family which I had collected in Porto Rico. In reply he wrote as follows:—"I am delighted to have all those *Arthonias*, and they will be invaluable to me both for the Porto Rico work and the North American Flora." After his appointment as assistant professor of Cryptogamic Botany at Harvard, he wrote thus:—"I now hope that I shall be able to realize my dream of writing a manual of the lichens of eastern North America. With daily access to the Tuckerman and the Farlow Herbaria I ought, certainly, to be able to accomplish something worth while in the lichens."

At this point it will be interesting to analyze some of the views of this young and efficient worker. He and the writer were in accord respecting the fungal nature of lichens; but he believed that the ascomycetous lichens should be kept distinct on account of their peculiar form of parasitism and the resulting peculiarities in vegetative structure. Regarding other lichens than ascomycetous, Doctor Riddle was in accord with my views, believing that these lichens, in which the vegetative structure had not been greatly modified by the peculiar. form of parasitism, should be distributed. The young lichenist held very nearly the same view as Doctor Farlow, differing from that of Karl M. Wiegand, under whom he worked at Wellesley College, in that the latter botanist believed that all lichens should be distributed. However, I find from our correspondence that Doctor Riddle had reached his conclusion, which he regarded a tentative one, wholly independently of those with whom he had studied. I may say further that passage from the belief that some lichens should be distributed to the belief that all should be distributed is an easy one, especially since vegetative structure should not play a very important part in classification.

We can arrive at some of Doctor Riddle's views through a consideration of his rather detailed review of "The Lichens of Minnesota." He was in agreement with respect to general treatment, generic and specific limitations, the citations of type species, uniform treatment of forms of sub-specific rank, the inclusion of few new species, and most other considerations. He did not agree regarding the sequence of orders, but would place the pyrenomycetous lichens first and modify the position of other coordinate groups accordingly. He said that "for those who hold the view that the lichens represent lines of evolution in a different direction from other fungi, a position for the lowly organized *Pyrenolochenes* at the beginning of the system seems more natural." I held the position that the lichens had branched off from other ascomycetes at several points, and that their arrangement should be similar to that of other ascomycetes, carpologic structure taking precedence over vegetative organization in determining position. With respect to the matter of specific names, Doctor Riddle regarded me "ultraconservative." I may quote him as follows:—"where an author of high standing shows that a specific name should be replaced by another on the basis of well-founded priority, there seems to be no good reason for not accepting the new name." I was in agreement with the principle involved in the criticism and readily admit that there was much ground for it.

In our correspondence, there was some discussion of lichenists and the literature of lichenology. Doctor Riddle was, of course, much interested in the voluminous publications of Doctor Vainio on tropical lichens, these papers being the best thing ever published on the subject and their author knowing more about tropical lichens than any other lichenist of the past or the present. I am not certain that the two men met at Helsingfors, but I think that they did. I suppose that Doctor Müller (Müller-Argau) and Doctor Zahlbruckner came next in esteem. Doctor Nylander was mentioned several times, but I cannot give Doctor Riddle's estimate of this man, who was in his prime in the days of Doctor Tuckerman. I am certain that the American lichenist, Doctor Tuckerman, was highly regarded, as he richly deserved.

I know no place in America where one could have a more excellent environment for gaining a wide knowledge of mycology, including lichenology, than Harvard University at the time when Doctor Riddle was there. Here were two mycologists of ripe experience and a breadth of knowledge of the whole field scarcely surpassed. Added to this were the unusually fine collections. To this favorable environment came a man admirably qualified to take advantage of it. The result was a rapid growth into a lichenist possessing a wide knowledge of the field; and it is my judgment that no more able or broadly trained man has graced this field of botanical research in my time. True, others have lived longer and have therefore been able to accomplish more; but it is doubtful whether any worker has gained more rapidly in ability to accomplish good results in research in a field in which such accomplishment is much needed. Doctor Riddle showed a genuine love for systematic lichenology, and this rested on a broad training in biological science and special training in modern mycology. He was becoming more and more interested in tropical lichens, and he would doubtless have contributed greatly to our knowledge of the tropical groups, had not his life-work been cut short at the time when he was ready to accomplish his best results. If the work on the lichens of Porto Rico, the manuscript on the *Arthoniaceae*, and the manual on the lichens of Northeastern North America could have been finished, mycological science would have been greatly enriched.

While a student at Harvard, the subject of this memoir had the advantage of excellent training in Mycology and had done a good deal of work on fungi, though he had published little. After his return to Harvard he again began to give time to fungi other than lichens. Regarding this he wrote as follows:—"I have done a good deal of work on fungi this year, especially the *Spheriales;* and that together with organizing my courses here has kept me pretty busy." The last paper to appear, "Observations on the genus *Acrospermum,*" is the only publication that resulted from these latest activities, which were the natural result of being placed in charge of courses in mycology. In these days when such courses do not attract many students, he wrote thus:—"I have enjoyed my work this year immensely. My advanced course in mycology has been larger and more interested than I had expected." This was written at the close of the first year of the professorship at Harvard and only seven months before his death.

Cut down in the prime of life, Doctor Riddle left, in the papers cited below, a considerable number of new species and much in way of general notes and in critical treatment of certain groups. This was all published in about 14 years, and it is probable that results would have followed much more rapidly in the next decade, had this efficient worker been spared. There is need of many such able workers in lichenology, and it is earnestly to be hoped that they may appear.

The remuneration is mainly such as our departed friend sought,—the enjoyment of the work and the pleasure of doing something that will aid others in securing desired information and in obtaining a larger view and a better enjoyment of nature.

Appended below is a list of works published by the subject of this memoir. Perhaps notes, including lists of new species, should have been included; but workers would wish to consult the sources of information in any event. Therefore, such an addition to this memoir seems scarcely necessary.

Doctor Riddle's Botanical Publications.

Contributions to the cytology of the *Entomophthoraceae;* preliminary discussion. Rhodora **8:** 67, 68. 1906.

On the cytology of the *Entomophthoraceae.* Proc. Am. Acad. Arts Sci. **42:** 177–195. pl. 1–3. 1906.

Notothylas orbicularis in Massachusetts. Rhodora **9:** 219, 220. 1907.

Notes on some lichens from the Gaspé Peninsula. Rhodora **11:** 100–103. 1909.

A key to the species and principal varieties of *Cladonia* occurring in New England. Rhodora **11:** 212–214. 1909.

Preliminary lists of New England plants,—XXIII. *Cladoniaceae.* Rhodora **11:** 215–219. 1909.

The North American species of *Stereocaulon.* Bot. Gaz. **50:** 285–304. f. 1–9. 1910.

"The Lichens of Minnesota," a review. Bryologist **13:** 97–100. 1910.

The rediscovery of *Parmelia lophyrea,* Bryologist **14:** 35. 1911.

The lichen-flora of the Santa Cruz Peninsula, a review. Bryologist **14:** 6, 7. 1911.

An enumeration of the lichens collected by Clara Eaton Cummings in Jamaica. Mycologia **4:** 125–140. 1912.

Review: A recent contribution to the ecology of mosses. Bryologist **15:** 67–69. 1912.

Report on the lichens in N. L. Britton: The vegetation of Mona Island. Ann. Missouri Bot. Gard. **2:** 35, 36 and 51–53. 1915.

An undescribed species of *Cetraria*. Bryologist **16**: 17, 28. 1915.

The lichens of Bermuda. Bull. Torr. Bot. Club **43**: 145–160. 1916.

Report on the lichens in N. L. Britton: The vegetation of Anegada Island. Mem. New York Bot. Gard. **6**: 579, 580. 1916.

Some noteworthy lichens from Jamaica. Bull. Torr. Bot. Club **44**: 321–330. pl. 21. 1917.

The genus *Parmeliopsis* of Nylander. Bryologist **20**: 69–76. f, 1, 21. pl. 20. 1917.

Pyrenothrix nigra gen. et sp. nov. Bot. Gaz. **64**: 513–515. f. 1–4. 1917.

Some extensions of range. Bryologist **21**: 50. 1918.

Report on the lichens in N. L. Britton: The flora of the American Virgin Islands. Mem. Brooklyn Bot. Gard. **1**: 109–115. f. 1–3. 1918.

Chapter on lichens in N. L. Britton: The flora of Bermuda. 470–479. New York, Charles Scribner's Sons, 1918.

William Gibson Farlow. Rhodora **22**: 1–8. 1920.

Observations on the genus *Acrospermum*. Mycologia **12**: 175–181. pl. 11. 1920.

BRUCE FINK.

WALLACE CLEMENT WARE SABINE[1] (1868–1919).

Fellow in Class I, Section 2, 1894.

Our colleague, Wallace Clement Ware Sabine, was born in Richwood, Ohio, June 13, 1868. According to tradition, four racial strains were joined in him, each of his four names representing some family of his ancestors, one Scotch, one Dutch, one English, one French. The Sabines, supposed to be of Huguenot stock, came to Ohio from New England in the early part of the nineteenth century. The Wares, his mother's family, of English Quaker antecedents, came there about the same time, probably from New Jersey. Of his father's father, John Fletcher Sabine, the son of a circuit preacher, we are told, "He was of such gentle disposition that in manhood he renounced the stern faith of his father and came to believe that 'all men would be saved.'" . . . "He died at the age of eighty-nine, with mind as vigorous and clear as in youth, with a remarkably retentive memory. His wife was Euphemia Clement, a gentle, industrious, reliable woman. Hylas Sabine was their oldest son."

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[1] Much of what is given in this paper is contained in a *Minute* presented to the Harvard Faculty of Arts and Sciences on March 18, 1919. A much fuller biography of Professor Sabine has been written, by the author of the present paper, for the National Academy of Sciences and it should appear in print during the year 1925.

Of his mother's father, Jacob Reed Ware, it is written, "He was one of the early, ardent abolitionists and lived on the most direct line from Southern slavery to freedom in Canada." . . . "Untiring of body, alert of mind, and exceedingly strong of purpose, he lived in perfect health, with such simple habits that at the age of ninety-eight, without disease, he fell asleep." "J. R. Ware married Almira Wallace, a woman of force and uprightness. Anna Ware was their first daughter."

To those who knew Sabine well this brief family history is deeply significant. Gentleness, courtesy, rectitude, untiring energy, fixity of purpose that was like the polarity of a magnet, all these traits we find in him. It is interesting and impressive to see how the individualism and stern conscience that made his ancestors on the one side Quakers in England and on the other side, probably, Protestants in France found expression in him, under changed intellectual conditions. He was of the very stuff of which martyrs are made; in fact, he died a martyr to his sense of duty, but, with an austerity of morals and a capacity for devotion which none of his conspicuously religious forefathers could have surpassed, he held aloof, silently but absolutely, from all public profession of religious creed, and he took small part in religious observances.

As a child he was allowed to develop without forcing, but such was the natural vigor of his mind that he gained the degree of A.B. at Ohio State University at the age of eighteen. He is said not to have specialized in his college studies, but he had in Professor T. C. Mendenhall an inspiring teacher of physics, and his early interest in scientific matters is shown by the fact that he attended a meeting of the American Association for the Advancement of Science held in Philadelphia in 1884, when he was sixteen years old. On leaving Ohio State University in 1886 he came to Harvard as a graduate student in mathematics and physics, and he received the Harvard

A.M. in 1888. From 1887 to 1889 he held a Morgan Fellowship, but in the latter year he became an Assistant in Physics. Rather early in his Harvard residence he was taken by Professor Trowbridge as partner in a photographic study of the oscillating electric discharge, and he showed a remarkable aptitude for work of this kind, requiring high experimental skill, yet he never became a candidate for the Ph. D. Absorption in the work of teaching prevented him for several years from engaging deeply in further work of research. He spent his energy and his talents in building up courses of laboratory work, designing and making apparatus for instruction and in every way practicing with devotion the profession of a teacher. It is not too much to say that, for the fifteen years preceding his taking the duties of a Deanship, he was the most effective member of the Department of Physics in giving inspiration and guidance to individual students of promise. This was due in part to his comparative youth, though no one of the Department was repellently old; in part to his sympathetic willingness to give help and to spend much time in giving help, though others were not lacking in this quality. It was perhaps due mainly to the fact that, while he was no more deeply versed than others in the profundities of physics and mathematics, he had a peculiarly clear vision for the right kind of experimental problem and for the best way of attacking it, and his students instinctively, it may be, perceived this.

For a long time he seemed to be content to remain in comparative obscurity, while directing others into paths of conspicuous achievement. He was made Assistant Professor of Physics in 1895, after six years of teaching, in which he published little or nothing descriptive of research. This was partly because he had a most severe standard for what a research paper should be; it should describe some piece of work so well done that no one would ever have to investigate this particular matter again. To this standard he held true, with the result that his published papers were remarkably few and remarkably significant.

One might have expected him, when he found time for research, to take up some problem in light, for that had seemed to be his chief field of interest; but accident, and a sense of duty, turned him to a different quarter. The Fogg Art Museum at Harvard, on its completion in 1895, proved to have an auditorium that was monumental in its acoustic badness, and President Eliot, not fully realizing the importance of the step he was taking, but acting with his usual sure judgment of men, called upon Sabine to find a remedy, as a practical service to the University. With this warrant for diverting some of his energy from teaching, Sabine entered upon an investigation which proved to be his most conspicuous scientific work. Though he was dealing with a new structure, he was attacking a practical problem as old as the institution of public buildings. It had never been solved before in any thorough-going manner. He did solve it, and he did this not by virtue of any extraordinary resources given by modern science. He did it in such a way as to show that it might have been done by a man like him centuries before. Not only did he cure the defect of the particular room that first engaged his attention; he went on with his study till he could tell in advance what the acoustic qualities of a projected auditorium would be; and his visible instruments in all this achievement were organ pipes, common fabrics and materials, and the unaided human ear.

Was it, then, so easy and simple a thing to do? Did he merely happen to find the solution of a difficulty thousands of years old? No. He succeeded by reason of a combination of qualities, among which were unending patience and untiring energy. He must work in the small hours of the night, when other men had ceased from their noisy labors and when street-cars were infrequent; he must, for certain ends, work only in the summer, when windows could be kept open; in the early summer, before the crickets began their nightly din. He must work with the most scrupulous regard for conditions that to another might seem trivial. He once threw away the observations of months because he had failed to record the clothes he wore while at his work. Such was the difficulty of his undertaking, on the more physical side, and such the rigor of his devotion to it. We say of such a man, it is a pity he died so young; if he had taken care of himself, had been regular in his meals and in his hours of sleep, he would have had a long as well as a useful life. Yes; but a man must work according to his nature, and Sabine's temperament was not that of the ordinary man, not that of the ordinary scientific investigator. Some of the high things he did could not have been done by a man who must be regular at his meals and regular in his hours of sleep. When we remember how long the plagues he grappled with had baffled the efforts of others, and with what intensity of labor he exorcised them, it seems not irreverent or unfitting to recall the words: "This kind goeth not out but by prayer and fasting."

The establishment of a Graduate School of Applied Science, in place of the undergraduate Lawrence Scientific School which had existed at Harvard for a long time, was the result of a movement led by Sabine in 1906.

He took the Deanship of the Scientific School reluctantly, at the urgent request of President Eliot, but he threw himself into the duties of the office with characteristic energy, devotion, and elevation of ideals. It was his ambition to make this school as good as any school of applied science anywhere, and he strove for that end.

When this Deanship ended, he returned gladly to the work of teaching and research,[1] and but for the war he would probably have had before him a long career of growing usefulness and fame, and would have lived to a vigorous old age according to the habit of his ancestors. But from that fiery furnace into which other men were drawn by millions he could not hold himself back. He would have felt recreant if he had escaped unscathed. Going to France in 1916, with the intention of giving a course of lectures as Exchange Professor at the Sorbonne in the fall, he engaged during the summer in humanitarian work which took him to Switzerland. Overtaxing himself in this, he was attacked during the fall, while in Paris, by a disease which nearly ended his life and compelled the postponement of his Sorbonne lectures. When he was able to be moved, he went back to Switzerland, this time as a patient; but he gained strength, studying French constantly meanwhile, and in the spring of 1917 gave his lectures, on architectural acoustics, in Paris. These ended, he went through some months of extreme activity in the technical science service of the allied governments. Returning to America in the late fall, he went on with similar work in Washington, and elsewhere, coming to Cambridge for his lectures every week, eating and sleeping where and when he could, always too busy for the surgical operation which his physical condition demanded. He refused military rank, declaring, with that severity of judgment which sometimes verged upon intolerance, that the uniform should be worn only by those who were subject to the dangers and labors of the front. But he risked his life constantly, and at last fatally, in the service of the country and of Harvard University.

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[1] A special research laboratory was built for him at Geneva, Illinois, by his friend Colonel Fabyan, an energetic and successful merchant of Chicago. This laboratory still bears Sabine's name, and investigations in acoustics are carried on there, with the support of Colonel Fabyan, by Doctor Paul Sabine, a cousin of Wallace.

We have known in him a rare spirit, and we reverence his memory.

In 1900 Sabine married Miss Jane Downs Kelly, originally of New Bedford, Mass. She was a physician of established reputation in Boston before her marriage, and she continued to practice after marriage, especially in connection with the Children's Hospital. The fact that she found time and energy for this professional occupation, while performing with rare competence the duties of a housekeeper and mother of a family, is sufficient evidence of her unusual combination of qualities.

Two daughters, children such as one might hope to see from such parents, were the issue of this marriage—Janet, born in 1903, and Ruth in 1905. In them Sabine's fervently affectionate nature and his fatherly pride rejoiced. The younger, not thoroughly strong and sorely afflicted by the loss of her father, died suddenly in 1922.

EDWIN H. HALL.

JOHN TROWBRIDGE (1843–1923).

Fellow in Class I, Section 2, 1871.

John Trowbridge was born in Boston on August 5th, 1843, the son of John Howe Trowbridge and Adeline Trowbridge. At the age of eighteen, after attending the Bostin Latin School, he entered the Lawrence Scientific School by special arrangement, without any previous scientific training whatsoever. In spite of this disadvantage and the further handicap of a serious financial burden, he graduated with the degree of S.B. summa cum laude, in 1865. This brilliant success at Harvard doubtless decided the choice of his profession. Decision must have been difficult, since his interests at that time lay fully as much in the direction of art and literature, which remained delightful avocations, as in the direction of science.

From 1866 to 1869 he was tutor in Physics in Harvard College, and during the following year he served as Assistant Professor of Physics in the Massachusetts Institute of Technology. He returned in 1870 to Harvard, where he remained at first as assistant professor, and afterwards as full professor, until the date of his resignation in 1910,—a continuous service of forty years. He received the degree of S.D. in 1873, in 1888 was appointed Rumford Professor, and on his resignation became Rumford Professor Emeritus. For twenty-two years he was Director of the Jefferson Physical Laboratory.

He was a member of the National Academy of Sciences and the American Philosophical Society, and a Fellow of the American Academy of Arts and Sciences, serving as president of the latter body from 1908 to 1915. He was a member of the International Committee on Electrical Units at one time and an editor of the American Journal of Science for a considerable period.

On the 20th of June, 1877, he married Mrs. Gray, the widow of Thomas W. Gray, of Boston, whose young daughter, now Mrs. Edmund M. Parker, helped to brighten his life. Mrs. Trowbridge died in 1907 and his own death occurred in his eightieth year, on the 18th of February, 1923.

The earlier part of the long period during which Professor Trowbridge was a member of the teaching staff of Harvard College was characterized by the development of laboratory methods in teaching, and by the recognition of research as one of the fundamental activities of the department of Physics. Keenly alive to the lack of adequate facilities for the advancement of his chosen field along these lines at Harvard, Trowbridge projected a great physical laboratory and found the means to construct it. When the laboratory was begun, models for such a building were altogether lacking in this country, nevertheless, so excellent was its design that it still affords good facilities for teaching and research; it forms a lasting monument to the genius of the man who planned it.

Throughout his whole academic life, Trowbridge was a constant contributor to scientific literature. Besides a considerable number of reviews on the work of others, he published more than seventy original articles in the American Journal of Science, while his papers in the Proceedings of this Academy number thirty two.

Not unnaturally his earlier papers were devoted to matters connected with the class room,—the design of special instruments for the measurement of electric currents and the like; but almost from the beginning he was attracted by the problems of pure science. He possessed two most important gifts, a sort of scientific clairvoyance which enabled him to foresee the importance of certain lines of study, often before their full value was appreciated by his contemporaries, and the power to inspire his students with a love of research. Under his guidance many men who have won distinction in science took up original problems for the first time.

Some of the subjects to which he devoted himself and a few of the names of his collaborators may serve to indicate the use to which he put his talents. He published a number of papers dealing with magnetism which formed a starting point for a series of valuable studies by the late Benjamin O. Peirce. Probably animated by the work of Heinrich Hertz, he made, in collaboration with William Duane, a determination of the velocity of electric waves on wires, using the revolving mirror to determine the frequency, a method which at that time was a novelty. He devoted himself to spectroscopy, publishing with Professor T. W. Richards, papers on the spectra of Argon, on the multiple spectra of gases and on the ohmic resistance of gases. He inspired the late W. C. Sabine, then a graduate student, to enter this same field, thereby initiating a course of research in the Jefferson Laboratory which has continued to bear fruit to the present day. After X-Rays had been discovered, he contributed several articles on this fascinating form of radiation.

During the progress of these researches he realized the importance of a constant source of high potential; accordingly he caused the great storage battery to be constructed, which, unique in its time, is still in constant use.

Professor Trowbridge's personality was manifested not only in the intellectual activity which he exhibited, inspired, and fostered, but also in unselfish and constant devotion to the needs of his students and colleagues. Both traits of his character contributed toward the sentiment of respect and affection with which his memory will ever be cherished by those who came under his influence.

THEODORE LYMAN.

GEORGE CHANDLER WHIPPLE (1866–1924).

Fellow in Class I, Section 4, 1914.

The late George Chandler Whipple, professor of Sanitary Engineering at Harvard University, was born at New Boston, N. H., March 2nd, 1866, of New England lineage. He was a direct descendant of Matthew Whipple, who resided at Bocking, Essex, England, and whose son John settled at Ipswich, Mass., in 1637, taking a farm in Hamilton, Mass., the boundaries of which are still visible. On his mother's side, he descended from William Chandler, who came from England and settled in Roxbury, Mass., also in the year 1637.

When George C. Whipple was three years old, his parents moved to Chelsea, Mass. He attended the Williams School there, until 1880, and then the Chelsea High School, from which he graduated in 1884, and where he met Mary Rayner, whom he married nine years after his graduation.

In 1885, he entered the Freshman class of the Massachusetts Institute of Technology, in the Department of Civil Engineering. He took the degree of S.B. in Civil Engineering, June 1889. In his senior year, he came under the instruction of the inspiring leader in the science of public health,—Prof. W. T. Sedgwick. Young Whipple decided to make his life work in the sanitary branch of Civil Engineering. Immediately after graduating, he commenced his teaching career, by becoming an assistant in the M.I.T. Summer School of Engineering at Boston.

Whipple then came under the direction of Desmond Fitzgerald, a division superintendent of the Boston Waterworks, and a pioneer in the study of micro-organisms in potable waters. Whipple took up his work at the Chestnut Hill Laboratory, close by the Chestnut Hill Reservoir of the Boston Water Works. Here he applied himself whole-heartedly to the biological and chemical study of the city's water supply. He was soon the biologist in charge of the laboratory, and established for himself a wide reputation in the science and art of water-supply examination. For several years he was consulting water-supply biologist to the city of Lynn, Mass.

In 1897, Whipple resigned from the directorship of the Chestnut Hill Laboratory, and was appointed biologist and director of what is now probably the largest municipal water laboratory in the world; i.e. the Brooklyn, N. Y., Laboratory, near the Mount Prospect reservoir of the New York City Water Department. After the consolidation of water supplies for Greater New York in 1900, he had charge of the examination of all its waters. It was in the midst of such labors that he published, in 1899, his well-known text book —"The Microscopy of Drinking Water." Almost all of the 250 illustrations of micro-organisms in its appended plates were drawn by his own hand from his own examinations. After having passed through three editions, this is still a standard text-book.

In 1904, Whipple entered private engineering practice with Mr. Allen Hazen, also a M.I.T. graduate engaged in water-supply engineering, under the firm title of Hazen and Whipple, New York. He retained membership in this firm to the end of his life. In this connection, he made numerous studies of city water supplies and sewage disposals. His books on "The Value of Pure Water," "Typhoid Fever," "State Sanitation," "Vital Statistics," and "Fresh Water Biology," all testify to his active interests in sanitary engineering and public health. He served on many important commissions dealing with the solution of sanitary problems, both in this country and abroad. He was a prolific writer in his chosen field, and contributed largely for thirty years to its literature. He also specialized in the apparatus used for researches in water supply. Among other devices, he invented and developed a very practical portable instrument for measuring deep-water temperatures—the thermophone.

In 1907, Whipple was appointed Professor of Water Supply at the Brooklyn Polytechnic Institute. In 1911, he resigned to become Gordon McKay Professor of Sanitary Engineering at Harvard University, extending this work in 1915, to the Massachusetts Institute of Technology during the period of coöperation between the two institutions.

Whipple originated the plan of the Harvard-Technology School of Public Health. He succeeded with the late Prof. W. T. Sedgwick, and Dr. Milton Rosenau, in organizing this school in 1913, serving himself as its Secretary. For nine years this school did excellent work, granting Certificates in Public Health to many students who are today professionally occupied with Public Health in different parts of the world. The school came to an end with the opening of the Harvard School of Public Health in 1922.

During the thirteen years that Professor Whipple taught at Harvard University, his civic and scientific interests were many. For nine years he served on the Public Health Council of Massachusetts, refraining during that period of public office, from private practice in the State of Massachusetts. During the World War, he went, in 1917, to Russia, as Deputy Commissioner for the American Red Cross, with the rank of Major, returning the same year to instruct the Students' Army Training Corps at Harvard.

As a teacher, Whipple exerted a widespread, inspiring influence. Students came to his courses from all parts of the world. He was an earnest believer in the importance of sanitary engineering to the welfare of mankind. He preached at all times the doctrine of careful scientific study and the patient application of scientific knowledge to public benefit. He was at all times an advocate of schools of public health. When in Europe, in 1920, on sabbatical leave from Harvard, and acting as Director of Sanitation of the League of Red Cross Societies, he interested the Swiss authorities in the formation of a school of public health in Switzerland. A few days before his death, he was informed of the opening, at Basle, of the first Swiss School of Public Health, as a direct outcome of his incentive. In that same year of foreign service, he made a study of typhus fever in Roumania.

He also served as President of the Anti-Mosquito Association of the State of Massachusetts. A few months before his death, he accepted an invitation to become a Director of the Boston School of Physical Education.

In his own city of Cambridge, Mass., he served as chairman of the city's Sanitary Commission, a director of the Y. M. C. A., and of the Anti-Tuberculosis Association. He always advocated good citizenship and set his friends a shining example in that direction.

Prof. Whipple was a member of the American Society of Civil Engineers, President, in 1917, of the Boston Society of Civil Engineers, and a Fellow of the American Academy of Arts and Sciences. He was also an Honorary Fellow of the Royal Sanitary Institute of Great Britain, and a Fellow of the Royal Microscopic Society.

In personality, he was of a bright, sunny, and engaging disposition, making friends everywhere by his earnest, honest, and genial character. He had keen sympathies and a gift of social understanding, which endeared him to his students, his colleagues and his friends. He was an ardent advocate of his specialty, without being intrusive, and was frankly outspoken without wounding susceptibilities. He was a good friend, comrade and companion, happy in his home life, and seeking to make his home a source of happiness to all his friends. He brought sunshine, courtesy, humor, and keen insight, into all his discussions.

His tastes were cultivated and catholic. He was devoted to a wide range of literature, and was a votary of music. Although much of a student, he loved sunshine and the open air. Without being an athlete, he was fond of outdoor sports, especially of walking and canoeing. He was a member of the Appalachian Mountain Club, and climbed many lesser mountains in this country and in Europe. He was a keen observer, taking delight in geological, botanical and biological studies. Among his recreations were gardening, bowling and billiards. To each and every occupation he brought full and vigorous action. He was a tireless worker, and his life was full of accomplishment for the love of the deed done.

Quite unexpectedly, without premonition, and in the full vigor of his activities, he died suddenly of heart failure, on November 27th, 1924. His fame was still rising, and his many friends had expected for it a long and honored future culmination.

He is survived by his widow, Mary Rayner, a daughter, Mrs. Gerald M. Keith of New Haven, Conn., and a son, Rayner Whipple, a student at Bowdoin College.

A. E. Kennelly.

ROBERT WHEELER WILLSON (1853–1922).

Fellow in Class I, Section 1, 1896.

It is with deep regret that we have to record the loss to Astronomy of Professor Robert Wheeler Willson, whose sudden death on the first day of November (1922) terminated a long career of service, devotion, and benefaction to science, to education, and especially to Harvard University, his Alma Mater, for whom he always held the warmest affection and where he was actively connected with the faculty for more than a quarter of a century, serving successively in the capacities of tutor, instructor, assistant professor, and professor; and, for nearly fifteen years, a member of its administrative board.

In tracing the early events in his life, we learn that he was born in West Roxbury, Massachusetts, July 20, 1853, the eldest son of Reverend Edmund B. and Martha Buttrick Willson, his mother's grandfather being Major John Buttrick, who led the Concord fight in the Revolutionary War.

When Robert Willson was six years old, his parents removed to Salem, where his father became minister of the North Church, Unitarian, holding the pastorate for thirty-seven years, the remainder of his life. During the Civil War, he was granted leave of absence by his parish and served as chaplain of the 24th Massachusetts Regiment, an incident of which Professor Willson seldom spoke, but which made a strong appeal to his sense of loyal patriotism and devotion to duty.

His early education was in the public schools of Salem where he showed marked ability as a student, graduating with honors at the age of fifteen. His constant association with the port of Salem, during the impressionable years of his career, doubtless early instilled that fondness for things nautical which he kept through life, and which, later in his career, was to lead his attention to the application of Astronomy, to Navigation and the development of improved instruments and methods in this cause.

In the fall of 1869 Willson entered Harvard University, one of the youngest members of his class, graduating with the class of 1873, and receiving the degree of Bachelor of Arts cum laude and with highest honors in Physics and Chemistry.

Little opportunity was then open to the undergraduate to learn Astronomy. In 1872–3, a short course in Astronomy was given by Professor Lovering in connection with a course in Acoustics. Willson was early fascinated by the subject; and when the opportunity came upon graduation to assist Dr. B. A. Gould at the Argentine National Observatory, he gladly accepted, and went immediately to South America, where he spent a year in the work of the Observatory there. The following year he returned to Cambridge, to become assistant in the Harvard College Observatory under the direction of Joseph Winlock, and was greatly impressed with the assignment of duty to the then great 15-inch Equatorial.

During the years 1875–81, he served as tutor in Physics at Harvard. It was during this period that the occasion was offered of joining a party to observe the total eclipse of the sun on the 29th of July in 1878. The eclipse track crossed this country, extending from Alaska to the Gulf of Mexico, and we find Willson at Fort Worth, Texas, in company with Waldo, Seagrave, Reese, and others. Results of this expedition were published in the report on "Fort Worth Eclipse Observations." In 1881 he was called to Yale as Assistant Astronomer in the Winchester Observatory, and in December of that year he married Annie Downing West, of Salem, Massachusetts, who survives him. While at Yale, he had the unusual opportunity of being able to participate in a campaign organized for observing the transit of Venus on December 5, 1882, from which the solar parallax was redetermined.

Little opportunity for taking advanced degrees in Physics and Astronomy was to be had in America at that time; and in 1884 Willson went to Germany for further study, receiving the degree of Doctor of Philosophy magna cum laude from the University of Würzberg in 1886. Shortly after returning to this country, he collaborated with Professor Benjamin O. Peirce in the investigation of problems of heat conductions; and in 1891 was appointed instructor in Physics and Astronomy at Harvard. At the time of his appointment, there was no course in Astronomy given at the University, except as comprised in a course in spherical Trigonometry with applications, given by the Department of Mathematics. With opportunity for teaching in his chosen field, he immediately began to build up substantial courses in

Astronomy. In 1899, he received the appointment of Assistant Professor of Astronomy; and in 1903 was made full Professor, which position he held until his retirement in 1919 as Professor Emeritus.

With the rapid spread of laboratories of Physics, Chemistry, and other sciences, Professor Willson visualized a laboratory for Astronomy, and was a pioneer in the devising of apparatus and methods which should accomplish for the student of Astronomy what laboratory work in other fields was doing for the allied sciences and in 1903, the present Astronomical Laboratory of Harvard University was established.

Professor Willson's enthusiasm for his favorite science won him many friends among his students as well as his professional associates. He believed firmly in the educational advantages of Astronomy, and deplored the great ignorance often displayed by otherwise educated people concerning the most obvious of astronomical phenomena. As a teacher, he gave of his time unstintingly to clarify the more obscure parts of his subject, and was a master and a genius in the development of apparatus to demonstrate in a moment what otherwise hardly, if ever, could be satisfactorily explained. Other institutions, learning of the novel development in Astronomy at Harvard, began to establish laboratory work in Astronomy, and increasingly looked to him for suggestions and guidance in the acquisition of material and the development of methods.

It was his constant desire, not only to provide the best of equipment for laboratory instruction at Harvard, but to have such material and methods available for courses elsewhere. While engaged in such work, he found time to embody most of his ideas and methods in his book, "Laboratory Astronomy," published by Ginn and Company, which appeared in two successive editions. At a later date, he published "Sunrise and Sunset" in the United States, the results of a series of calculations, the remuneration from which went to the laboratory for the purchase of a Riefler Sidereal Clock, the highest grade astronomical timepiece manufactured.

In his courses in practical Astronomy, which were brought to a high degree of efficiency, he numbered among his pupils A. Hamilton Rice and Donald B. MacMillan, in whose training for explorations in remote parts of the earth he had taken no small part. It was most natural that when the Harvard Travelers' Club decided to publish a "Handbook of Travel," Professor Willson was selected for author of the section on "Determination of Position by Astronomical Observations," one of the most important sections of the volume.

While Professor Willson's chief life work was concerned with the teaching of Astronomy, which he ever regarded as of equal importance with research, he was by no means idle as an investigator. Among his earlier contributions should be mentioned the development of rotating and conical prisms for the positional observations of the sun, reducing the sun's disk to a small spot in the field of the telescope, thereby making possible observations of its transit with nearly the same facility as of a star image. For a number of years, he was at work on the design and construction of a new form of bubble sextant which should make unnecessary the use of a horizon in the measurement of altitudes at sea or land; and when the war brought exacting demands for navigational instruments for aviation, the Willson Bubble Sextant was thoroughly tried out and found to be superior to any similar invention. Professor Russell, of Princeton, basing his remarks on more than a thousand observations in flight, has said of it: "The bubble sextant appears to leave little to be desired as an instrument for aerial navigation . . . and gives a precision of observation much surpassing the limit set by the small irregularities in even the best piloting." Other instruments facilitating air navigation were devised by Professor Willson, and included a computing rule and box sight for determining ground speed.

Professor Willson's interest in aviation was aroused at the time of the aviation meet at Cambridge in 1910, when he assumed the responsibility of determining accurately by triangulation the record flights for altitude of Brookins and Grahame White in the demonstrations at Squantum. Detailed results of these determinations, which were of great interest at that time were published in the Proceedings of the American Academy in May 1911.

The variety of his interests led him to gain familiarity with the early writings of the Maya peoples of Central America. An examination of the astronomical tables of the Dresden Codex suggested to him the possibility of determining with considerable certainty the epoch of the calendar. Reasoning first from the evidence of the eclipse tables, and later from the pages of the planetary tables, he devoted a large part of his spare time from 1915 to the last year of his life, in carrying through an exhaustive study involving many long and laborious computations.

The results of his investigations, which he prepared for publication but a few weeks before his death, led him to the conclusion that the zero day of the Maya Calendar 4 Ahau 8 Cumhu was probably the same as August 29, 3512 B. C., as ordinarily reckoned by the chronologist. The detailed work of his investigation is comprised in "Astronomical Notes on Maya Codices" by Robert Wheeler Willson, a volume published by the Harvard University Press.

Professor Willson's early astronomical training lay chiefly in the older branches of Astronomy known as meridian circle work, calling for great exactness in observation and the utmost vigilance for instrumental and observational errors. This undoubtedly fostered a habit of critical examination of every piece of scientific work before unquestioned acceptance on his party of any results; and he subjected results from his own endeavors to the same critical standard of judgment which he demanded of others. This habit of searching inquiry in many instances deterred him from making public preliminary results of long investigations in many lines, which another with a less hesitant attitude would have published freely. He was never content with the partial solution of any problem. However important the intermediate step might be, or however long he may have been at work upon a given investigation, he was always hoping for the complete solution or the perfected method, and appeared dissatisfied at publishing anything less.

Professor Willson was much interested in the early history of Astronomy and its allied subjects, read Latin with comparative ease, and found pleasant relaxation in the collecting of rare volumes, early printings, and autographed copies of the masterpieces of science. He prized especially highly a presentation copy of Galileo's Dialogues, published in 1632, inscribed presumably in the handwriting of its author.

Apart from Astronomy, he was deeply interested in the public good. In 1917, he was elected a member of the special committee of the Boston Chamber of Commerce for considering the Daylight Saving

Plan, and used his technical knowledge and skill in the accumulation of evidence to favor its adoption for what he believed the public good.

His quick perception, breadth of knowledge, and wealth of experience kept his mind always active. The work he had planned to accomplish after retirement fully occupied him to the end, and the unfinished tasks he was obliged to leave stand as silent witnesses to his cherished ambitions and his unmitigated zeal. His attainments as a scientist and as an educator won him honored recognition. He was a member of Phi Beta Kappa, the American Astronomical Society, a fellow of the American Association for the Advancement of Science, and of the American Academy of Arts and Sciences since the date of his election, April 8, 1896.

As a teacher, he was highly esteemed by his students and by his colleagues. His genial personality, unfailing sense of good humor, and indefatigable optimism won many friends and transformed many a hopeless circumstance into noteworthy achievement. Those who worked with him and came to know him intimately found in him a wise counselor and a trusted friend.

H. T. STETSON.

American Academy of Arts and Sciences.

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[1] Died July 31, 1925.

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(Corrected to December 1, 1925.)

FELLOWS.—577.

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George Russell Agassiz	Boston
Raymond Clare Archibald	Providence, R. I.
Solon Irving Bailey	Cambridge
George David Birkhoff	Cambridge
Ernest William Brown	New Haven, Conn.
William Elwood Byerly	Cambridge
Florian Cajori	Berkeley, Cal.
William Wallace Campbell	Berkeley, Cal.
Julian Lowell Coolidge	Cambridge
George Cary Comstock	Beloit, Wis.
Leonard Eugene Dickson	Chicago, Ill.
Philip Fox	Evanston, Ill.
Fabian Franklin	New York, N. Y.
Edwin Brant Frost	Williams Bay, Wis.
William Caspar Graustein	Cambridge
Frank Lauren Hitchcock	Belmont
Edward Vermilye Huntington	Cambridge
Dunham Jackson	Minneapolis, Minn.

Oliver Dimon Kellogg	Cambridge
Edward Skinner King	Cambridge
Carl Otto Lampland	Flagstaff, Ariz.
Willem Jacob Luyten	Cambridge
George Abram Miller	Urbana, Ill.
John Anthony Miller	Swarthmore, Pa.
Samuel Alfred Mitchell	University, Va.
Clarence Lemuel Elisha Moore	Newton
Eliakim Hastings Moore	Chicago, Ill.
Frank Morley	Baltimore, Md.
Forest Ray Moulton	Chicago, Ill.
Henry Bayard Phillips	Boston
William Henry Pickering	Cambridge
Charles Lane Poor	New York, N. Y.
Roland George Dwight Richardson	Providence, R. I.
Henry Norris Russell	Princeton, N. J.
Frank Schlesinger	New Haven, Conn.
Harlow Shapley	Cambridge
Vesto Melvin Slipher	Flagstaff, Ariz.
Frederick Slocum	Middletown, Conn.
Virgil Snyder	Ithaca, N. Y.
Joel Stebbins	Madison, Wis.
William Edward Story	Worcester
Henry Taber	Worcester
Harry Walter Tyler	Newton
Oswald Veblen	Princeton, N. J.
Alfred North Whitehead	Cambridge
Frederick Shenstone Woods	Newton

CLASS I., SECTION II.—*Physics*.—51.

Joseph Sweetman Ames	Baltimore, Md.
Samuel Jackson Barnett	Pasadena, Cal.
Carl Barus	Providence, R. I.
Louis Agricola Bauer	Washington, D. C.
Percy Williams Bridgman	Cambridge
George Ashley Campbell	New York, N. Y.
Leslie Lyle Campbell	Boston

Emory Leon Chaffee	Cambridge
Daniel Frost Comstock	Boston
William David Coolidge	Schenectady, N. Y.
Henry Crew	Evanston, Ill.
Harvey Nathaniel Davis	Cambridge
Arthur Louis Day	Washington, D. C.
William Johnson Drisko	Winchester
William Duane	Boston
Alexander Wilmer Duff	Worcester
Arthur Woolsey Ewell	Worcester
William Suddards Franklin	Cambridge
Harry Manley Goodwin	Brookline
George Ellery Hale	Pasadena, Cal.
Edwin Herbert Hall	Cambridge
Hammond Vinton Hayes	Boston
Paul Alphonse Heymans	Brookline
John Charles Hubbard	New York, N. Y.
Gordon Ferrie Hull	Hanover, N. H.
Charles Clifford Hutchins	Brunswick, Me.
Frederic Eugene Ives	Philadelphia, Pa.
James Edmund Ives	Washington, D. C.
William White Jacques	Boston
Edwin Crawford Kemble	Cambridge
Norton Adams Kent	Cambridge
Frank Arthur Laws	Salem
Henry Lefavour	Boston
Theodore Lyman	Brookline
Ernest George Merritt	Ithaca, N. Y.
Albert Abraham Michelson	Chicago, Ill.
Dayton Clarence Miller	Cleveland, O.
Robert Andrews Millikan	Pasadena, Cal.
Harry Wheeler Morse	Stanford University, Cal.
Edward Leamington Nichols	Ithaca, N. Y.
Charles Ladd Norton	Boston
George Washington Pierce	Cambridge
Michael Idvorsky Pupin	New York, N. Y.
Frederick Albert Saunders	Cambridge

John Stone Stone	San Diego, Cal.
Maurice deKay Thompson	Brookline
Elihu Thomson	Swampscott
David Locke Webster	Palo Alto, Cal.
Edwin Bidwell Wilson	Brookline
Robert Williams Wood	Baltimore, Md.
John Zeleny	New Haven, Conn.

CLASS I., SECTION III.—*Chemistry.*—52.

Wilder Dwight Bancroft	Ithaca, N. Y.
Gregory Paul Baxter	Cambridge
Arthur Alphonzo Blanchard	Cambridge
Marston Taylor Bogert	New York, N. Y.
Bertram Borden Boltwood	New Haven, Conn.
William Crowell Bray	Berkeley, Cal.
Russell Henry Chittenden	New Haven, Conn.
Arthur Messinger Comey	Cambridge
James Bryant Conant	Cambridge
Tenney Lombard Davis	Cambridge
Charles William Eliot	Cambridge
Henry Fay	Boston
George Shannon Forbes	Cambridge
Edward Curtis Franklin	Palo Alto, Cal.
Frank Austin Gooch	New Haven, Conn.
Lawrence Joseph Henderson	Cambridge
Charles Loring Jackson	Boston
Walter Louis Jennings	Worcester
Grinnell Jones	Cambridge
Frederick George Keyes	Cambridge
Elmer Peter Kohler	Cambridge
Charles August Kraus	Worcester
Arthur Becket Lamb	Cambridge
Irving Langmuir	Schenectady, N. Y.
Gilbert Newton Lewis	Berkeley, Cal.
Warren Kendall Lewis	Newton
Arthur Dehon Little	Brookline
Charles Frederic Mabery	Cleveland, O.

Duncan Arthur MacInnes	Cambridge
Forris Jewett Moore	Cambridge
George Dunning Moore	Worcester
Edward Mueller	Cambridge
Samuel Parsons Mulliken	Cambridge
Charles Edward Munroe	Forest Glen, Md.
James Flack Norris	Boston
Arthur Amos Noyes	Pasadena, Cal.
William Albert Noyes	Urbana, Ill.
Thomas Burr Osborne	New Haven, Conn.
Samuel Cate Prescott	Brookline
Ira Remsen	Baltimore, Md.
Robert Hallowell Richards	Jamaica Plain
Theodore William Richards	Cambridge
Martin André Rosanoff	Pittsburgh, Pa.
Miles Standish Sherrill	Winchester
Harry Monmouth Smith	Brookline
Julius Oscar Stieglitz	Chicago, Ill.
Henry Paul Talbot	Newton
Richard Chase Tolman	Washington, D. C.
William Hultz Walker	Bridgeton, Me.
Willis Rodney Whitney	Schenectady, N. Y.
Robert Seaton Williams	Cambridge
Alpheus Grant Woodman	Watertown

Class I., Section IV.—*Technology and Engineering.*—46.

Henry Larcom Abbot	Cambridge
Comfort Avery Adams	Cambridge
Bernard Arthur Behrend	Boston
William Herbert Bixby	Chicago, Ill.
Francis Tiffany Bowles	Boston
Charles Francis Brush	Cleveland, O.
William Hubert Burr	New Canaan, Conn.
Vannevar Bush	Chelsea
John Joseph Carty	New York, N. Y.
Harry Ellsworth Clifford	Newton

Arthur Powell Davis	Washington, D. C.
Theodore Harwood Dillon	Cambridge
Gano Dunn	New York, N. Y.
William Frederick Durand	Palo Alto, Cal.
Charles Leavitt Edgar	Brookline
Frederic Harold Fay	Boston
Desmond FitzGerald	Brookline
John Ripley Freeman	Providence, R. I.
George Washington Goethals	New York, N. Y.
John Hays Hammond	New York, N. Y.
Ira Nelson Hollis	Worcester
William Hovgaard	Brookline
Hector James Hughes	Cambridge
Alexander Crombie Humphreys	New York, N. Y.
James Robertson Jack	Cambridge
Dugald Caleb Jackson	Cambridge
Lewis Jerome Johnson	Cambridge
Arthur Edwin Kennelly	Cambridge
Gaetano Lanza	Philadelphia, Pa.
William Henry Lawrence	Boston
Charles Thomas Main	Winchester
Lionel Simeon Marks	Cambridge
Edward Furber Miller	Newton
Frederick Law Olmsted	Brookline
Charles Francis Park	Taunton
William Barclay Parsons	New York, N. Y.
Harold Pender	Philadelphia, Pa
Albert Sauveur	Cambridge
Peter Schwamb	Arlington
Henry Lloyd Smyth	Watertown
Charles Milton Spofford	Brookline
Samuel Wesley Stratton	Cambridge
George Fillmore Swain	Brookline
Edward Pearson Warner	Cambridge
Robert Simpson Woodward	Washington, D. C.
Joseph Ruggles Worcester	Boston

CLASS II.—*Natural and Physiological Sciences.*—197.

SECTION I.—*Geology, Mineralogy, and Physics of the Globe.*—53.

Wallace Walter Atwood	Worcester
George Hunt Barton	Cambridge
Norman Levi Bowen	Washington, D. C.
Isaiah Bowman	New York, N. Y.
Thomas Chrowder Chamberlin	Chicago, Ill.
Henry Helm Clayton	Canton
Herdman Fitzgerald Cleland	Williamstown
William Otis Crosby	Jamaica Plain
Reginald Aldworth Daly	Cambridge
Edward Salisbury Dana	New Haven, Conn.
William Morris Davis	Cambridge
Benjamin Kendall Emerson	Amherst
William Ebenezer Ford	New Haven, Conn.
James Walter Goldthwait	Hanover, N. H.
Louis Caryl Graton	Cambridge
Herbert Ernest Gregory	New Haven, Conn.
William Jackson Humphreys	Washington, D. C.
Ellsworth Huntington	New Haven, Conn.
Robert Tracy Jackson	Peterborough, N. H.
Thomas Augustus Jaggar	Honolulu, H. I.
Douglas Wilson Johnson	New York, N. Y.
Arthur Keith	Washington, D. C.
James Furman Kemp	New York, N. Y.
Alfred Church Lane	Cambridge
Esper Signius Larsen, Jr.	Cambridge
Andrew Cowper Lawson	Berkeley, Cal.
Charles Kenneth Leith	Madison, Wis.
Waldemar Lindgren	Brookline
Frederic Brewster Loomis	Amherst
Alexander George McAdie	Readville
Kirtley Fletcher Mather	Cambridge
John Campbell Merriam	Washington, D. C.
William John Miller	Los Angeles, Cal.
Charles Palache	Cambridge

Percy Edward Raymond	Lexington
William North Rice	Middletown, Conn.
Austin Flint Rogers	Palo Alto, Cal.
Robert Wilcox Sayles	Chestnut Hill
Waldemar Theodore Schaller	Washington, D. C.
Charles Schuchert	New Haven, Conn.
William Berryman Scott	Princeton, N. J.
Hervey Woodburn Shimer	Hingham
Frank Bursley Taylor	Fort Wayne, Ind.
Thomas Wayland Vaughan	La Jolla, Cal.
Charles Doolittle Walcott	Washington, D. C.
Robert DeCourcey Ward	Cambridge
Charles Hyde Warren	New Haven, Conn.
David White	Washington, D. C.
Herbert Percy Whitlock	New York, N. Y.
Bailey Willis	Palo Alto, Cal.
Arthur Winslow	Boston
John Eliot Wolff	Pasadena, Cal.
Frederick Eugene Wright	Washington, D. C.

Class II., Section II.—*Botany.*—37.

Oakes Ames	North Easton
Joseph Charles Arthur	Lafayette, Ind.
Irving Widmer Bailey	Cambridge
Liberty Hyde Bailey	Ithaca, N. Y.
Edward Wilber Berry	Baltimore, Md.
Nathaniel Lord Britton	New York, N. Y.
Douglas Houghton Campbell	Palo Alto, Cal.
George Perkins Clinton	New Haven, Conn.
John Merle Coulter	Chicago, Ill.
Bradley Moore Davis	Ann Arbor, Mich.
Edward Murray East	Jamaica Plain
Rollins Adams Emerson	Ithaca, N. Y.
Alexander William Evans	New Haven, Conn.
Merritt Lyndon Fernald	Cambridge
Robert Almer Harper	New York, N. Y.
Albert Spear Hitchcock	Washington, D. C.

John George Jack	East Walpole
Edward Charles Jeffrey	Cambridge
Fred Dayton Lambert	Tufts College
Jacob Goodale Lipman	New Brunswick, N. J.
Burton Edward Livingston	Baltimore, Md.
George Richard Lyman	Morgantown, W. Va.
Elmer Drew Merrill	Berkeley, Cal.
Winthrop John Vanleuven Osterhout	New York, N. Y.
Charles Vancouver Piper	Washington, D. C.
Alfred Rehder	Jamaica Plain
Benjamin Lincoln Robinson	Cambridge
Charles Sprague Sargent	Brookline
William Albert Setchell	Berkeley, Cal.
Arthur Bliss Seymour	Cambridge
Erwin Frink Smith	Washington, D. C.
John Donnell Smith	Baltimore, Md.
Elvin Charles Stakman	St. Paul, Minn.
William Codman Sturgis	New York, N. Y.
Roland Thaxter	Cambridge
William Trelease	Urbana, Ill.
William Henry Weston, Jr.	Cambridge

CLASS II., SECTION III.—*Zoology and Physiology.*—64.

Nathan Banks	Cambridge
Thomas Barbour	Boston
Francis Gano Benedict	Boston
Henry Bryant Bigelow	Concord
Robert Payne Bigelow	Brookline
Edwin Garrigues Boring	Cambridge
William T. Bovie	Milton
Edward Allen Boyden	Newton Center
John Lewis Bremer	Boston
Charles Thomas Brues	Boston
Hermon Cary Bumpus	Duxbury
Walter Bradford Cannon	Cambridge
Thorne Martin Carpenter	Boston
William Ernest Castle	Belmont

Charles Value Chapin	Providence, R. I.
Benjamin Preston Clark	Boston
Samuel Fessenden Clarke	Williamstown
Edwin Grant Conklin	Princeton, N. J.
Manton Copeland	Brunswick, Me.
Joseph Augustine Cushman	Sharon
William Healey Dall	Washington, D. C.
Charles Benedict Davenport	Cold Spring Harbor, N. Y.
Samuel Randall Detwiler	Cambridge
Raymond Dodge	Middletown, Conn.
Gilman Arthur Drew	Woods Hole
Cecil Kent Drinker	Boston
Herbert McLean Evans	Berkeley, Cal.
Alexander Forbes	Milton
Samuel Henshaw	Cambridge
Samuel Jackson Holmes	Berkeley, Cal.
Leland Ossian Howard	Washington, D. C.
Herbert Spencer Jennings	Baltimore, Md.
Charles Willison Johnson	Brookline
Charles Atwood Kofoid	Berkeley, Cal.
Frederic Thomas Lewis	Waban
Ralph Stayner Lillie	Cleveland, O.
Richard Swann Lull	New Haven, Conn.
Edward Laurens Mark	Cambridge
Ernest Gale Martin	Palo Alto, Cal.
Albert Davis Mead	Providence, R. I.
Gerrit Smith Miller	Washington, D. C.
Edward Sylvester Morse	Salem
Herbert Vincent Neal	Tufts College
Henry Fairfield Osborn	New York, N. Y.
George Howard Parker	Cambridge
William Patten	Hanover, N. H.
Raymond Pearl	Baltimore, Md.
John Charles Phillips	Wenham
Henry Augustus Pilsbry	Philadelphia, Pa.
Herbert Wilbur Rand	Cambridge
Arthur Clarence Redfield	Boston

William Emerson Ritter	La Jolla, Cal.
Alexander Grant Ruthven	Ann Arbor, Mich.
Percy Goldthwait Stiles	Newtonville
John Eliot Thayer	Lancaster
William Lyman Underwood	Belmont
Addison Emory Verrill	Westville, Conn.
John Broadus Watson	New York, N. Y.
Arthur Wisswald Weysse	Boston
William Morton Wheeler	Jamaica Plain
Harris Hawthorne Wilder	Northampton
Edmund Beecher Wilson	New York, N. Y.
Frederick Adams Woods	New York, N. Y.
Robert Mearns Yerkes	Washington, D. C.

CLASS II., SECTION IV.—*Medicine and Surgery.*—43.

Nathaniel Allison	Boston
Edward Hickling Bradford	Boston
Charles Macfie Campbell	Cambridge
Alexis Carrel	New York, N. Y.
Henry Asbury Christian	Boston
Stanley Cobb	Ponkapoag
Rufus Cole	New York, N. Y.
Harvey Cushing	Boston
David Linn Edsall	Cambridge
Simon Flexner	New York, N. Y.
Joseph Lincoln Goodale	Boston
Robert Battey Greenough	Boston
Ross Granville Harrison	New Haven, Conn.
William Henry Howell	Baltimore, Md.
Reid Hunt	Boston
Henry Jackson	Boston
Elliott Proctor Joslin	Boston
William Williams Keen	Philadelphia, Pa.
Roger Irving Lee	Cambridge
Frank Burr Mallory	Brookline
William James Mayo	Rochester, Minn.
Samuel Jason Mixter	Boston

Robert Bayley Osgood	Boston
Francis Weld Peabody	Boston
William Lambert Richardson	Boston
Milton Joseph Rosenau	Brookline
Frederick Cheever Shattuck	Boston
Theobald Smith	Princeton, N. J.
Charles Wardell Stiles	Washington, D. C.
Richard Pearson Strong	Boston
Edward Wyllys Taylor	Boston
William Sydney Thayer	Baltimore, Md.
Ernest Edward Tyzzer	Wakefield
Frederick Herman Verhoeff	Boston
Henry Pickering Walcott	Cambridge
John Warren	Boston
John Collins Warren	Boston
William Henry Welch	Baltimore, Md.
Benjamin White	Boston
Francis Henry Williams	Boston
Simeon Burt Wolbach	Boston
Horatio Curtis Wood	Philadelphia, Pa.
Hans Zinsser	Boston

CLASS III.—*Moral and Political Sciences.*—183.

SECTION I.—*Theology, Philosophy and Jurisprudence.*—44.

Thomas Willing Balch	Philadelphia, Pa.
Simeon Eben Baldwin	New Haven, Conn.
Joseph Henry Beale	Cambridge
Charles Henry Brent	Buffalo, N. Y.
Howard Nicholson Brown	Boston
Edmund Burke Delabarre	Providence, R. I.
Frederic Dodge	Belmont
Edward Staples Drown	Cambridge
William Harrison Dunbar	Cambridge
William Herbert Perry Faunce	Providence, R. I.
William Wallace Fenn	Cambridge
Frederick Perry Fish	Brookline

Paul Revere Frothingham	Boston
George Angier Gordon	Boston
Alfred Hemenway	Boston
William Ernest Hocking	Cambridge
Charles Evans Hughes	New York, N. Y.
Frederick John Foakes Jackson	New York, N. Y.
William Lawrence	Boston
Frederick Lawton	Boston
William Caleb Loring	Boston
Nathan Matthews	Boston
William McDougall	Cambridge
Edward Caldwell Moore	Cambridge
John Bassett Moore	New York, N. Y.
George Herbert Palmer	Cambridge
Charles Edwards Park	Boston
Leighton Parks	New York, N. Y.
Francis Greenwood Peabody	Cambridge
George Wharton Pepper	Philadelphia, Pa.
Roscoe Pound	Belmont
Elihu Root	New York, N. Y.
James Hardy Ropes	Cambridge
Arthur Prentice Rugg	Worcester
Austin Wakeman Scott	Cambridge
Henry Newton Sheldon	Boston
Moorfield Storey	Boston
William Howard Taft	Washington, D. C.
William Jewett Tucker	Hanover, N. H.
William Cushing Wait	Medford
Eugene Wambaugh	Cambridge
Edward Henry Warren	Boston
Winslow Warren	Dedham
Samuel Williston	Belmont

Class III., Section II.—*Philology and Archæology.*—43.

Francis Greenleaf Allinson	Providence, R. I.
William Rosenzweig Arnold	Cambridge
Maurice Bloomfield	Baltimore, Md.

Franz Boas	New York, N. Y.
Ingersoll Bowditch	Jamaica Plain
Carl Darling Buck	Chicago, Ill.
Eugene Watson Burlingame	New Haven, Conn.
Edward Capps	Princeton, N. J.
George Henry Chase	Cambridge
Walter Eugene Clark	Chicago, Ill.
Roland Burrage Dixon	Cambridge
Franklin Edgerton	Philadelphia, Pa.
Jesse Walter Fewkes	Washington, D. C.
Jeremiah Denis Mathias Ford	Cambridge
Pliny Earle Goddard	New York, N. Y.
Charles Hall Grandgent	Cambridge
Louis Herbert Gray	Lincoln, Neb.
William Chase Greene	Cambridge
Charles Burton Gulick	Cambridge
Roy Kenneth Hack	Cincinnati, O.
William Arthur Heidel	Middletown, Conn.
George Lincoln Hendrickson	New Haven, Conn.
Bert Hodge Hill	Athens, Greece
Elijah Clarence Hills	Berkeley, Cal.
William Henry Holmes	Washington, D. C.
Edward Washburn Hopkins	New Haven, Conn.
William Guild Howard	Cambridge
Ales Hrdlicka	Washington, D. C.
Eugene Xavier Louis Henry Hyvernat	Washington, D. C.
Carl Newell Jackson	Cambridge
Hans Carl Gunther von Jagemann	Cambridge
James Richard Jewett	Cambridge
Alfred Louis Kroeber	Berkeley, Cal.
Kirsopp Lake	Cambridge
Henry Roseman Lang	New Haven, Conn.
Charles Rockwell Lanman	Cambridge
John Livingston Lowes	Cambridge
David Gordon Lyon	Cambridge
Clifford Herschel Moore	Cambridge
George Foot Moore	Cambridge

Hanns Oertel	Munich, Germany
Chandler Rathfon Post	Cambridge
Edward Kennard Rand	Cambridge
George Andrew Reisner	Cambridge
Edward Robinson	New York, N. Y.
Fred Norris Robinson	Cambridge
Rudolph Schevill	Berkeley, Cal.
Herbert Weir Smyth	Cambridge
Franklin Bache Stephenson	Washington, D. C.
Charles Cutler Torrey	New Haven, Conn.
Alfred Marston Tozzer	Cambridge
Clark Wissler	New York, N. Y.
James Haughton Woods	Cambridge

Class III., Section III.—*Political Economy and History.*—44.

Wilbur Cortez Abbott	Cambridge
Brooks Adams	Quincy
Charles McLean Andrews	New Haven, Conn.
John Spencer Bassett	Northampton
Carl Lotus Becker	Ithaca, N. Y.
Charles Jesse Bullock	Cambridge
Thomas Nixon Carver	Cambridge
Edward Channing	Cambridge
John Bates Clark	New York, N. Y.
Archibald Cary Coolidge	Boston
Richard Henry Dana	Cambridge
Clive Day	New Haven, Conn.
Davis Rich Dewey	Cambridge
Ephraim Emerton	Cambridge
Henry Walcott Farnam	New Haven, Conn.
Max Farrand	New Haven, Conn.
William Scott Ferguson	Cambridge
Irving Fisher	New Haven, Conn.
Worthington Chauncey Ford	Cambridge
Edwin Francis Gay	Cambridge
Frank Johnson Goodnow	Baltimore, Md.
Evarts Boutell Greene	Champaign, Ill.

Arthur Twining Hadley	New Haven, Conn.
Albert Bushnell Hart	Cambridge
Charles Homer Haskins	Cambridge
Charles Downer Hazen	New York, N. Y.
George La Piana	Cambridge
Abbot Lawrence Lowell	Cambridge
William MacDonald	New York, N. Y.
Charles Howard McIlwain	Cambridge
Roger Bigelow Merriman	Cambridge
Samuel Eliot Morison	Concord
William Bennett Munro	Boston
Charles Lemuel Nichols	Worcester
James Ford Rhodes	Boston
Michael I. Rostovtzeff	Madison, Wis.
William Milligan Sloane	New York, N. Y.
John Osborne Sumner	Boston
Frank William Taussig	Cambridge
Frederick Jackson Turner	Madison, Wis.
Claude Halstead Van Tyne	Ann Arbor, Mich.
George Grafton Wilson	Cambridge
George Parker Winship	Charles River
Allyn Abbott Young	Cambridge

CLASS III., SECTION IV.—*Literature and the Fine Arts.*—42.

Irving Babbitt	Cambridge
George Pierce Baker	New Haven, Conn.
William Sturgis Bigelow	Boston
Le Baron Russell Briggs	Cambridge
Charles Allerton Coolidge	Boston
Frederick Shepherd Converse	Boston
Samuel McChord Crothers	Cambridge
Wilberforce Eames	New York, N. Y.
Edward Waldo Emerson	Concord
William Emerson	Cambridge
Arthur Fairbanks	Cambridge
Frank Edgar Farley	Middletown, Conn.
Edward Waldo Forbes	Cambridge

Kuno Francke	Cambridge
Daniel Chester French	New York, N. Y.
Horace Howard Furness	Philadelphia, Pa.
Robert Grant	Boston
Morris Gray	Boston
Chester Noyes Greenough	Cambridge
James Kendall Hosmer	Minneapolis, Minn.
Mark Antony DeWolfe Howe	Boston
Archer Milton Huntington	New York, N. Y.
George Lyman Kittredge	Cambridge
William Coolidge Lane	Cambridge
John Ellerton Lodge	Boston
Charles Martin Tornov Loeffler	Medfield
Charles Donagh Maginnis	Brookline
Albert Matthews	Boston
Harold Murdock	Brookline
William Allan Neilson	Northampton
William Lyon Phelps	New Haven, Conn.
Arthur Kingsley Porter	Cambridge
Herbert Putnam	Washington, D. C.
Denman Waldo Ross	Cambridge
Paul Joseph Sachs	Cambridge
Ellery Sedgwick	Boston
Henry Dwight Sedgwick	Cambridge
Richard Clipston Sturgis	Boston
Charles Howard Walker	Boston
Owen Wister	Philadelphia, Pa.
George Edward Woodberry	Beverly
Charles Henry Conrad Wright	Cambridge

ASSOCIATES.

Edwin Hale Abbot	Cambridge
Charles Francis Adams	Concord
Francis N. Balch	Lincoln
Charles F. Batchelder	Cambridge
William B. Cabot	Boston

Henry W. Cunningham	Boston
Charles Ernest Fay	Somerville
Francis Russell Hart	Boston
William James	Cambridge
Everett Morss	Boston
Andrew James Peters	Boston
Anthony John Philpott	Arlington
Charles Henry Taylor	Boston
Edwin Sibley Webster	Brookline
Benjamin Loring Young	Weston

FOREIGN HONORARY MEMBERS.—62.

(Number limited to seventy-five.)

CLASS I.—*Mathematical and Physical Sciences.*—23.

SECTION I.—*Mathematics and Astronomy.*—7.

Johann Oskar Backlund	Leningrad
Arthur Stanley Eddington	Cambridge
Jacques Salomon Hadamard	Paris
Godfrey Harold Hardy	Oxford
Tullio Levi-Civita	Rome
Charles Emile Picard	Paris
Charles Jean de la Vallée Poussin	Louvain

CLASS I., SECTION II.—*Physics.*—7.

Svante August Arrhenius	Stockholm
Albert Einstein	Berlin
Sir Joseph Larmor	Cambridge
Hendrik Antoon Lorentz	Haarlem
Max Planck	Berlin
Sir Ernest Rutherford	Cambridge
Sir Joseph John Thomson	Cambridge

CLASS I., SECTION III.—*Chemistry.*—4.

Fritz Haber	Berlin
Henri Louis Le Chatelier	Paris
Wilhelm Ostwald	Leipsic
William Henry Perkin	Oxford

CLASS I., SECTION IV.—*Technology and Engineering.*—5.

Ferdinand Foch	Paris
Joseph Jacques Césaire Joffre	Paris
Maurice d'Ocagne	Paris
Vsevolod Evgenievich Timonoff	Leningrad
William Cawthorne Unwin	London

CLASS II.—*Natural and Physiological Sciences.*—21.

SECTION I.—*Geology, Mineralogy, and Physics of the Globe.*—10.

Frank Dawson Adams	Montreal
Charles Barrois	Lille
Waldemar Christofer Brögger	Christiania
Viktor Goldschmidt	Heidelberg
Albert Heim	Zürich
John Horne	Edinburgh
Emmanuel de Margerie	Paris
Gustaf Adolf Frederik Molengraaff	Delft
Sir William Napier Shaw	London
Johan Herman Lie Vogt	Trondhjem

CLASS II., SECTION II.—*Botany.*—4.

John Briquet	Geneva
Adolf Engler	Berlin
Ignatz Urban	Berlin
Hugo de Vries	Lunteren

CLASS II., SECTION III.—*Zoology and Physiology.*—4.

George Albert Boulenger	Brussels
Maurice Caullery	Paris
Sir Edwin Ray Lankester	London
George Henry Falkiner Nuttall	Cambridg

CLASS II., SECTION IV.—*Medicine and Surgery.*—3.

Sir Thomas Barlow, Bart	London
Francis John Shepherd	Montreal
Sir Charles Scott Sherrington	Oxford

CLASS III.—*Moral and Political Sciences.*—18.

SECTION I.—*Theology, Philosophy and Jurisprudence.*—3.

Rt. Hon. Arthur James Balfour, Earl of Balfour	Prestonkirk
Raymond Poincaré	Paris
Rt. Hon. Sir Frederick Pollock, Bart.	London

Class III., Section II.—*Philology and Archæology.*—7.

Wilhelm Dörpfeld	Athens
Karl Friedrich Geldner	Marburg
Henri Guy	Grenoble
Hermann Georg Jacobi	Bonn
Arthur Anthony Macdonell	Oxford
Alfred Percival Maudslay	Hereford
Ramon Menendez Pidal	Madrid

Class III., Section III.—*Political Economy and History.*—3.

Adolf Harnack	Berlin
Henri Pirenne	Ghent
Rt. Hon. Sir George Otto Trevelyan, Bart.	London

Class III., Section IV.—*Literature and the Fine Arts.*—5.

George Brandes	Copenhagen
Thomas Hardy	Dorchester
Jean Adrien Antoine Jules Jusserand	Paris
Rudyard Kipling	Burwash
Sir Sidney Lee	London

STATUTES AND STANDING VOTES.

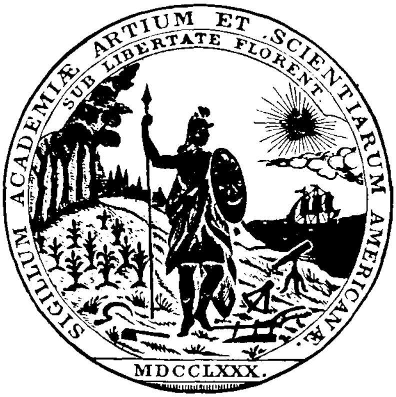
STATUTES.

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919, *February* 8, *April* 12, *and December* 13, 1922, *February* 14, *March* 14, *and October* 10, 1923.

CHAPTER I.

THE CORPORATE SEAL.

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:


SIGILLUM ACADEMIÆ ARTIUM ET SCIENTIARUM AMERICANÆ
SUB LIBERTATE FLORENT.
MDCCLXXX.

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v. art. 3; chap. vi. art 2.

CHAPTER II

FELLOWS AND FOREIGN HONORARY MEMBERS AND DUES

ARTICLE 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

CLASS I. *The Mathematical and Physical Sciences*

Section 1. Mathematics and Astronomy
Section 2. Physics
Section 3. Chemistry
Section 4. Technology and Engineering

CLASS II. *The Natural and Physiological Sciences*

Section 1. Geology, Mineralogy, and Physics of the Globe
Section 2. Botany
Section 3. Zoölogy and Physiology
Section 4. Medicine and Surgery

CLASS III. *The Moral and Political Sciences*

Section 1. Theology, Philosophy, and Jurisprudence
Section 2. Philology and Archæology
Section 3. Political Economy and History
Section 4. Literature and the Fine Arts

ARTICLE 2. The number of Fellows shall not exceed Six hundred, of whom not more than Four hundred shall be residents of Massachusetts, nor shall there be more than Two hundred and ten in any one Class.

ARTICLE 3. The number of Foreign Honorary Members shall not exceed Seventy-five. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Twenty-five in any one Class.

ARTICLE 4. If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii. art. 2.

ARTICLE 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments. Any Fellow shall also be exempt from annual dues who has paid such dues for forty years, or, having attained the age of seventy-five, has paid dues for twenty-five years.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any Fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. x, art. 2.

ARTICLE 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three-fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii.; chap. vi. art. 1; chap. ix. art. 1, 7; chap. x. art. 2.

CHAPTER III

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS

The procedure in the election of Fellows and Foreign Honorary Members shall be as follows:

Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by two Fellows of that Section or by three Fellows of any Sections, and sent to the Corresponding Secretary accompanied by a statement of the qualifications of the nominee and brief biographical data.

Notice shall be sent to every Fellow not later than the fifteenth of January in each year, reminding him that all nominations must be in the hands of the Corresponding Secretary before the fifteenth of February following.

A list of the nominees, giving a brief account of each, with the names of the nominators, shall be sent to every Fellow with a request that he return the list with such confidential comments and indications of preference as he may choose to make.

All the nominations, with any comments thereon and with expressions of preference on the part of the Fellows, shall be referred to the appropriate Class Committees, which shall canvass them, and report their recommendations in writing to the Council before the Stated Meeting of the Academy in April.

Elections of Fellows and Foreign Honorary Members shall be made by the Council before the Annual Meeting in May, and announced at that meeting.

Persons nominated in any year, but not elected, may be carried over to the list of nominees for the next year at the discretion of the Council, but shall not be further continued unless renominated.

See Chap. ii.; chap. vi. art. 1; chap. ix. art 1.

CHAPTER IV.

OFFICERS.

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, and a Librarian, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. x. art. 1.

ARTICLE 2. If any officer be unable, through death, absence, or disability, to fulfil the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

ARTICLE 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Commitees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi. art. 2.

ARTICLE 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi. art. 2.

ARTICLE 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V.

THE PRESIDENT.

ARTICLE 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii. art. 7; chap. iv. art. 1, 3; chap. vi. art. 2; chap. vii. art. 1; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1.

CHAPTER VI.

THE SECRETARIES.

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii. art. 7; chap. iii. art. 2, 3; chap. iv. art. 1; chap. ix. art. 6; chap. x. art. 1; chap. xi. art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meeting of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow and by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv. art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i.; chap. ii. art. 7; chap. iv. art. 3, 4, 5; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1, 3.

ARTICLE 3. The Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII.

THE TREASURER AND THE TREASURY.

ARTICLE 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

Article 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

Article 3. He shall give such security for the trust reposed in him as the Academy may require.

Article 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the funds of the Academy.

Article 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

Article 6. The Financial Year of the Academy shall begin with the first day of April.

Article 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without the previous consent and approval of the Council.

See Chap. ii. art. 4, 5; chap. vi. art. 2; chap. ix. art. 6; chap. x. art. 1, 2, 3; chap. xi. art. 1.

CHAPTER VIII.

THE LIBRARIAN AND THE LIBRARY.

ARTICLE 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

ARTICLE 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

ARTICLE 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

ARTICLE 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

ARTICLE 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

ARTICLE 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

Article 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii. art. 6; chap. x. art. 1, 2.

CHAPTER IX.

The Council.

Article 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections, reporting its action to the Academy.

See Chap. iii. art. 2, 3; chap. x. art. 1.

Article 2. Seven members shall constitute a quorum.

Article 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

Article 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

Article 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

ARTICLE 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. x. art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii. art. 4, 5, 8; chap. iv. art. 1, 2; chap. vi. art. 1; chap. vii. art. 1; chap. xi. art. 1, 4.

CHAPTER X.

STANDING COMMITTEES.

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii. art. 2.

ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance,* to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and of changing the investments thereof in their discretion.

See Chap. iv. art. 3; chap vii. art. 1, 4; chap. ix. art. 6.

(ii) *The Rumford Committee,* to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books, publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iii) *The Cyrus Moors Warren Committee,* to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iv) *The Committee of Publication,* to consist of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which the Publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

Two hundred extra copies of each paper accepted for publication in the Proceedings or the Memoirs shall be placed at the disposal of the author without charge.

See Chap. iv. art. 3; chap. vi. art. 1, 3; chap. ix. art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv. art. 3; chap. viii. art. 1, 2; chap. ix. art. 6.

(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv. art. 1, 3; chap. ix. art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv. art. 3; chap. ix. art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv. art. 3; chap. vii. art. 1; chap. ix. art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii. art. 1, 7; chap. ix. art. 6.

CHAPTER XI.

MEETINGS, COMMUNICATIONS, AND AMENDMENTS.

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

ARTICLE 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

ARTICLE 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two-thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

ARTICLE 8. Standing Votes may be passed, amended, or rescinded *at a Stated Meeting, or at a Special Meeting called for the purpose,* by a vote of two-thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii. art. 5, 8; chap. iii; chap. iv. art. 3, 4, 5; chap. v. art. 1; chap. vi. art. 1, 2; chap. ix. art. 8.

STANDING VOTES.

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. The Council, under such rules respecting nominations as it may prescribe, may elect as Associates of the Academy a limited number of men of mark in affairs or of distinguished service in the community.

Associates shall be entitled to the same privileges as Fellows, but shall not have the right to vote.

The admission fee and annual dues of Associates shall be the same as those of Fellows residing within fifty miles of Boston.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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